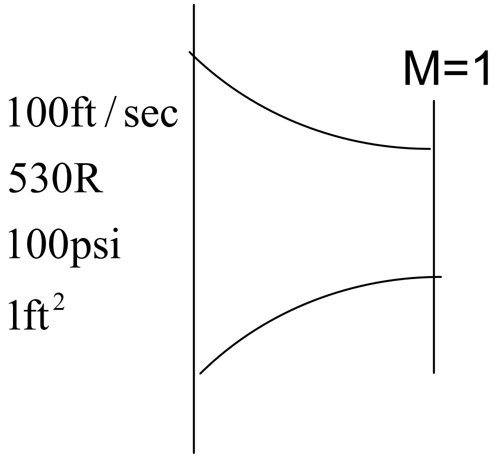
Problem Set 11 (PS 11) Due Tuesday April 11

11.1 Air flowing in a 1 foot square duct at 100 ft/sec, 530 R and 100 psi is expanded to choked conditions through a converging nozzle. What is the area, velocity, mass flow, volume flow and Mach Number at the nozzle throat ? If the throat area is kept constant and the duct pressure is increased to 125 psi and then to 150 psi how do the throat velocity, volume flow, mass flow and pressure change ?

11.2 Air is expanded from the duct conditions of problem 11.1 in a converging diverging nozzle to a back pressure of 70 psi. What is the exit area of the diverging portion of the nozzle ? What would happen if the back pressure downstream of the diverging portion exit were above and below 70 psi.

11.3 With a back pressure of 80 psi in the nozzle of problem 11.2, with the throat area kept the same, at what area ratio (A_{shock}/A^*) would a shock form ?

11.1



$$a_1 = \sqrt{\gamma RT}$$

$$a_1 = \sqrt{1.4 \times 32.2 \times 530 \times 53.35}$$

$$a_1 = 1129 \text{ft/sec}$$

$$M_1 = V_1/a_1 = 100/1129 = .0886$$

$$\rho_1 = \frac{p}{RT} = \frac{100 \times 144}{53.35 \times 530} = .509 \text{lb/ft}^3$$

isentropic @ $M_1 = .0886$

$$\frac{p_O}{p_1} = 1.0055, \ \frac{T_O}{T_1} = 1.0016, \ \frac{\rho_O}{\rho_1} = 1.0039$$

$$\frac{A_1}{A_t} = \frac{A_1}{A^*} = 6.5625$$

$$A_t = A_1/6.5625 = .1523 \text{ ft}^2$$

isentropic @ $M_t = 1$

$$\frac{T_O}{T^*} = 1.200, \frac{p_O}{p^*} = 1.839 \frac{\rho_O}{\rho^*} = 1.577$$

$$T_t = \frac{T_t}{T_O}\frac{T_O}{T_1}T_1 = \frac{1}{1.2} \times 1.0016 \times 530 = 442.2$$

$$a_t = \sqrt{1.4 \times 32.2 \times 53.35 \times 442.2} = 1031.1$$

$$V_t = 1031.3 \text{ft / sec}$$

$$Q_t = A_t \times V_t = .1532 \times 1031.3 = 157.1 \text{ft}^3 \text{ / sec}$$

$$T_t = \frac{T_t}{T_O}\frac{T_O}{T_1}T_1$$

$$p_t = \frac{p_t}{p_O}\frac{p_O}{p_1}p_1 = \frac{1}{1.577} \times 1.0055 \times 100$$

$$p_t = 63.76 \text{psi}$$

$$m_t = Q_t V_t = 157.1 \times .3239 = 50.89 \text{lb / sec}$$

11.1

$$\frac{m_1}{m_2} = \frac{(\rho AV)_1}{(\rho AV)_2} = \frac{\rho_1}{\rho_2} = \frac{p_1}{RT_1}\frac{R_2T_2}{p_2}$$

$$@\, p_1 = 125\text{psim} = 50.89\frac{125}{100} = 63.61\,\text{lb/sec}$$

$$p_t = \frac{p_t}{p_O}\frac{p_O}{p_1}p_1 = \frac{1}{1.577}\times 1.0055\times 125 = 63.76\text{ psi}$$

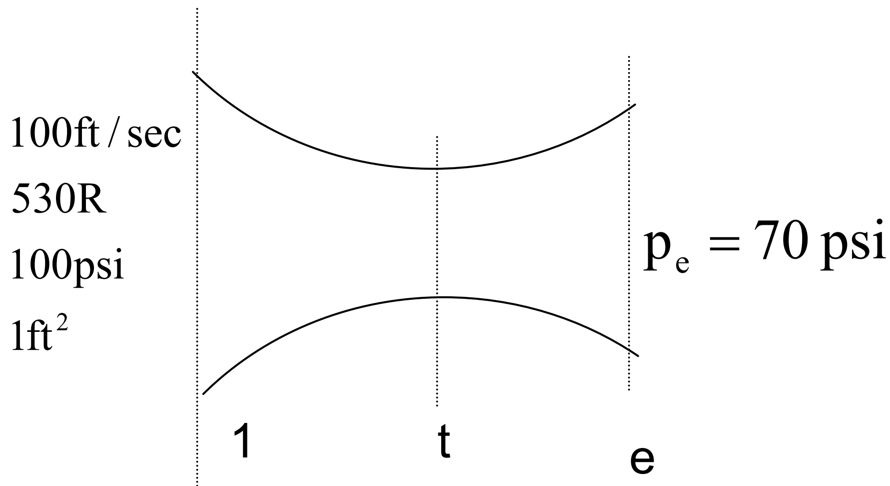
$$Q_t = 157.1\,\text{ft}^3/\text{sec}$$

$$@\, p_1 = 150\text{ psi},\ \ m = 50.89\frac{150}{100} = 76.33\text{ lb/sec}$$

$$p_t = \frac{p_t}{p_O}\frac{p_O}{p_1}p_1 = \frac{1}{1.577}\times 1.0055\times 150 = 95.64\text{ psi}$$

$$Q_t = 157.1\text{ ft}^3/\text{sec}$$

11.2



At lower pressures
exit approaches sonic

isentropic @ $M_1 = .0886$

$$\frac{p_O}{p_1} = 1.0055, \frac{T_O}{T_1} = 1.0016, \frac{\rho_O}{\rho_1} = 1.0039$$

$$\frac{A_1}{A_t} = \frac{A_1}{A^*} = 6.5625$$

$$A_t = A_1/6.5625 = .1523\ \text{ft}^2$$

$$\frac{p_O}{p_e} = \frac{p_O}{p_1}\frac{p_1}{p_e} = \frac{100}{70} \times 1.0055 = 1.4364$$

isentropic @ $\frac{p_O}{p_e} = 1.4364$

$$\frac{A_e}{A^*} = 1.068, \quad M_e = .74$$

$$A_e = \frac{A_e}{A^*}\frac{A^*}{A_1}A_1 = 1.068 \times \frac{1}{6.5625} = .1627\ \text{ft}^2$$

11.3

_no shocks would occur in the nozzle