STATE UNIVERSITY OF NEW YORK AT BUFFALO

Department of Mechanical and Aerospace Engineering

MEA 589 Diffraction, Microscopy and Spectroscopy Techniques

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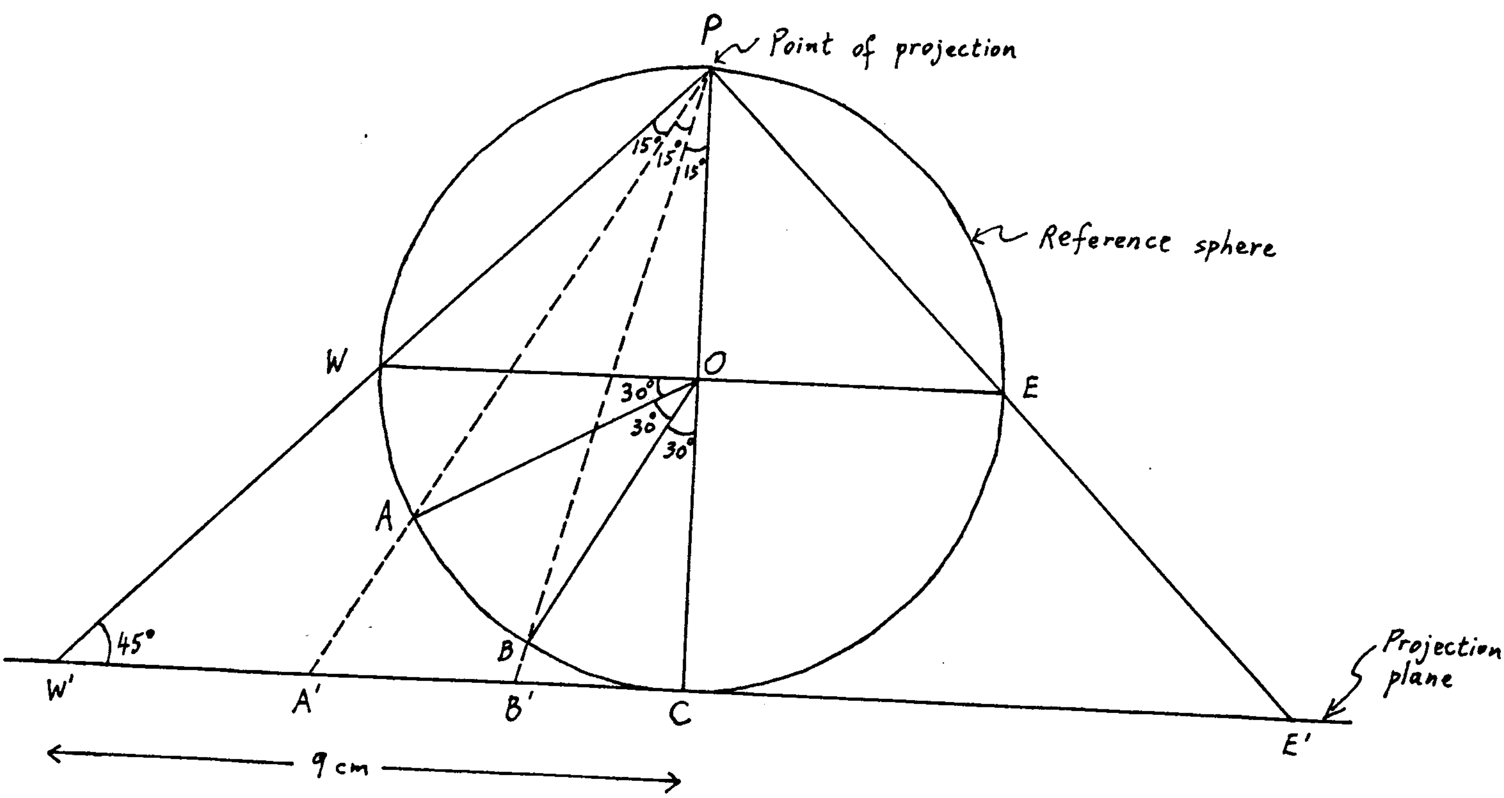
Homework No. 5

SOLUTION

Problem 1

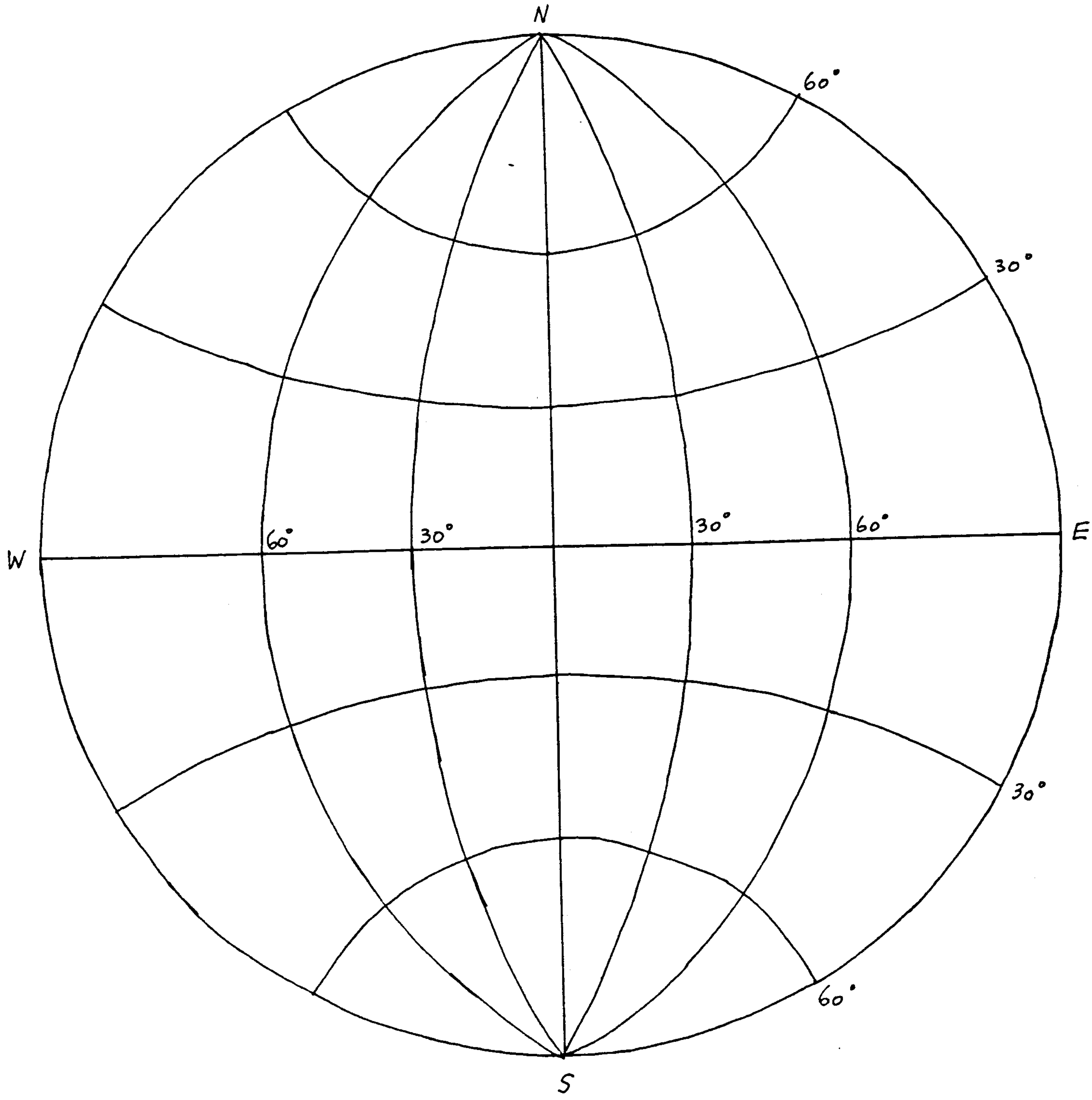


By using the geometry theorem which says that the angle subtended by an arc at the center of the circle is twice that subtended by the same arc at a point on the circumference,

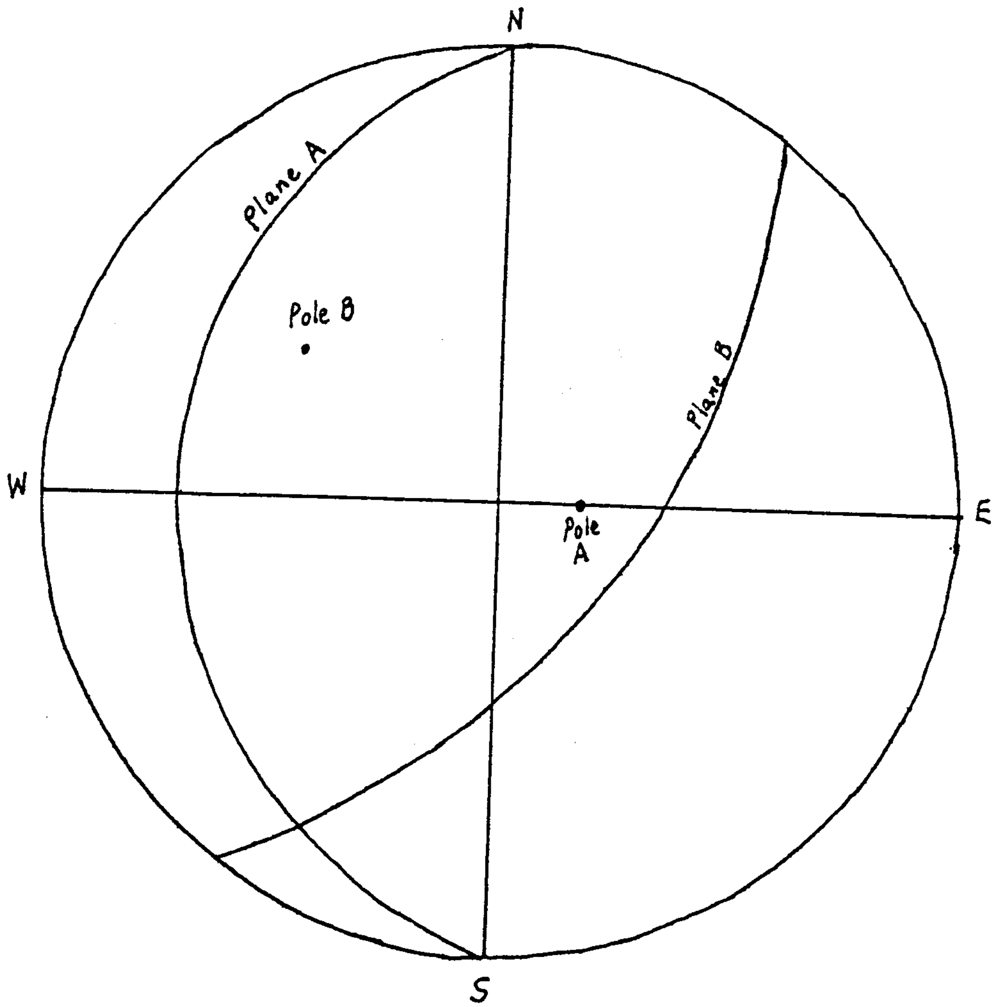
$$\angle AOB = 2(\angle APB)$$
$$\angle BOC = 2(\angle BPC)$$
$$\angle WOA = 2(\angle WPA).$$

Since $\angle AOB = \angle BOC = \angle WOA = 30°$,
$\angle APB = \angle BPC = \angle WPA = 15°$.

$PC = W'C \tan 45° = W'C = 9$ cm.
$B'C = PC \tan 15° = (9 \text{ cm})(0.2679) = 2.41$ cm
$A'C = PC \tan 30° = (9 \text{ cm})(0.5774) = 5.20$ cm

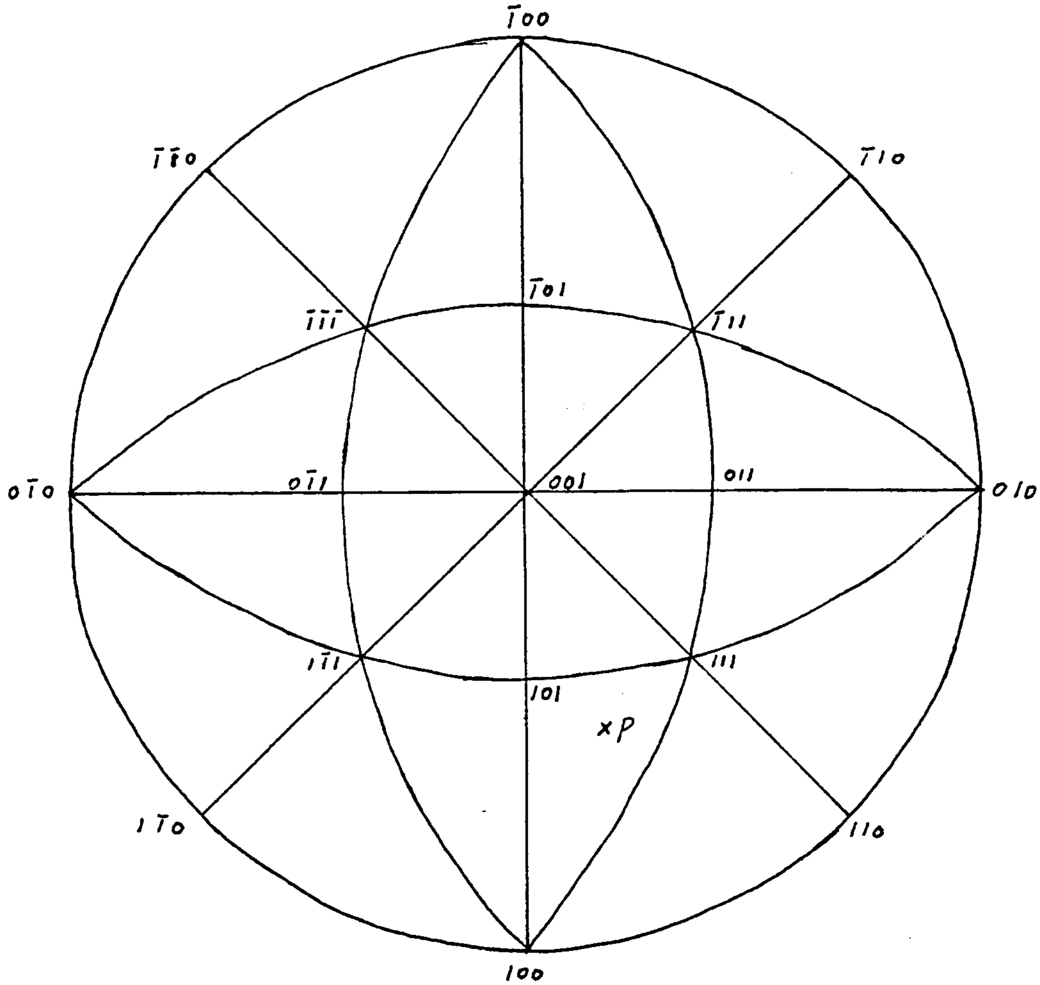


Problem 2



The trace of a plane is located at a great circle 90° from the pole of the plane. To obtain the trace of B, rotate the the projection on top of the Wulff net (Fig. 2-29, Cullity) until the pole of B sits on the equator of the underlying Wulff net. To obtain the angle between the poles of A and B, rotate the projection on top of the Wulff net until both poles sit on the same meridian. The angle thus found is 74°.

Problem 3



P is 37° from 100. From Table 2-3 on p.75 of Cullity, we know that the plane that is 37° from 100 is 321 in a cubic crystal.

Using the method involving direction cosines, :

Angle between P and 100 $= \rho = 36.7°$

" " P and 010 $= \sigma = 74.4°$

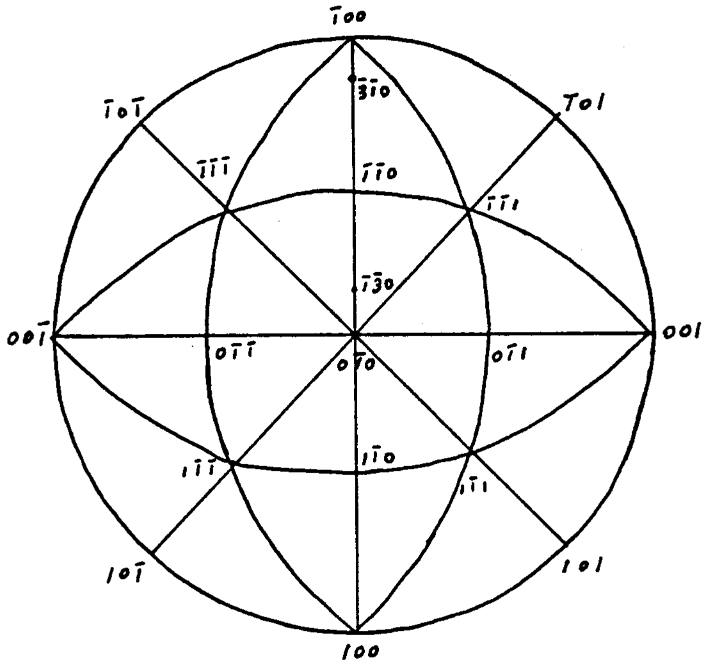
" " P and 001 $= \tau = 56.5°$

For the cubic system,

$$h : k : l = \cos\rho : \cos\sigma : \cos\tau$$
$$= 0.8020 : 0.2693 : 0.5520$$
$$= 2.98 : 1 : 2.05$$
$$= 3 : 1 : 2$$

Thus, the Miller indices are 312.

Problem 4



Crystal orientation is specified as
$[\bar{1}00]$ axis vertically upward,
$[0\bar{1}0]$ axis along beam toward x-ray tube,
$[001]$ axis horizontal and parallel to the film.

The angle between the $[0\bar{1}0]$ direction and the normal to the $(\bar{3}\bar{1}0)$ plane is

$$\alpha = \cos^{-1}\frac{[0\bar{1}0]\cdot[\bar{3}\bar{1}0]}{|[0\bar{1}0]|\,|[\bar{3}\bar{1}0]|} = \cos^{-1}\frac{1}{\sqrt{10}} = 71.6^\circ$$

Thus the angle between the $[0\bar{1}0]$ direction and the $(\bar{3}\bar{1}0)$ plane is

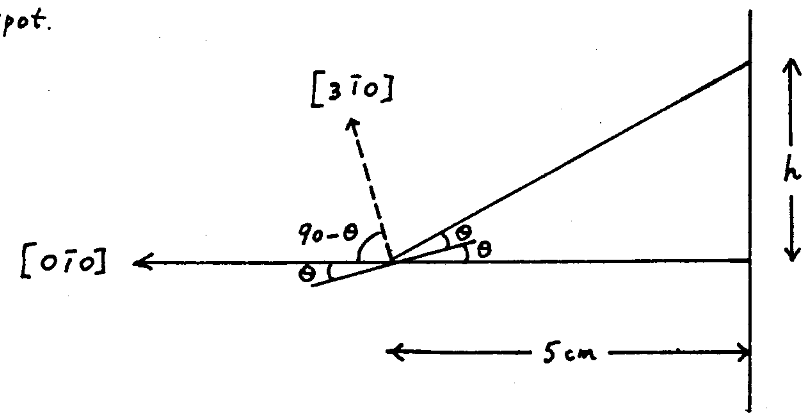
$$\theta = 90^\circ - \alpha = 18.4^\circ$$

$$\lambda = 2d\sin\theta = \frac{2a}{\sqrt{h^2+k^2+l^2}}\sin\theta = \frac{(2)(4\,\text{Å})}{\sqrt{10}}\sin 18.4^\circ = \underline{\underline{0.80\,\text{Å}}}$$

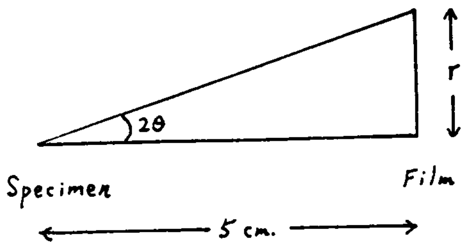
The $[\bar{3}\bar{1}0]$ direction is in the plane containing the $[\bar{1}00]$ and $[0\bar{1}0]$ directions. Thus the $(\bar{3}\bar{1}0)$ spot is directly above the primary beam spot on the film. The angle between the diffracted beam and the primary (transmitted) beam is 2θ. Therefore, the $(\bar{3}\bar{1}0)$ spot is located at

$$h = 5\text{ cm}\tan(2\theta) = \underline{\underline{3.75\text{ cm}}}$$

above the primary beam spot.



Problem 5



$$\lambda_{SWL} = \frac{12.4 \times 10^3}{40 \times 10^3} \text{ Å} = 0.310 \text{ Å}$$

$$r_{min} = (5 \text{ cm}) \tan (2\theta)_{min}$$

$$\sin \theta_{min} = \frac{\lambda_{SWL}}{2d}$$

(111)

$$d = \frac{a}{\sqrt{h^2+k^2+l^2}} = \frac{4.0497 \text{ Å}}{\sqrt{3}} = 2.338 \text{ Å}$$

$$\sin \theta_{min} = \frac{0.310}{2(2.338)} = 0.0663$$

$$\theta_{min} = 3.78^\circ$$

$$(2\theta)_{min} = 7.56^\circ$$

$$r_{min} = (5 \text{ cm}) \tan 7.56^\circ = \underline{\underline{0.66 \text{ cm.}}}$$

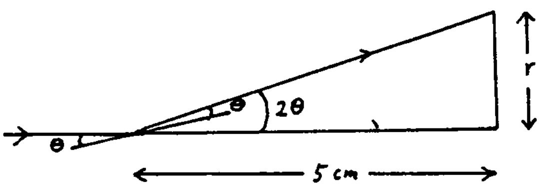
(200)

$$d = \frac{a}{\sqrt{h^2+k^2+l^2}} = \frac{4.0497 \text{ Å}}{2} = 2.0249 \text{ Å}$$

$$\sin \theta_{min} = \frac{0.310}{2(2.0249)} = 0.07655$$

$$r_{min} = (5 \text{ cm}) \tan (2\theta)_{min} = \underline{\underline{0.78 \text{ cm.}}}$$

Problem 6



$\theta = 3^\circ$

$d = 2.338$ Å (From Problem 1)

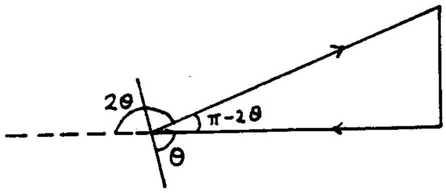
$\lambda = 2d \sin \theta$

$\quad = 2(2.338 \text{ Å}) \sin 3^\circ = 0.2455$ Å

The minimum tube voltage V corresponds to $\lambda_{SWL} = 0.2455$ Å. Thus,

$$V = \frac{12.40 \times 10^3}{\lambda_{SWL}} = \frac{12.40 \times 10^3}{0.2455} V = \underline{\underline{50.5 \text{ kV}}}$$

Problem 7



(a) $\lambda_{SWL} = \dfrac{12.4 \times 10^3}{50 \times 10^3}\ \text{Å} = 0.248\ \text{Å}$

$d = 2.338\ \text{Å}$ (From Problem 1)

$\theta = 88°$

$n\lambda = 2d \sin\theta$

$= 2\,(2.338\ \text{Å}) \sin 88° = 4.6733\ \text{Å}$

$n_{max} = \dfrac{4.6733\ \text{Å}}{\lambda_{SWL}} = 18.84 \Rightarrow n_{max} = 18$

$n_{min} = \dfrac{4.6733\ \text{Å}}{\lambda_{min}} = \dfrac{4.6733\ \text{Å}}{2\ \text{Å}} = 2.337 \Rightarrow n_{min} = 3$

The 3rd to 18th orders of reflection are present.

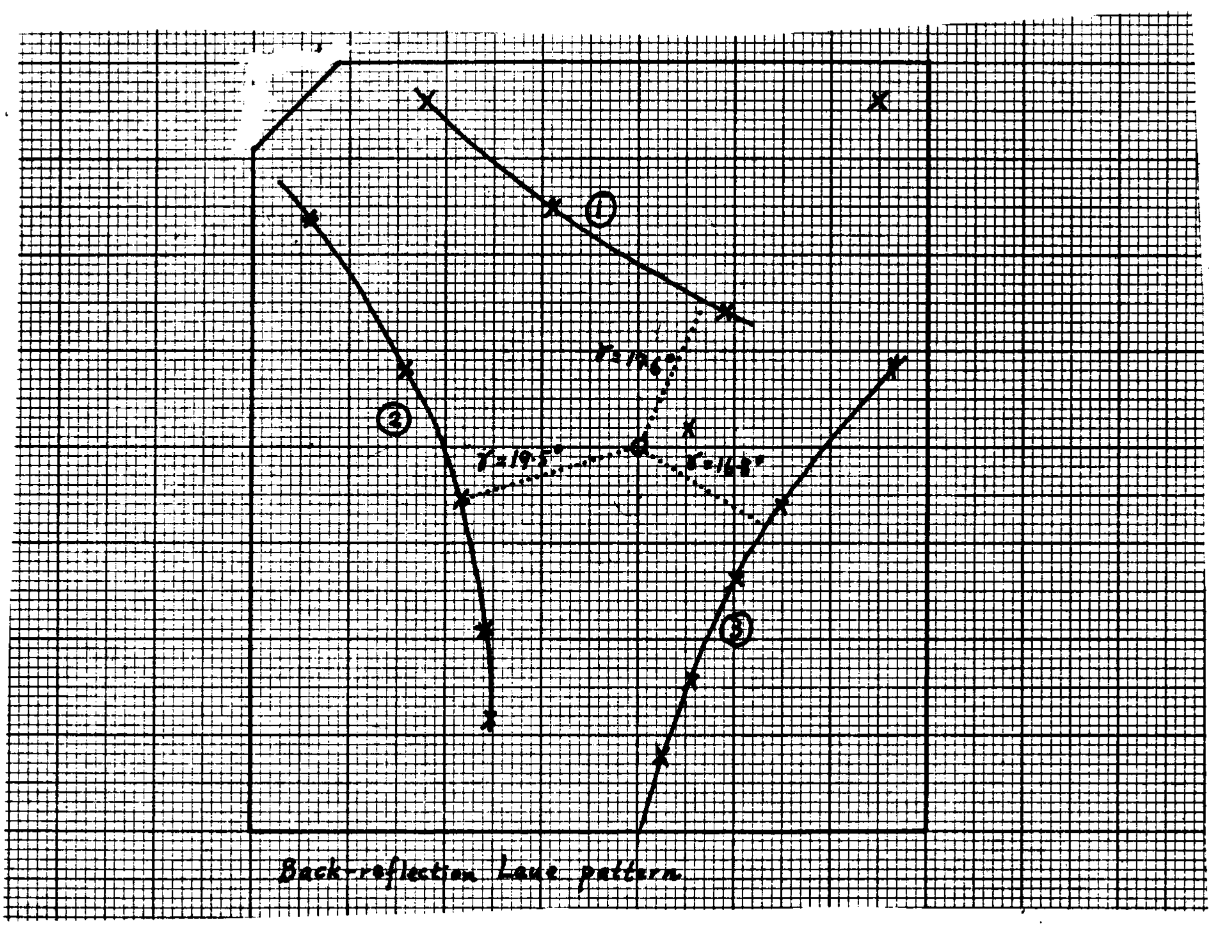
(b) $\lambda_{SWL} = \dfrac{12.4}{40} = 0.310\ \text{Å}$

$n_{max} = \dfrac{4.6733}{0.310} = 15.08 \Rightarrow n_{max} = 15$

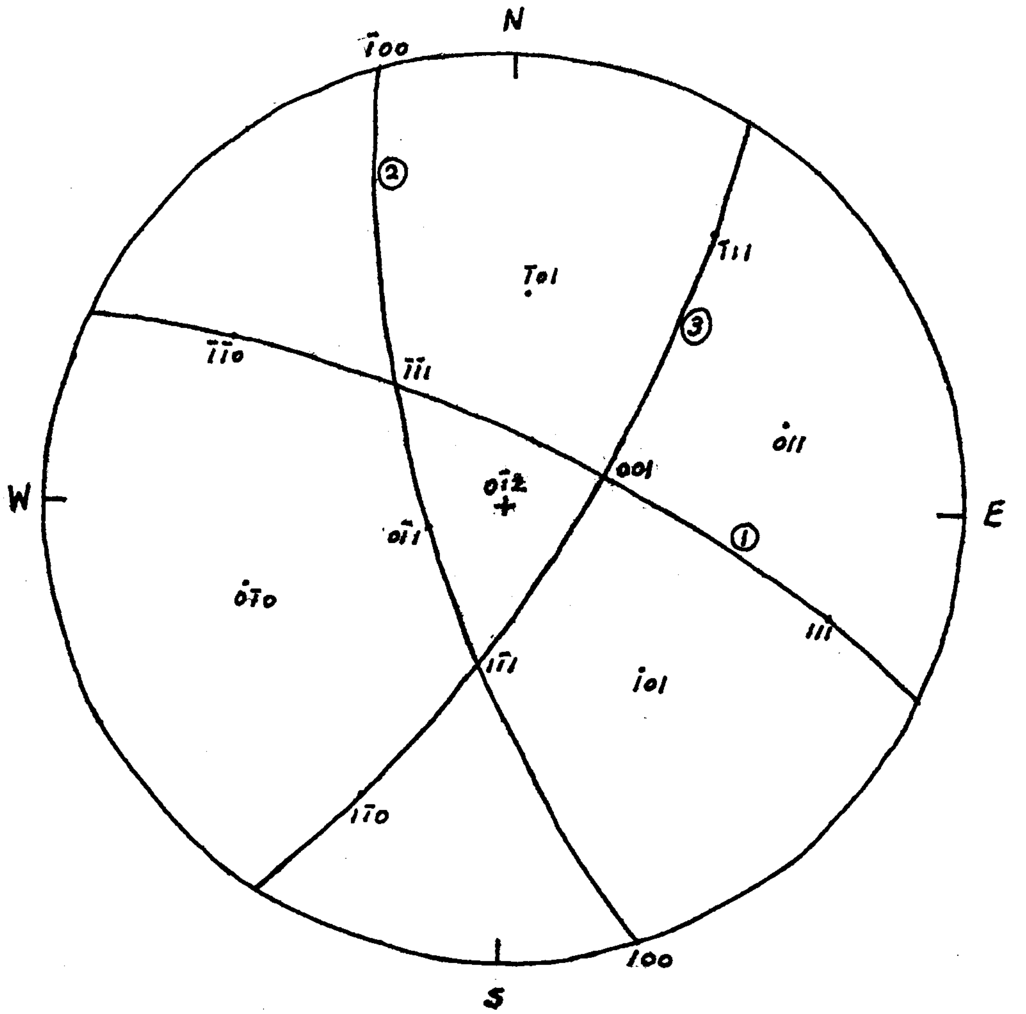
$n_{min} = 3$ (From (a)).

Thus, the 3rd to 15th orders of reflection are present.

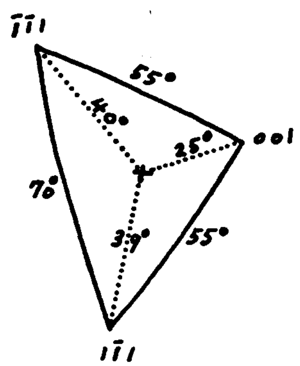
Problem 8



Back-reflection Laue pattern



Stereographic projection



Unit stereographic triangle

Crystal orientation : $0\bar{1}2$

{100} poles : 8° N 24° E
72° S 90° E
16° S 60° W
73° N 90° W