MAE 552
Heuristic Optimization

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Lecture #32

4/19/02

Fuzzy Logic

Fuzzy Logic

References:

"NeuroFuzzy Adaptive Modeling and Control", Martin Brown and Chris Harris, Prentice Hall, 1994

http://www.seattlerobotics.org/encoder/mar98/fuz/flindex.html

Fuzzy Logic

Background:

The optimization problems we are used to are in the form:

Min: $F(\bar{x}) = x_1 + 2x_2 + \ldots$

S.T. $g(\bar{x}) : x_1 + x_2^2 - \ldots \leq 0$

Fuzzy Logic

So these formulations are given in precise mathematical terms.

For example, if we are optimizing a beam for some load and we put a constraint in our formulation that states that the stress must be less than 30,000 psi, then a beam for which the max stress is 30,001 psi is considered infeasible.

Really, there is no practical difference between 30,000 psi and 30,001 psi.

Fuzzy Logic

So many real world problems are better stated in imprecise terms.

Such terms imply that a particular range of values are considered acceptable and that the level of acceptability is dependent on where a particular value lies in that range.

Fuzzy Logic

For example, some fuzzy statements are as follows:

"The beam carries a large load"

- fuzziness implied by the word "large"

"The beam carries a load of 1000 lbs with a probability of 0.8"

- fuzziness implied by the probabilistic nature of the load.

Fuzzy Set Theory

Consider ***X*** to be a set of all possible members of a class. In that sense, it represents the entire *universe* for that class.

The elements of class ***X*** are denoted by ***x***.

Also consider ***A*** to be a subset of ***X***.

Fuzzy Set Theory

We can describe membership in ***A*** with a characteristic function, $\mu_a(\quad)$, which can take on a value [0, 1].

A value of 0 indicates complete non-compliance with the premise of ***A***, and a value of 1 indicates complete compliance with the premise of ***A***.

Fuzzy Set Theory

Mathematically:

This is referred to as a *valuation set.*

Fuzzy Set Theory

Our subset ***A*** becomes a fuzzy set if we allow its valuation set to take on all values in [0, 1].

And we can thus define the set ***A*** by a collection of pairs comprised of a member value and its associated characteristic function value for ***A*** as follows.

Fuzzy Set Theory

So for example, let ***X*** represent all possible temperature settings for a thermostat and ***A*** represent all comfortable temperatures for human activity.

X may be:

X = { 62, 64, 66, 68, 70, 72, 74, 76, 78, 80 }

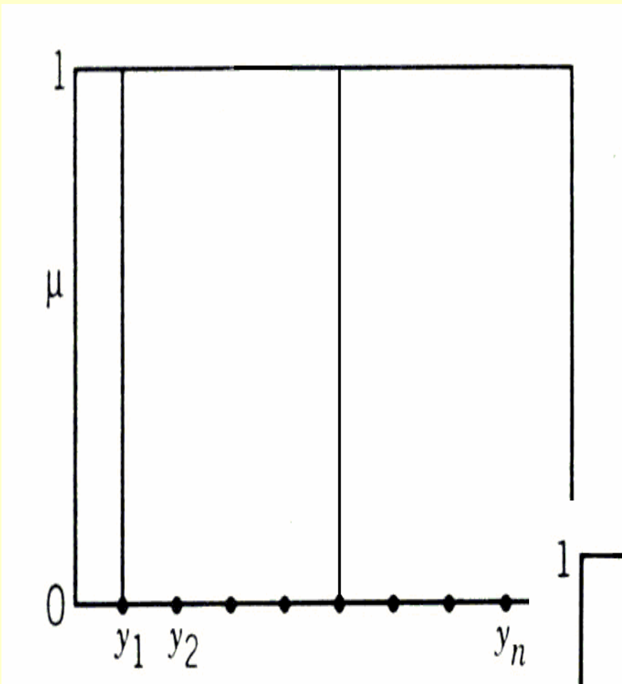
Fuzzy Set Theory

A may then look something like:

A = { (62, 0.2), (64, 0.5), (66, 0.8), (68, 0.95), (70, 0.85), (72, 0.75), (74, 0.6), (76, 0.4), (78, 0.2), (80, 0.1) }

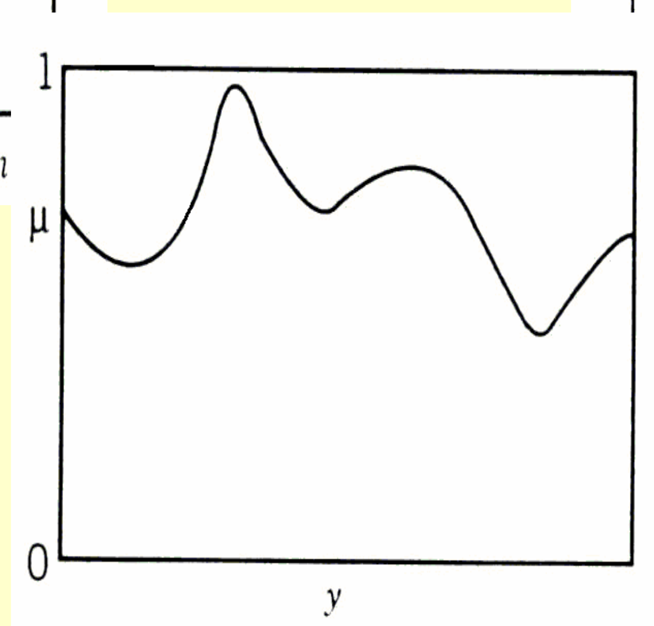
So we see that different temperatures satisfy the requirements of membership in ***A*** by different amounts.

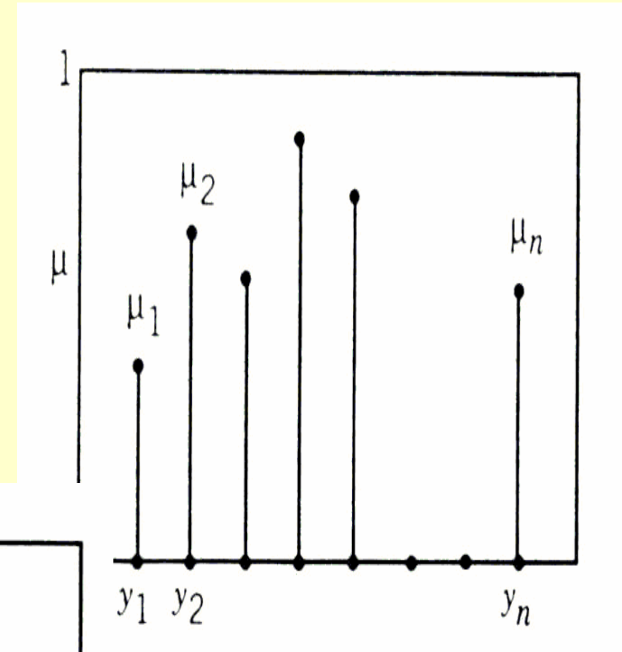
Fuzzy Sets vs. Crisp Sets



Discrete Crisp

Continuous Fuzzy

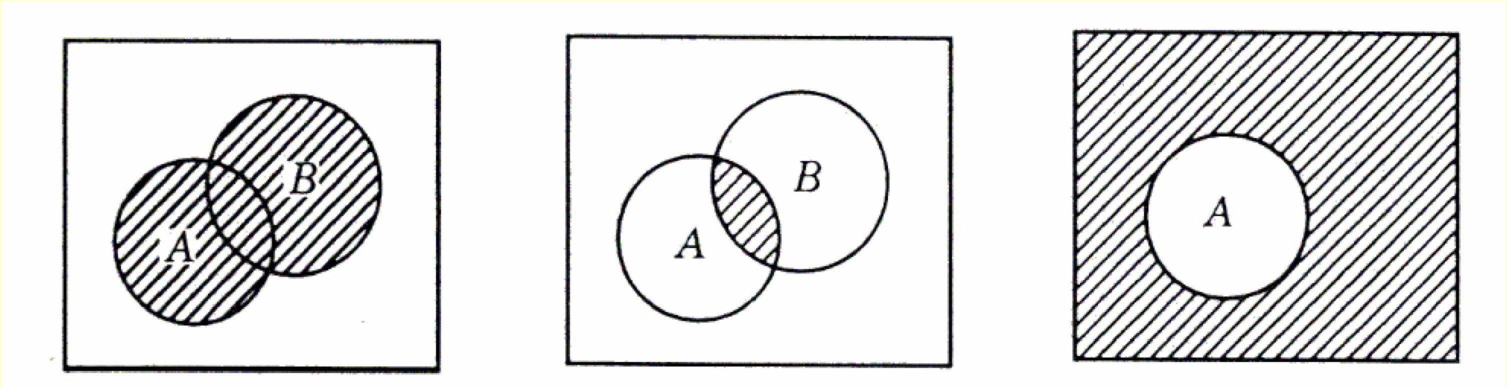




Discrete Fuzzy

Fuzzy Set Theory

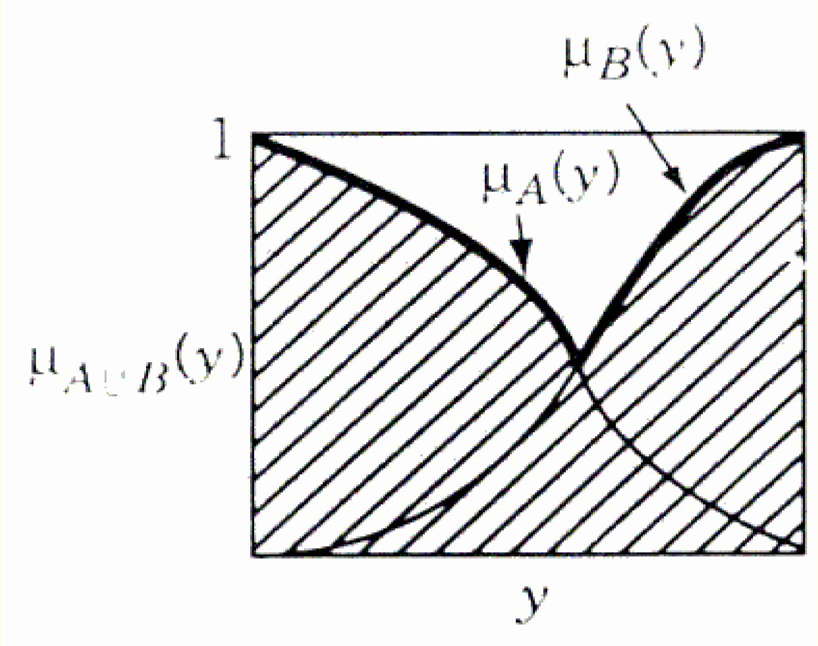
So there is some correlation between crisp sets and fuzzy sets. Do the same operations exist for fuzzy sets that exist for crisp sets (union, intersection, complement) as shown below?



Fuzzy Set Theory

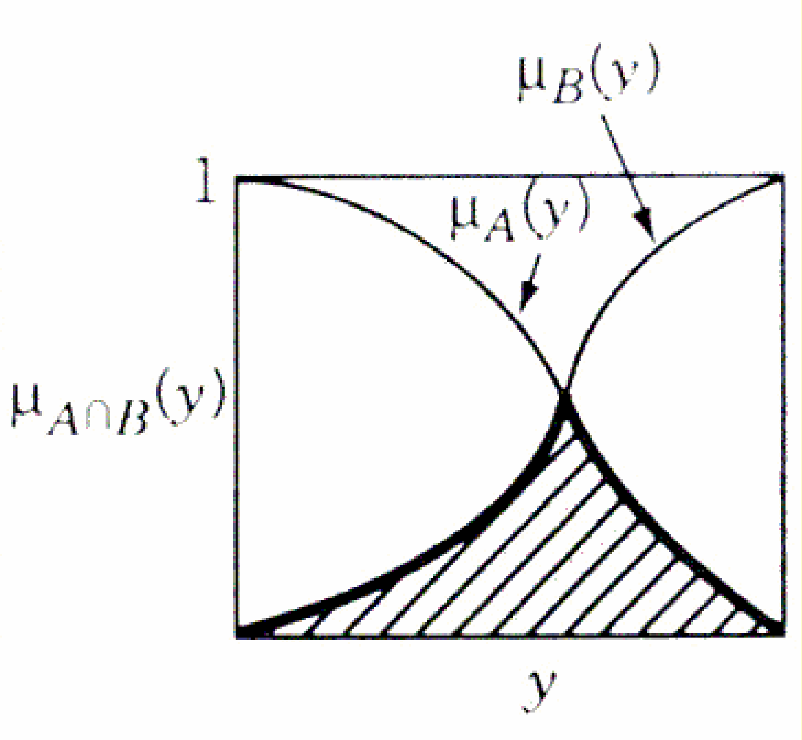
The answer is yes.

The figure below shows the fuzzy union of some fuzzy sets ***A*** and ***B.***



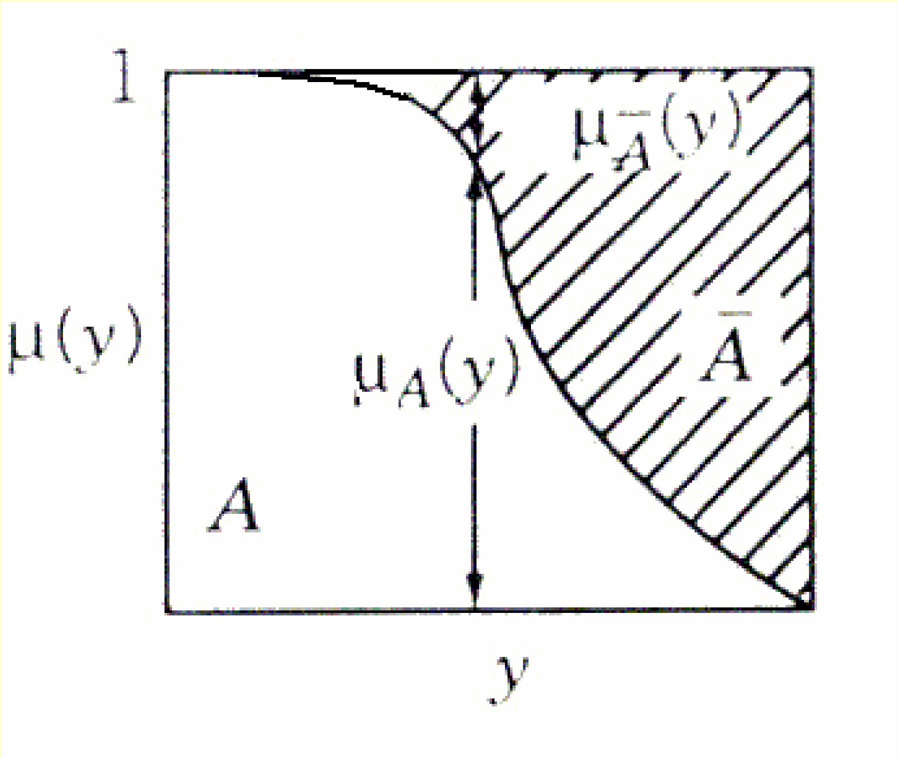
Fuzzy Set Theory

The figure below shows the fuzzy intersection of some fuzzy sets ***A*** and ***B.***



<u>Fuzzy Set Theory</u>

Finally, the figure below shows the fuzzy complement of some fuzzy set ***A***.



Fuzzy System Optimization

Our conventional optimization typically entails finding the set of design parameters that minimizes some objective function subject to some constraints.

For fuzzy systems, this notion has to be revised because we do not have a precise mathematical representation for our system.

Fuzzy System Optimization

Since our objective and constraint functions are characterized by membership functions in our fuzzy system, a design can be viewed as the intersection of these fuzzy functions.

Consider the following example.

Fuzzy System Optimization

Suppose we have an objective stated as:

"The depth of the crane girder (x) should be substantially greater than 80 in."

Our membership function for this statement may be something like:

Fuzzy System Optimization

Suppose we also have a constraint stated as:

"the depth of the crane girder (x) should be in the vicinity of 83 in"

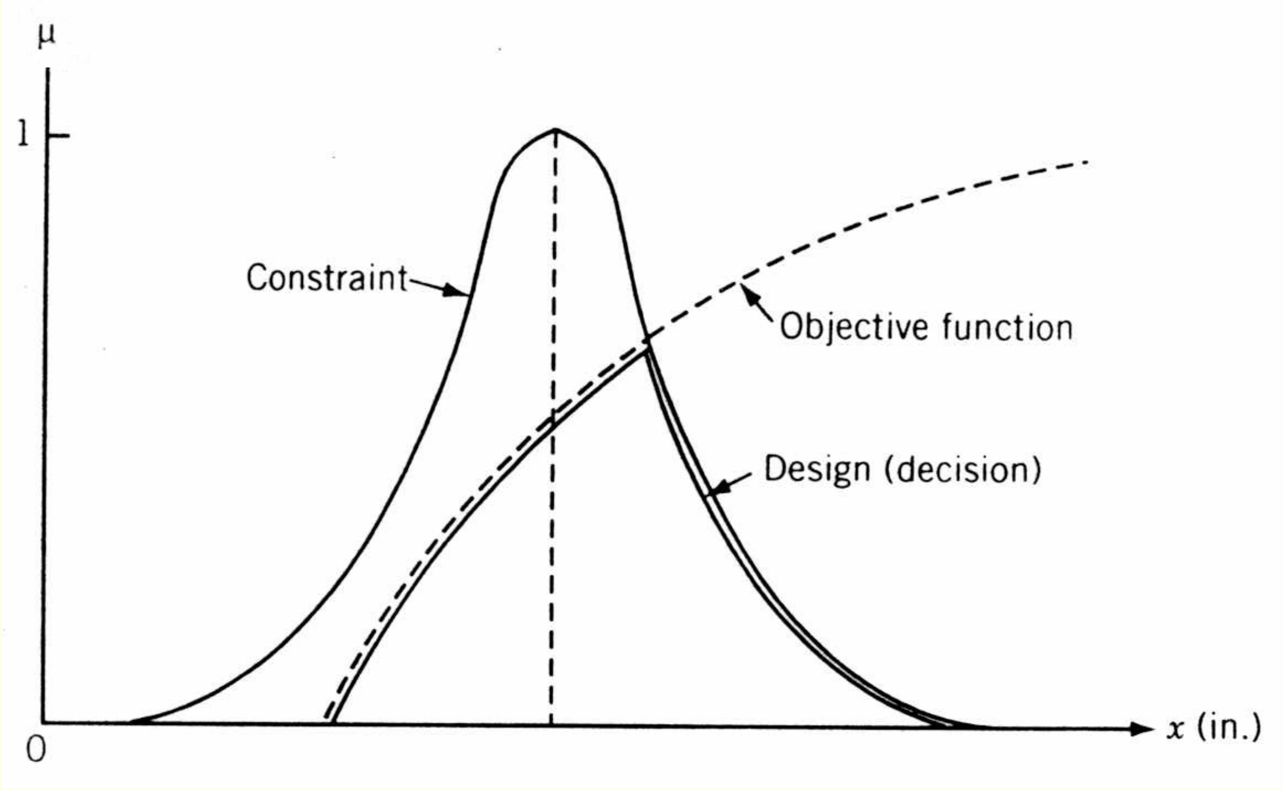
The corresponding membership function might be:

Fuzzy System Optimization

So the fuzzy intersection of these two functions is given by:

Fuzzy System Optimization

A plot containing the membership functions for both the objective and constraints is shown below.



Fuzzy System Optimization

The fuzzy feasible space is defined by the intersection of all the fuzzy constraint membership functions. It has a membership function:

Where G_j denotes the fuzzy set to which g_j should belong.

Fuzzy System Optimization

The optimal value is at the maximum intersection of the objective membership function and the fuzzy feasible space.