CE407 SEPARATIONS

Lecture 10

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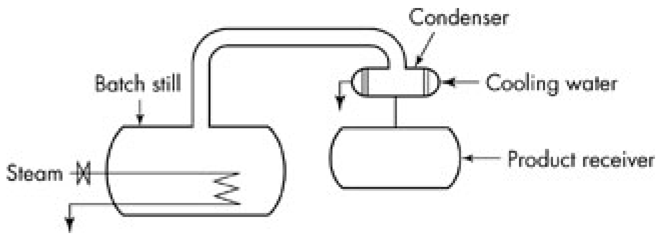
Binary Batch Distillation   McSH pp 724-727



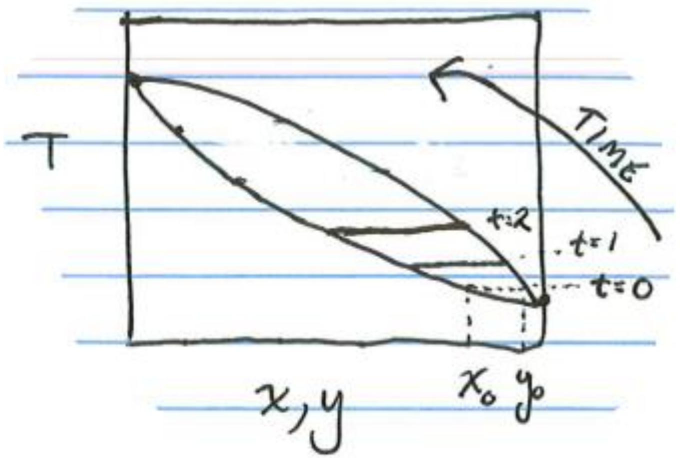
- Batch distillation is much simpler in terms of equipment, but the analysis is actually more involved…
- Define terms
  - A = light component
  - B = heavy component
  - **n** = total number of moles of liquid in the still pot
  - $\mathbf{n_A}$ = number of moles of liquid A in the still pot
  - x = mole fraction of component A in the still pot (liquid)
  - y = mole fraction of component A in the vapor coming off of the still pot

Binary Batch Distillation

- **x** and **y** values at any given time will be in equilibrium with one another
- A, being the lighter component, has a higher mole fraction in the vapor phase than it does in the liquid phase
- As a result, component A is being depleted from the liquid and **x** will drop with time
  - Unfortunately that means the mole fraction, **y**, of the vapor being generated is also dropping
- What is the rate of change of number of moles of A in the still pot?
- First express the number of moles of A in the still pot as:

$$n_A = x\,n$$

(# moles A = mole fraction A * total number of moles)



Binary Batch Distillation

- $dn_A = d(xn) = n\,dx + x\,dn$ when expressed in terms of liquid mole fraction
  - dn_A is rate of change of moles of A ***in the still pot***, the rate that moles of A are leaving when expressed in terms of liquid mole fraction
  - dx is rate of change of liquid mole fraction
  - dn is rate of change of total moles ***in still pot***
- $dn_A = y\,dn$ when expressed in terms of vapor mole fraction
- Note: the total moles leaving (dn) have a mole fraction **y** so we can express simply as $y\,dn$. The total moles leaving DO NOT have a mole fraction of **x** and therefore that expression is more complex.
- Obviously both expressions must equal one another

$$n\,dx + x\,dn = y\,dn$$

$$n\,dx = (y - x)\,dn$$

$$\frac{dx}{y - x} = \frac{dn}{n}$$

Binary Batch Distillation

$$\frac{dx}{y - x} = \frac{dn}{n}$$

- Integrate from time t_0 where $x = x_0$ and $n = n_0$ to an arbitrary time where $x = x$ and $n = n$
- To avoid mathematical confusion between the values x and n at the arbitrary time and the variables x and n as we integrate we will express the variables as x' and n'
- Express vapor mole fraction as $y = y(x')$ to explicitly indicate that the instantaneous value of y must be in equilibrium with the instantaneous value of x'
- $\int_{n_0}^{n} \frac{dn'}{n'} = \boxed{ln\left(\frac{n}{n_0}\right) = \int_{x_0}^{x} \frac{dx'}{y(x') - x'}}$ eq 21.86

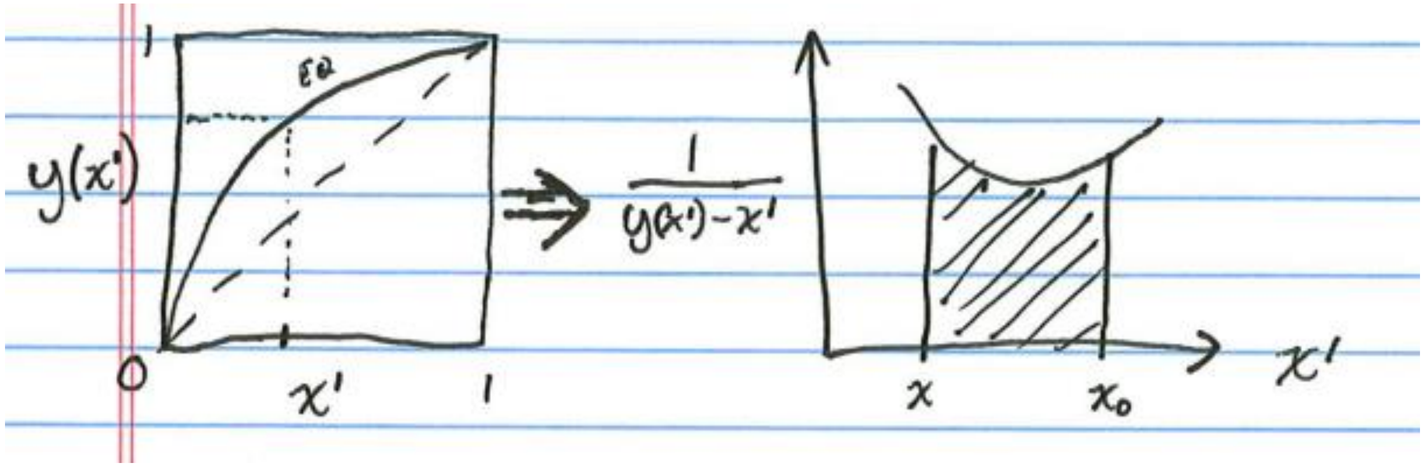
Rayleigh Equation

Rayleigh Equation

- The Rayleigh Equation gives the relationship between the total moles left in the still pot and the mole fraction of component A of the material left in the still pot

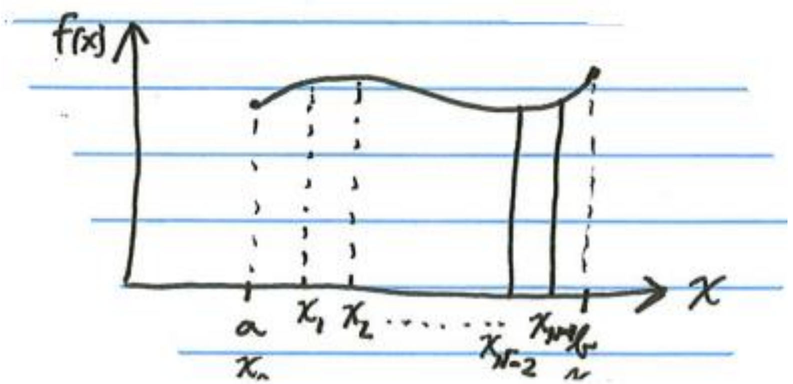
$$\ln\left(\frac{n}{n_0}\right) = \int_{x_0}^{x} \frac{dx'}{y(x') - x'}$$

- We need to interpret the integral on the right hand side of this equation
  - For various values of x' from x to x_0 read off $y(x')$ and calculate $\frac{1}{y(x') - x'}$
- We can now approximate the integral



Use Trapezoid Rule

- $\int_a^b f(x)dx \cong \left[\frac{1}{2}f(x_0) + f(x_1) + \cdots + f(x_{N-1}) + \frac{1}{2}f(x_N)\right]h$



- Where we break the range into N equally spaced slices
- $h = \frac{b-a}{N}$ is the thickness of each slice
- The area of a slice, **n**, is approximately $A_n = \left(\frac{f(x_{n-1})+f(x_n)}{2}\right)h$
  - Average height in that range times the width of the range
- Add up the slices $\sum = \left(\frac{f(x_0)+f(x_1)}{2}\right)h + \left(\frac{f(x_1)+f(x_2)}{2}\right)h + \cdots + \left(\frac{f(x_{N-1})+f(x_N)}{2}\right)h$

$$= \left[\frac{f(x_0)}{2} + f(x_1) + \cdots + f(x_{N-1}) + \frac{f(x_N)}{2}\right]h$$

- The greater a number N is, the more accurate the approximation

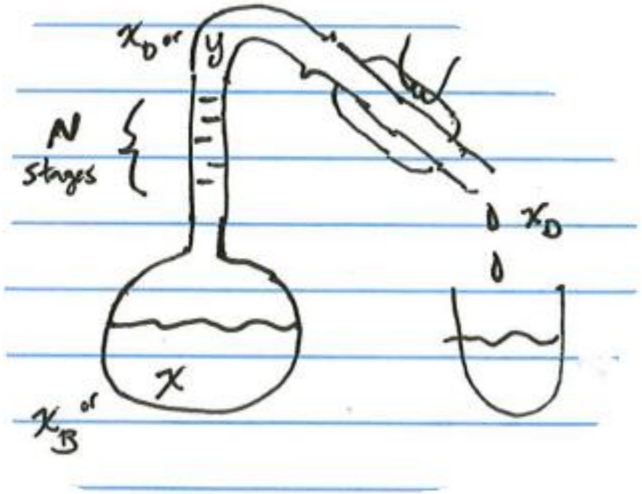
Binary Batch Distillation

- $ln\left(\frac{n}{n_0}\right) = \int_{x_0}^{x} \frac{dx'}{y(x') - x'}$
- Notice that our integral goes from x_0 to x and $x_0 > x$. Therefore our integral has a negative value.
- This makes sense because $\frac{n}{n_0} < 1$ and therefore *ln* ($\frac{n}{n_0}$) will be a negative number
- Our trapezoidal sum represents the absolute value of the integral, be sure to change the sign
- Method:
  - Pick a value of x and use trapezoidal approximation to estimate $\int_{x_0}^{x} \frac{dx'}{y(x') - x'}$
  - Calculate n using $ln\left(\frac{n}{n_0}\right) = \int_{x_0}^{x} \frac{dx'}{y(x') - x'}$
  - Repeat for various values of x. Create table of **x** and **n**
  - We now have the connection between # moles left in still pot and the mole fraction of the liquid left in the pot
  - Please see ***Binary Batch Distillation Examples*** in Notes for discussion of how we go from this knowledge of x vs n to an understanding of volume remaining and cumulative mole fraction of distillate

x	n
-	-
-	-
-	-

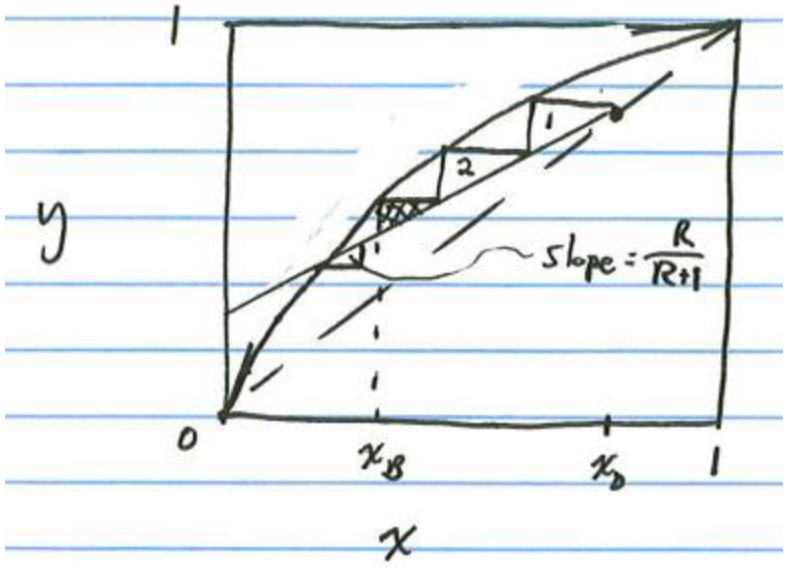
Batch Distillation with Reflux

- Improve separation by adding rectifying stages and reflux
- Improves purity of distilled product but not of the bottoms (material left in still pot)
- **y** refers to mole fraction above the **N** stages and is equal to $\mathbf{x_D}$, composition of condensed material leaving the still
- x refers to the mole fraction of liquid remaining in still pot and can be referred to as $\mathbf{x_B}$



Batch Distillation with Reflux

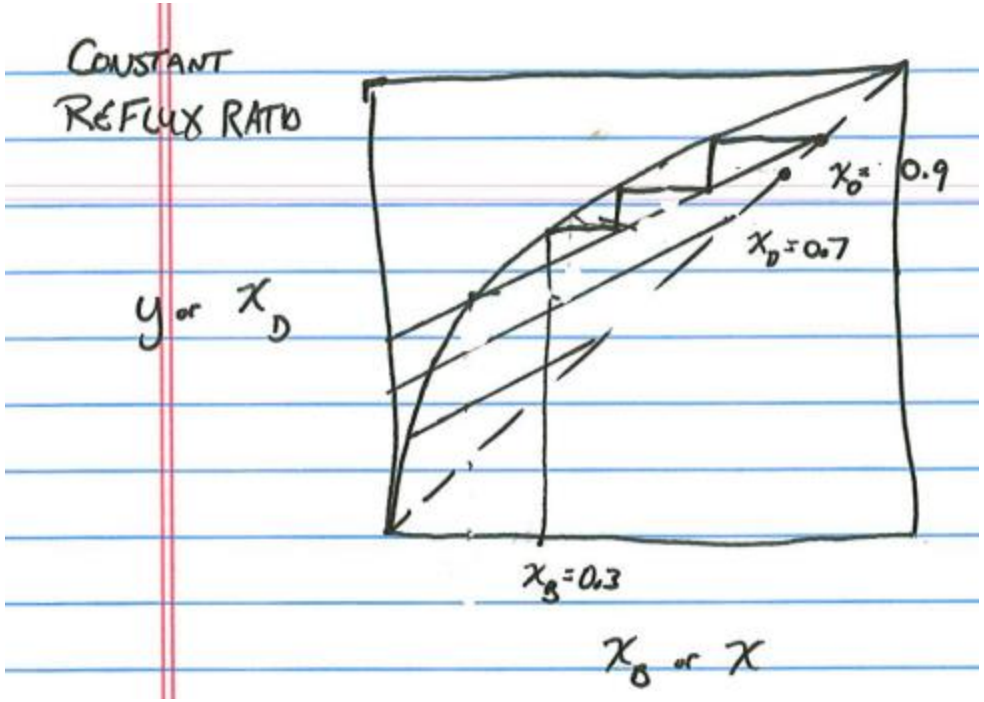
- Analyzing a still with a set number of stages
- Pick a value of $\mathbf{x_D}$ and determine the value of $\mathbf{x_B}$
- Draw rectifying line with slope = R/(R+1)
- In this example with 2 stages a third step is drawn in
  - This represents the step in the still pot itself (equivalent to the reboiler)
  - This allows you to determine what $\mathbf{x_B}$ will correspond to that value of $\mathbf{x_D}$
- Choose multiple values of $\mathbf{x_D}$ and get $\mathbf{x_B}$ for each
- Make a table of $\mathbf{x_B}$ vs $\mathbf{x_D}$
- Do the same steps w/Rayleigh equation, etc



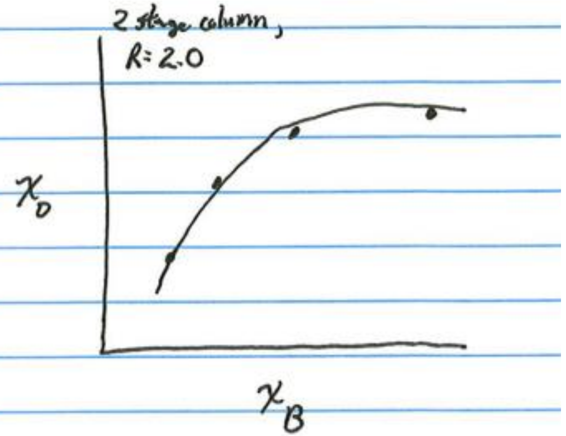
Batch Distillation with Reflux

Constant Reflux Rate

- x_D will change with time
- Draw multiple operating lines
  - All have same slope of R/(R+1)
- Step off # of steps corresponding to # of stages +1 for still pot
- Read off x_B
- Generate table of x_D vs x_B

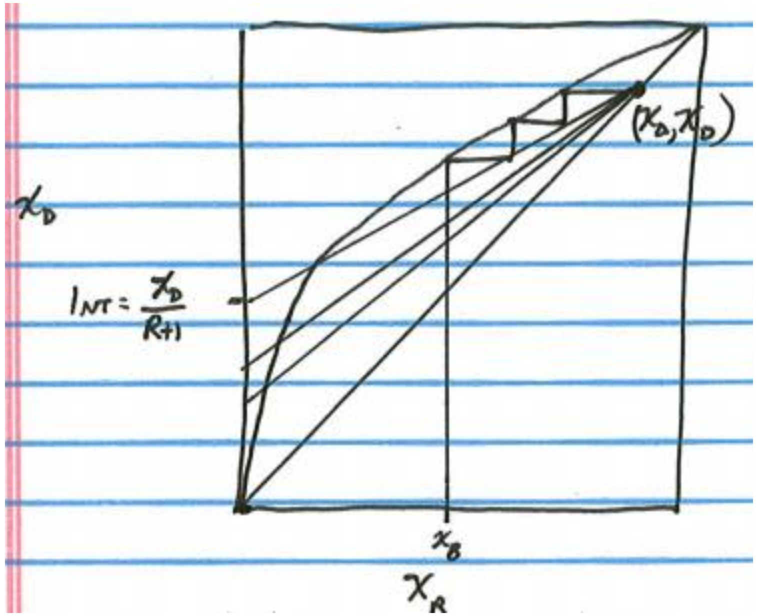


x_D	x_B
0.95	0.63
0.9	0.33
0.8	0.12
0.7	0.08



Batch Distillation with Reflux

Variable Reflux Rate



- $\mathbf{x_D}$ will be constant with time
- Draw various operating lines all originating from ($\mathbf{x_D}$, $\mathbf{x_D}$) each having a different slope
- Value of R for each line can be obtained from intercept = $\frac{x_D}{R+1}$
- Walk off the appropriate # of steps (= # stages+1) for each line to determine what $\mathbf{x_B}$ will corresponds to that reflux ratio
- Plot **R** vs $\mathbf{x_B}$ to show what **R** will be required for each $\mathbf{x_B}$ in order to maintain the desired $\mathbf{x_D}$