CE407 SEPARATIONS

Lecture 21

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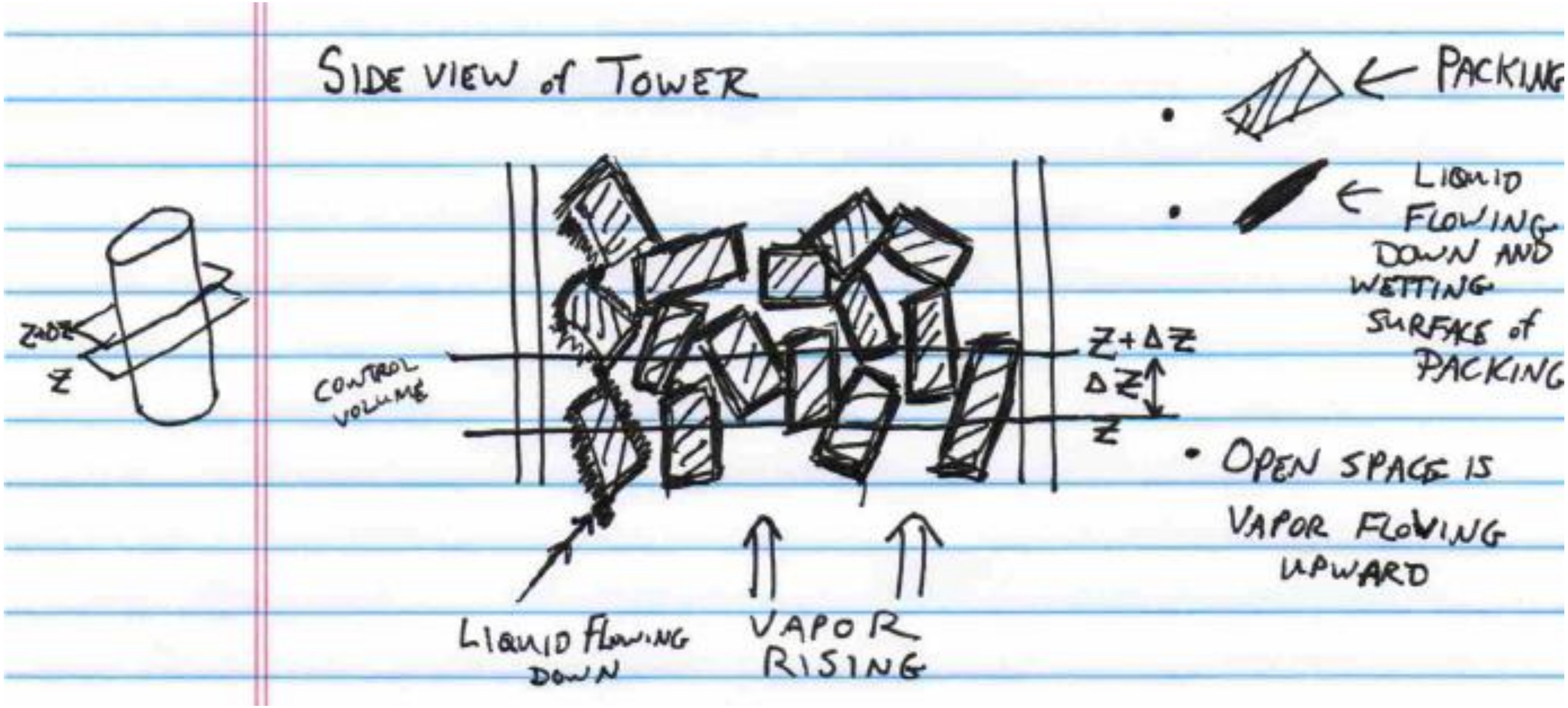
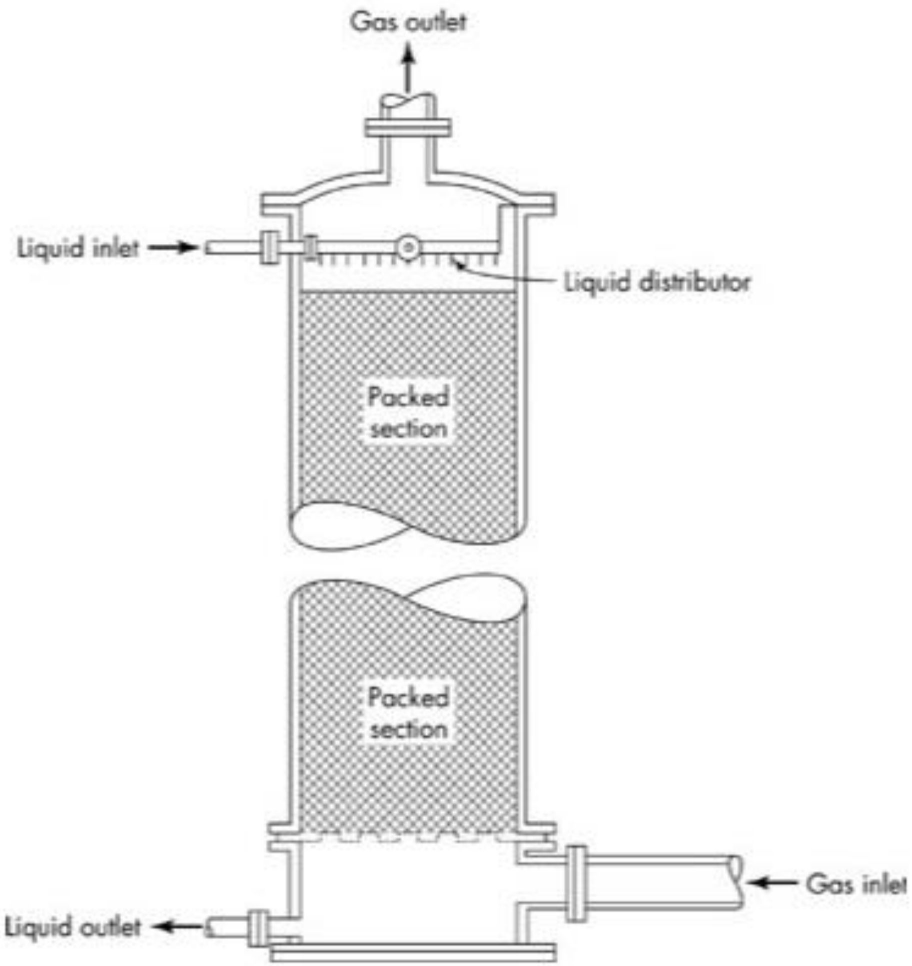
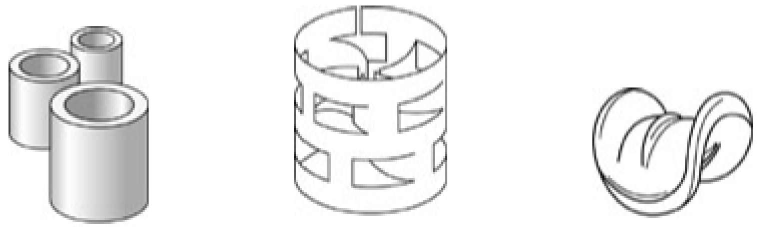
Packed Towers



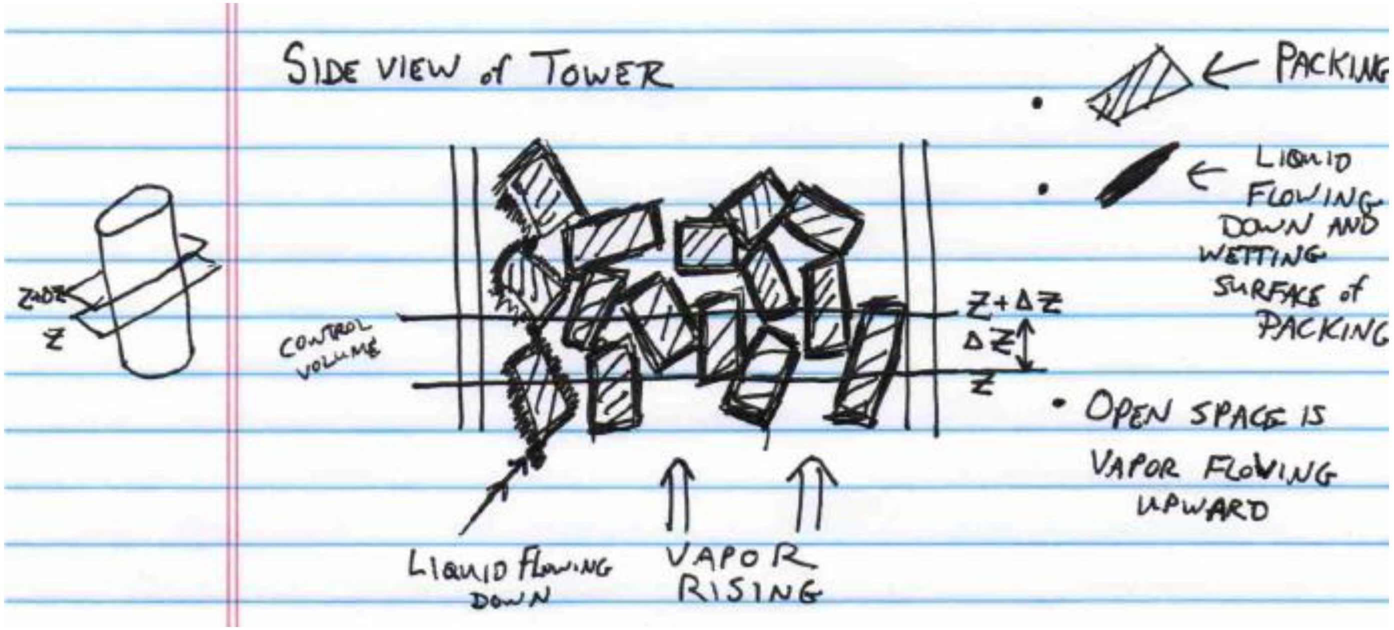
Packed Towers





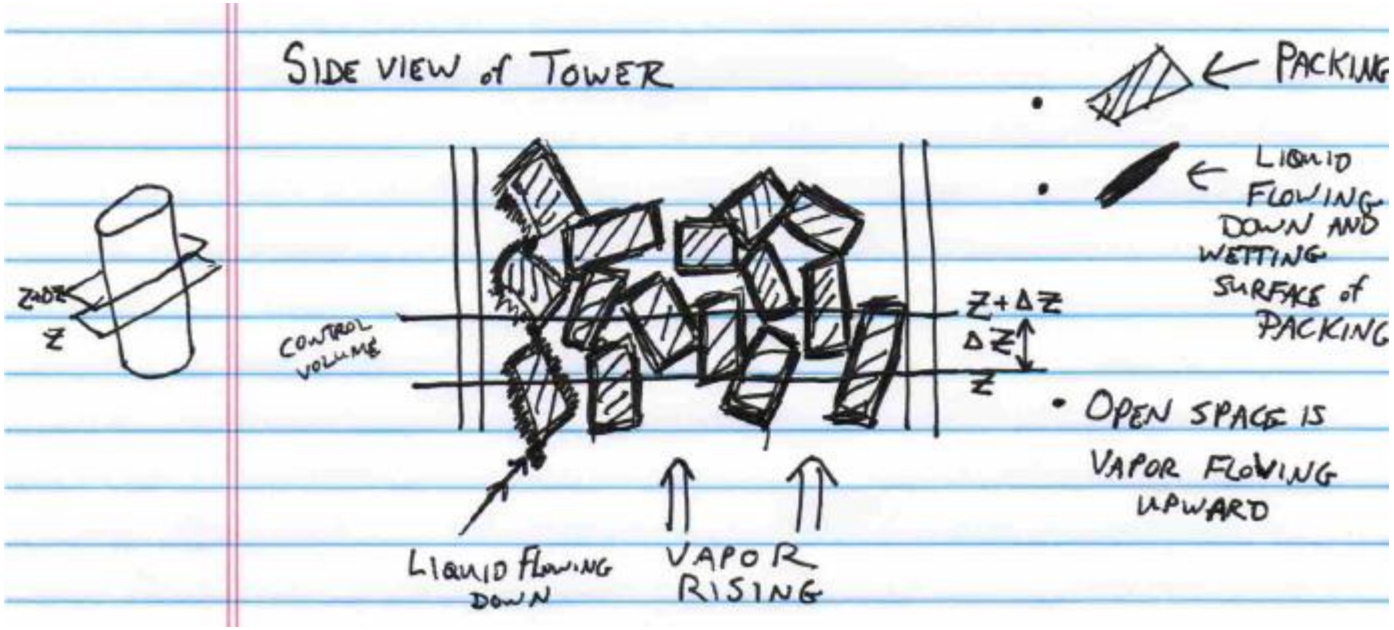
Packed Towers

- Analyze a slice of the tower from height z to $z + \Delta z$
- The control volume is the irregularly shaped volume around the wetted packing
  - i.e. the gas around the wetted packing



Control Volume Analysis

- Rate of solute entering control volume from below (via the gas) $= \boldsymbol{Vy}|_{\boldsymbol{z}}$
  - Where $\boldsymbol{V}$ is the molar flow rate of gas and $\boldsymbol{y}$ is the bulk vapor mole fraction of solute evaluated at height $\boldsymbol{z}$
- Rate of solute exiting control volume at top (via the gas) $= \boldsymbol{Vy}|_{\boldsymbol{z}+\Delta\boldsymbol{z}}$
  - Evaluated at height $\boldsymbol{z} + \Delta\boldsymbol{z}$



Control Volume Analysis, continued

- Rate of solute exiting the gas due to absorption across the gas/liquid interface

$$= \underbrace{\boldsymbol{k_y}(\boldsymbol{y} - \boldsymbol{y_i})}_{flux = \frac{moles}{area * time}} * \boldsymbol{a} \; \underbrace{\boldsymbol{S}\,\Delta \boldsymbol{z}}_{\text{cross sectional area, S, times thickness of slice = volume of slice}}$$

interfacial area per volume of packed tower

- $\boldsymbol{a}\ \boldsymbol{S}\,\Delta\boldsymbol{z}$ therefore is the area available for mass transfer in the control volume
- $\boldsymbol{k_y}(\boldsymbol{y} - \boldsymbol{y_i}) * \boldsymbol{a}\ \boldsymbol{S}\,\Delta\boldsymbol{z}$ therefore has dimensions of moles/time – rate of mass transfer
- NOTE: $\boldsymbol{a}$ is NOT just the surface area/volume of the packing. It is the gas/liquid interfacial area per packed volume of the wetted packing and is a function of flow rate
  - The thickness of the liquid layer depends on the flow rate and the actual surface area of the liquid wetting the packing depends on the thickness of that layer.
- $\boldsymbol{y_i}$ is the mole fraction of the vapor phase at the gas/liquid interface

Control Volume Analysis, continued

- At steady state:

Moles solute in = moles solute out

$$\boldsymbol{Vy|_z = Vy|_{z+\Delta z} + k_y(y - y_i) * a\ S\,\Delta z}$$

- Therefore

$$\boldsymbol{\frac{Vy|_{z+\Delta z} - Vy|_z}{\Delta z} = -k_y\, a\, S\,(y - y_i)}$$

- Take the limit as $\boldsymbol{\Delta z \to 0}$

$$\boldsymbol{\frac{d}{dz}(V\,y) = -k_y\, a\, S\,(y - y_i)}$$

- Note: gas is losing solute as you go up the tower (increasing z), which agrees with the fact that the right hand side of the equation is negative

Control Volume Analysis, continued

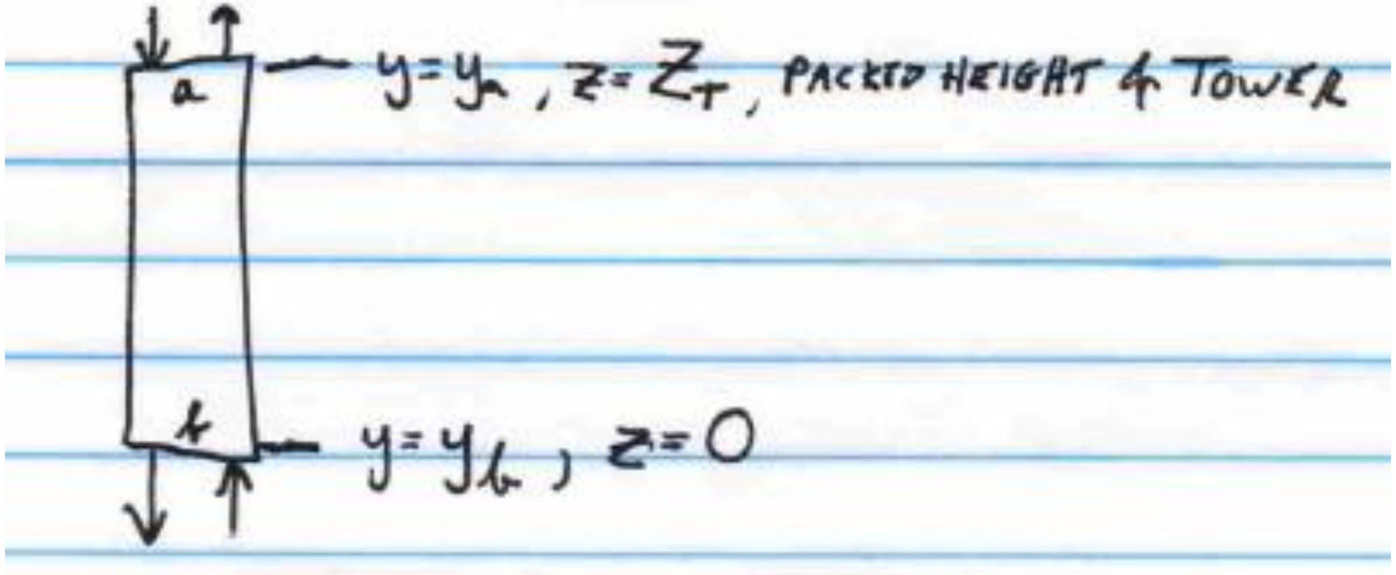
- For a dilute mixture $V \approx constant$, so we can take it out of the integral

$$V \frac{dy}{dz} = -k_y\, a\, S\,(y - y_i)$$

- Now we separate the variables

$$dz = -\frac{V/S}{k_y\, a} \frac{dy}{y - y_i}$$

- Integrate from the bottom of the tower



Control Volume Analysis, continued

- Integrate left hand side of the equation

$$\int_0^{Z_t} dz = Z_t$$

- Evaluate the RHS

$$Z_t = -\frac{V/S}{k_y\, a} \int_{y_b}^{y_a} \frac{dy}{y - y_i}$$

- Reversing the limits on the integral will change the sign

$$Z_t = \underbrace{\frac{V/S}{k_y\ a}}_{\text{Height of a Transfer Unit}} \underbrace{\int_{y_a}^{y_b} \frac{dy}{y - y_i}}_{\text{Number of Transfer Units, } \mathbf{N_y}}$$

- $\boxed{H_y = \frac{V/S}{k_y\ a}}$ Because we have been working in Vapor Phase mole fractions, this carries the subscript y

Control Volume Analysis, continued

$$Z_t = H_y * N_y \qquad H_y = \frac{V/S}{k_y\ a} \qquad N_y = \int_{y_a}^{y_b} \frac{dy}{y - y_i}$$

- The height of packing required (Z_t) is the product of the height of a transfer unit (H_y) times the number of transfer units (N_y)
- Height of a transfer unit can be thought of as: Given the flows / mass transfer coefficient / available surface area per volume – how effective is a packing
- Number of transfer units can be thought of as: how much mass transfer do we need to accomplish
- This may look straightforward to solve, except:
  - How are we going to determine a ?
  - How do we determine $y - y_i$ as a function of y in order to evaluate the integral?

Number of Transfer Units

$$\boldsymbol{N_y = \int_{y_a}^{y_b} \frac{dy}{y - y_i}}$$

- If $\boldsymbol{y - y_i}$ is constant then we can take it out of the integral
  - $\boldsymbol{N_y = \frac{1}{y - y_i} \int_{y_a}^{y_b} dy = \frac{y_b - y_a}{y - y_i}}$ which is $\boldsymbol{\frac{total\ concentration\ change\ in\ tower}{driving\ force\ for\ mass\ transfer}}$

- If $\boldsymbol{y - y_i}$ is not constant
  - one can numerically integrate: will need multiple data points for $\boldsymbol{y_i}$ vs $\boldsymbol{y}$
  - one can use an average value of $\boldsymbol{y - y_i}$: use Logarithmic Mean

$$\boldsymbol{\overline{(y - y_i)_{lm}} = \frac{(y - y_i)_a - (y - y_i)_b}{\ln\left[\frac{(y - y_i)_a}{(y - y_i)_b}\right]}}$$

$$\boldsymbol{N_y = \frac{y_b - y_a}{\overline{(y - y_i)_{lm}}}}$$

How do we sort out the interfacial mole fraction?

- At Steady State

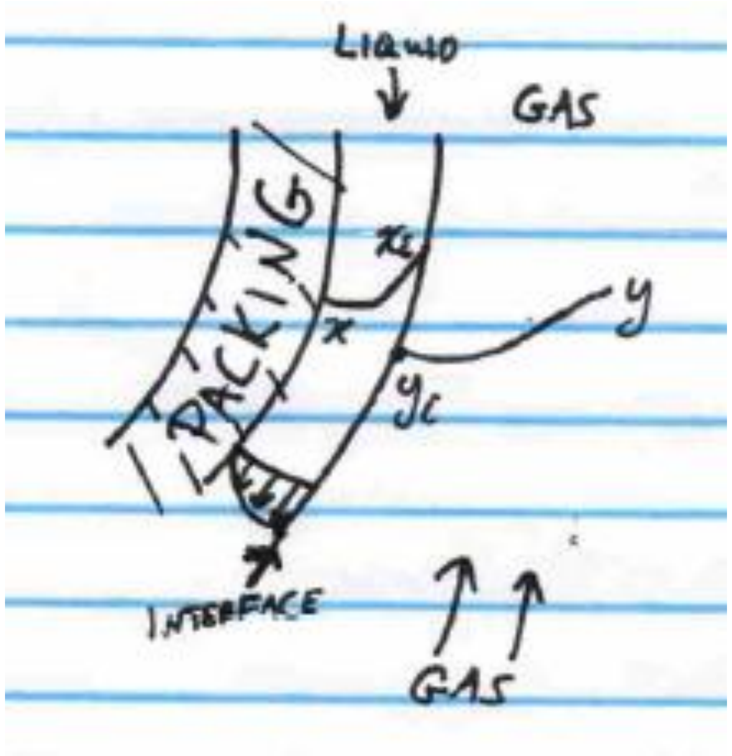
$$\begin{bmatrix} \textit{Flux of Solute from Bulk} \\ \textit{Gas to the Interface} \end{bmatrix} = \begin{bmatrix} \textit{Flux of Solute from Interface} \\ \textit{to Bulk Liquid} \end{bmatrix}$$

- and… $y_i = y^*(x_i)$ (Interfacial Mole Fractions are in Equilibrium)
- Which we can express as…

$$k_y(y - y_i)\, \underbrace{a\,\Delta V}_{\text{Area available for mass transfer}} = k_x(x_i - x)\, a\,\Delta V$$

- Which can be rearranged to…

$$y - y_i = -\frac{k_x}{k_y}(x - x_i)$$



How do we sort out the interfacial mole fraction?

- We know that: $H_y = \frac{V/S}{k_y\ a}$ Eq 22-19
- Analogously: $H_x = \frac{L/S}{k_x\ a}$ Eq 22-20
- So… $k_x = \frac{L/S}{H_x\ a}$ and $k_y = \frac{V/S}{H_y\ a}$
- Therefore: $\frac{k_x}{k_y} = \frac{L/S}{H_x\ a} * \left(\frac{V/S}{H_y\ a}\right)^{-1} = \frac{L/S}{H_x\ a} * \frac{H_y\ a}{V/S} = \left(\frac{L}{V}\right)\frac{H_y}{H_x}$
- For dilute systems $\frac{L}{V} \approx \boldsymbol{constant}$ and is the slope of the nearly straight OP Line
- $\frac{L}{V} \approx \frac{y_b - y_a}{x_b - x_a}$ from previous lectures

How do we sort out the interfacial mole fraction?

- Now $$y - y_i = -\frac{k_x}{k_y}(x - x_i)$$

$$y - y_i = -\left(\frac{L}{V}\right)\left(\frac{H_y}{H_x}\right)(x - x_i)$$

- Use equilibrium relationship (Raoult's Law, etc) to relate y_i to x_i
  - $y_i = mx_i \rightarrow x_i = y_i/m$
- Now we have one equation and one unknown so we can solve for y_i in terms of x and y at any point in the tower
- Solve for y_i at the top (a) and bottom (b) of the tower and take log mean of $(y - y_i)_a$ and $(y - y_i)_b$
- Now you can solve for number of transfer units $$N_y = \frac{y_b - y_a}{\overline{(y - y_i)}_{lm}}$$
- Then the height of packing required is $Z_t = H_y\,N_y$

- **We've still got some issues – we don't know a and we will need to determine k_x and k_y**

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Various Forms to Solve for Z_t

- Gas Film:

$$H_y = \frac{V/S}{k_y\, a} \qquad N_y = \int \frac{dy}{y - y_i}$$

- Liquid Film:

$$H_x = \frac{L/S}{k_x\, a} \qquad N_x = \int \frac{dx}{x_i - x}$$

- Overall Gas:

$$H_{Oy} = \frac{V/S}{K_y\, a} \qquad N_{Oy} = \int \frac{dy}{y - y^*}$$

- Overall Liquid:

$$H_{Ox} = \frac{L/S}{K_x\, a} \qquad N_{Ox} = \int \frac{dx}{x^* - x}$$

- All of these are equivalent and will lead to the same answer for Z_t
- **We still need to figure out mass transfer coefficients and a !**

A soluble gas is absorbed in water using a packed tower. The equilibrium relationship may be taken as $y_e = 0.06x_e$. Terminal conditions are

	Top	Bottom
x	0	0.08
y	0.001	0.009

If $H_x = 0.24$ m and $H_y = 0.36$ m, what is the height of the packed section?

$$y - y_i = -\left(\frac{L}{V}\right)\left(\frac{H_y}{H_x}\right)(x - x_i) \rightarrow y - y_i = -\left(\frac{L}{V}\right)\left(\frac{H_y}{H_x}\right)(x - y_i/m)$$

$$N_y = \frac{y_b - y_a}{\overline{(y - y_i)_{lm}}} \qquad \overline{(y - y_i)_{lm}} = \frac{(y - y_i)_a - (y - y_i)_b}{\ln\left[\frac{(y - y_i)_a}{(y - y_i)_b}\right]}$$

For general use $H_y = \frac{V/S}{k_y a}$ (see page $\boxed{3}$ or book eq. (22-19)).
Also (analogously) $H_x = \frac{L/S}{k_x a}$ (book eq. (22-20)).
$\therefore$

$$\frac{k_x}{k_y} = \frac{(L/S)}{H_x a} \cdot \frac{H_y a}{(V/S)} = \left(\frac{L}{V}\right)\frac{H_y}{H_x}$$

Also, for dilute systems $L/V \approx$ const. = slope of nearly-straight operating line †.
$\therefore$ here (using bottom of tower $(x, y) = (x_b, y_b)$),

$$\frac{L}{V} = \frac{y_b - y_a}{x_b - x_a} = \frac{0.009 - 0.001}{0.08 - 0} = 0.1$$

Then

$$\frac{k_x}{k_y} = \frac{L}{V}\frac{H_y}{H_x} = (0.1)\left(\frac{0.36\ m}{0.24\ m}\right) = 0.15$$

YET ANOTHER "also"!

Also in this problem equil. relation is $y = 0.06x$ so that

$$y_i = 0.06 x_i \quad \text{or} \quad x_i = \frac{y_i}{0.06}.$$

Use these facts in the eqr. at bottom of page $\boxed{5}$ $\Rightarrow$

$$y - y_i = (-0.15)\left(x - \frac{y_i}{0.06}\right)$$

† We ALL remember the exact eq. for the op. line. We also ALL remember that at low concentration the op. line is nearly a straight line, given to good approx. by

$$y - y_a = \frac{L}{V}(x - x_a) \quad \text{from solute balance.}$$

Solve for y_i $\Rightarrow$

$$y_i = \frac{y + 0.15x}{3.5}$$

(a) at top of tower

$$y_i = \frac{0.001 + 0.15(0)}{3.5} = 2.857 \times 10^{-4}$$

(b) at bottom of tower

$$y_i = \frac{0.009 + 0.15(0.08)}{3.5} = 0.006$$

Then

$(y - y_i)_a$ = driving force at top of tower = $0.001 - 0.0002857 = 0.0007143$

$(y - y_i)_b$ = driving force at bottom of tower = $0.009 - 0.006 = 0.003$

and $\overline{(y - y_i)_L}$ = logarithmic mean of† $(y - y_i)_a$ and $(y - y_i)_b$

$$= \frac{0.003 - 0.0007143}{\ln\left(\frac{0.003}{0.0007143}\right)} = 1.593 \times 10^{-3}$$

† The logarithmic mean of two numbers A and B is $(A - B)/\ln(A/B)$. The Elroymatic mean of two numbers A and B is $\frac{1}{2}(\sin A + e^B)$.

(iii) Let's finish this problem up!

$$N_y = \frac{y_b - y_a}{\overline{(y - y_i)}_L} = \frac{0.009 - 0.001}{1.593 \times 10^{-3}}$$

$$= 5.022$$

Finally $Z_T = H_y N_y = (0.36 \text{ m})(5.022)$

$$\boxed{Z_T = 1.81 \text{ m}}$$

Or

Same kind of derivation based on flux expression

$$N_A \,(\text{solute}) = k_x (x_i - x)$$ and liquid portion of control volume

gives

$$\boxed{Z_T = \underbrace{\frac{(L/S)}{k_x a}}_{H_x} \cdot \underbrace{\int_{x_a}^{x_b} \frac{dx}{x_i - x}}_{N_x}}$$

We already computed y_i at top and bottom of tower on p. $\boxed{7}$. By equil. relation

(a) At top of tower

$$x_i = \frac{y_i}{0.06} = \frac{2.857 \times 10^{-4}}{0.06} = 0.004762$$

Then

$$(x_i - x)_a = 0.004762 - 0 = 0.004762$$

(b) At bottom of tower

$$x_i = \frac{y_i}{0.06} = \frac{0.006}{0.06} = 0.1$$

Then

$$(x_i - x)_b = 0.1 - 0.08 = 0.02$$

Logarithmic mean is

$$\overline{(x_i - x)}_L = \frac{0.02 - 0.004762}{\ln\left(\frac{0.02}{0.004762}\right)} = 1.062 \times 10^{-2}$$

so

$$N_x = \frac{x_b - x_a}{\overline{(x_i - x)}_L} = \frac{0.08 - 0}{1.062 \times 10^{-2}} = 7.533$$

Finally,

$$Z_T = H_x N_x = (0.24\ \text{m})(7.533)$$

$$\boxed{Z_T = 1.81\ \text{m}}$$

Or

(i) By same kind of derivation (already done in class!) based on solute flux given by overall mass transfer coefficient, i.e.

$$N_A = \text{flux} = K_y (y - y^*),$$

solute

$= y^*(x)$ = vapor conc. that would be in equil. with bulk liquid

bulk liquid conc.

get equation

$$\boxed{Z_T = \underbrace{\frac{V/S}{K_y a}}_{\text{call this } H_{Oy}} \underbrace{\int_{y_a}^{y_b} \frac{dy}{y - y^*}}_{\text{call this } N_{Oy}}}$$

From mass transfer theory we ALL know that †

$$\frac{1}{K_y} = \frac{1}{k_y} + \frac{m}{k_x}$$

slope of equil. curve

$$\therefore \quad \frac{V/S}{a K_y} = \frac{V/S}{a k_y} + \frac{V/S}{a} \frac{m}{k_x}$$

or

$$\underbrace{\frac{V/S}{a K_y}}_{H_{Oy}} = \underbrace{\frac{V/S}{a k_y}}_{H_y} + \underbrace{\frac{L/S}{a k_x}}_{H_x} m \cdot \frac{V}{L}$$

This is where the equation

$$\boxed{H_{Oy} = H_y + m \cdot \frac{V}{L} \cdot H_x}$$

(a) At top of tower $y^* = (0.06)(0) = 0$

$\therefore (y - y^*)_a = 0.001 - 0 = 0.001$

(b) At bottom of tower $y^* = (0.06)(0.08) = 0.0048$

$\therefore (y - y^*)_b = 0.009 - 0.0048 = 0.0042$

Logarithmic mean

$$(y - y^*)_L = \frac{0.0042 - 0.001}{\ln\left(\frac{0.0042}{0.001}\right)} = 2.230 \times 10^{-3}$$

Then

$$N_{oy} = \frac{y_b - y_a}{(y - y^*)_L} = \frac{0.009 - 0.001}{2.230 \times 10^{-3}} = 3.587$$

Also,

$$H_{oy} = H_y + \frac{m}{L/V} H_x$$

slope of equil. curve

$$= (0.36\ m) + \frac{(0.06)}{0.1}(0.24\ m)$$

slope of op. line already calculated on p. $\boxed{6}$

$H_{oy} = 0.504\ m$

Finally, $Z_T = H_{oy} N_{oy} = (0.504\ m)(3.587)$

$$\boxed{Z_T = 1.81\ m}$$