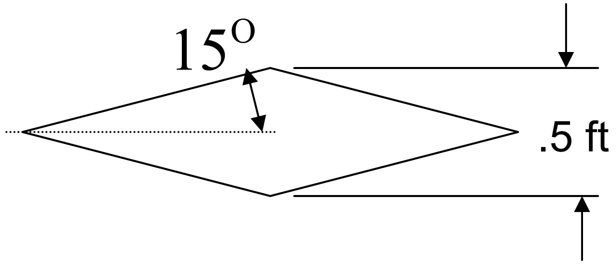
Problem Set 9 (PS 9) Due Tuesday March 28

9.1 Compute the lift, drag and lift/drag ratio for a flat plate positioned a 5 angles of attach from 0 to 25 degrees in an air flow at M= 2.5, 1 atm, 270 K. What happens to the flow at an angle of 30 degrees ? Qualitatively how does this affect the lift and drag ?

9.2 A diamond shaped air foil with a 15 degree half angle, a thickness of .5 ft and a 1 ft span is positioned at an angle of attack of 5 degrees in an air flow at M=2, p= 2116 lb/square foot. Calculate the lift, drag and lift drag ratio.

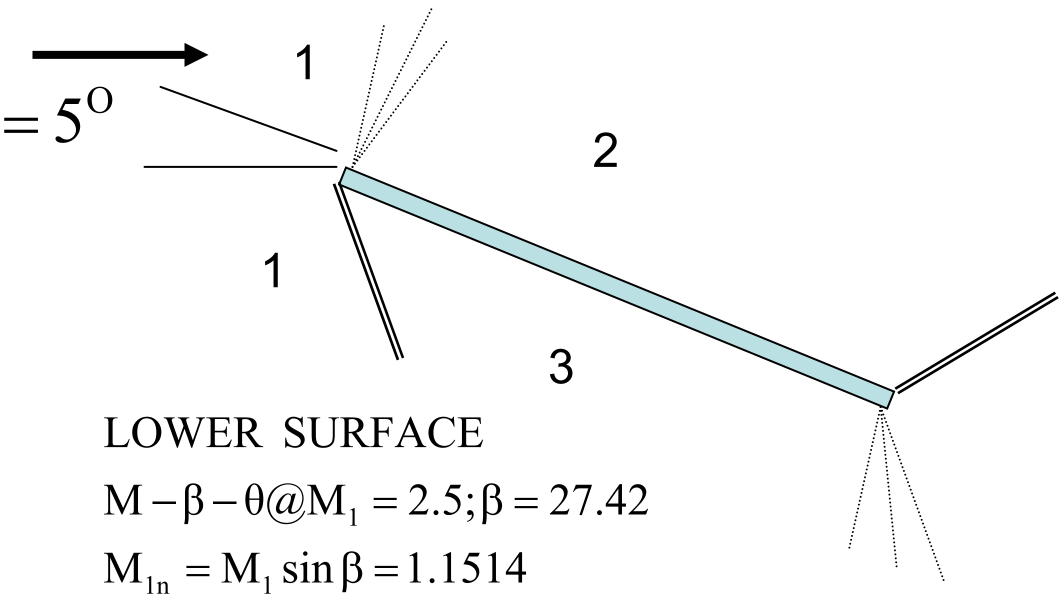


9.3 A supersonic flow with a Mach Number of 4 and a pressure of 1 atm is first deflected 10 degrees over an expansion corner and then deflected 10 degrees through a compression corner. The flow after the two corners is in the same direction as it was approaching the corners. Calculate the Mach Number and pressure of the flow downstream of these corners.

9.1

FLAT PLATE AT 5 DEGREE ANGLE OF ATTACK

$\alpha = 5^{O}$



UPPER SURFACE

$$\text{isentropic Table A.1} @ M_1 = 2.5;$$

$$\frac{p_{O1}}{p_1} = 17.085, \frac{T_O}{T_1} = 2.2502$$

$$\text{P\_M Table A.5} @ M_1 = 2.5$$

$$\nu_1 = 39.12$$

$$\nu_3 = \nu_1 + \theta_2 = 39.12 + 5 = 44.1$$

$$\text{P\_M Table A.5} @ \nu_3 = 44.1; M_3 = 2.7233$$

$$\text{isentropic Table A.1} @ M_3 = 2.7233;$$

$$\frac{p_{O3}}{p_3} = 24.13,$$

$$\frac{T_3}{T_1} = 2.4832$$

$$p_3 = 1.3799$$

$$T_3 = 296.3$$

LOWER SURFACE

$$M-\beta-\theta @ M_1 = 2.5; \beta = 27.42$$

$$M_{1n} = M_1 \sin\beta = 1.1514$$

$$\text{normal shock Table A.2} @ M_{1n} = 1.1514;$$

$$\frac{p_2}{p_1} = 1.3799,$$

$$\frac{T_2}{T_1} = 1.0974$$

$$p_2 = p_1 \times 1.3799 \times 1 = 1.3799,$$

$$T_2 = 1.0974 \times 270 = 296.3$$

$$\text{Lift} = (p_2 - p_1)\cos\alpha = .6693$$

$$\text{Drag} = (p_2 - p_1)\sin\alpha = .0586$$

$$\text{Lift}/\text{Drag} = 11.4$$

UPPER SURFACE

isentropic Table A.1 @ $M = 2.6$; $\frac{p_O}{p_1} = 17.17$

P – M Table A.5 @ $M_1 = 2.5$; $\nu_1 = 39.1$

at $\alpha = 5$, $\nu = \nu_1 + \theta = 44.1$

P – M Table A.5 @ $\nu = 44.1$; $M_1 = 2.7233$

isentropic Table A.1 @ $M = 2.7233$; $\frac{p_O}{p_3} = 24.13$

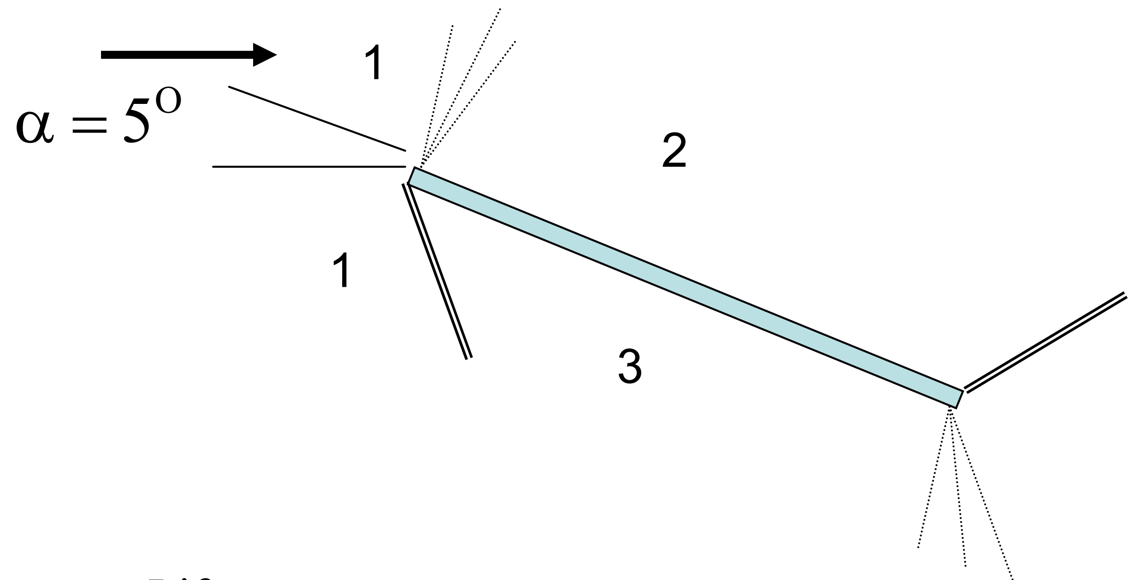
$$p_3 = p_1 \frac{p_O}{p_1} / \frac{p_O}{p_3} = 1 \times 17.17/24.15 = .711$$

α	ν	M	$\frac{p_O}{p_3}$	p_3
0	39.1	2.50	17.17	1 atm
5	44.1	2.7233	24.13	.7110
10	49.1	2.9674	34.98	.4885
15	54.1	3.2368	52.19	.3231
20	59.1	3.5376	80.45	.2123
25	64.1	3.877	128.7	.1328
30	69.1	4.265	214.9	.0795

LOWER SURFACE

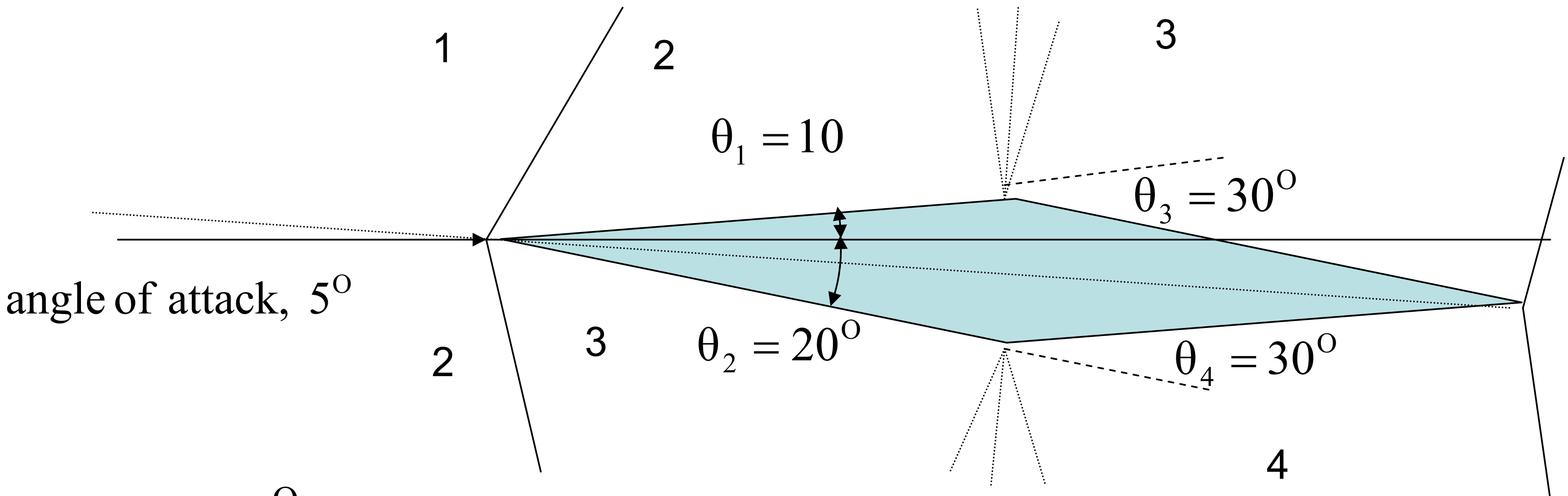
α	β	M_{1n}	$p_2 = \frac{p_2}{p_1} \times 1$
0			
5	27.42	1.1514	1.380
10	31.85	1.3193	1.864
15	36.94	1.503	2.468
20	42.89	1.702	3.211
25	50.25	1.922	4.143
30	detached oblique shock M_2		

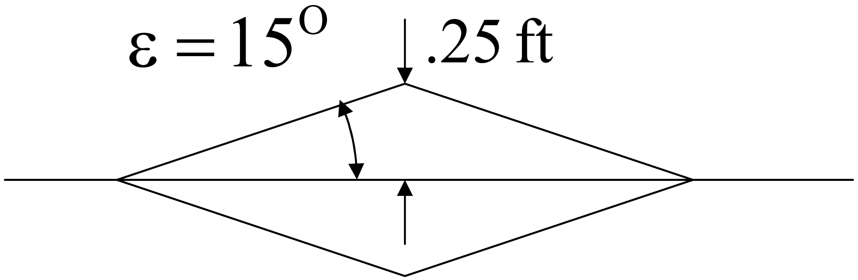
9.1



$$\text{Lift} = (p_3 - p_2)\cos\theta$$
$$\text{Drag} = (p_3 - p_2)\sin\theta$$

θ	p_2	p_3	Lift	Drag	$\frac{\text{Lift}}{\text{Drag}}$
0	1.	1.			
5	.711	1.380	.669	.0586	11.4
10	.4885	1.864	1.355	.239	5.67
15	.3231	2.468	2.067	.554	3.73
20	.2123	3.211	2.818	1.926	2.75
25	.1328	4.143	3.635	1.695	2.14





$$\theta_1 = \varepsilon - \alpha$$

$$\theta_2 = \varepsilon + \alpha$$

$$\theta_3 = \theta_4 = 180 - (180 - 2\varepsilon)$$

$$\theta_3 = 2\varepsilon$$

Upper Surface – region 1 - 2 - 3

$M_1 = 2.\theta$

	θ_2	10
$M - \beta - \theta$ Chart	β_2	39.31
normal shock	M_2	1.64
	p_2/p_1	1.707
	p_2	3611.2
	F_2	3488.4
P - M expansion	ν_2	16.07
isentropic flow	p_{O2}/p_2	4.518
	θ_3	30
$\nu_3 = \nu_2 + \theta_3$	ν_3	46.07
P - M expansion	M_3	2.816
isentropic flow	p_{O3}/p_3	27.8
$p_3 = p_1 \frac{p_3}{p_{O3}} \frac{p_{O3}}{p_2} \frac{p_2}{p_1}$	p_3	586.8
	F_3	566.8

Lower Surface – regions 4 - 5 - 6

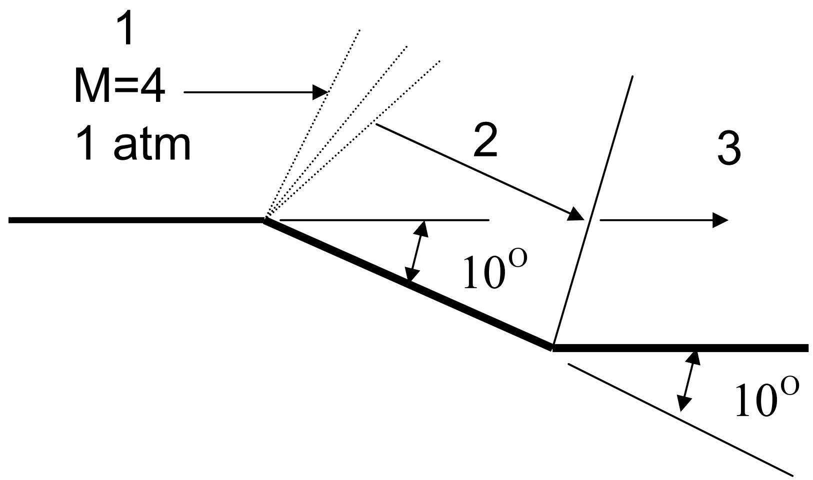
θ_5	20.
β_5	53.42
M_5	1.21
p_5/p_4	2.823
p_5	5973.5
F_5	5770.4
ν_5	3.811
p_{O5}/p_5	2.458
θ_6	30
ν_6	33.81
M_6	2.28
p_{O6}/p_6	12.142
p_6	1209.2
F_3	1168.

9.2

$$\text{Drag} = \sum F_x = \left(F_2\cos(90-\theta_2) + F_5\cos(90-\theta_5)\right)$$
$$-\left(F_3\cos(90-\theta_3+\theta_2) + F_6\cos(90-\theta_6+\theta_5)\right)$$
$$\text{Drag} = \sum F_x = \left(3488.4\cos(80) + 5770.4\cos(70)\right)$$
$$-\left(566.8\cos(70) + 1168\cos(80)\right)$$
$$\text{Drag} = \sum F_x = \left(605.7 - 1973.6\right) - \left(193.9 - 202.8\right)$$
$$\text{Drag} = \sum F_x = 2182$$

$$\text{Lift} = \sum F_y = \left(F_2\sin(90-\theta_2) + F_5\sin(90-\theta_5)\right)$$
$$-\left(F_3\sin(90-\theta_3+\theta_2) + F_6\sin(90-\theta_6+\theta_5)\right)$$
$$\text{Lift} = \sum F_y = \left(3435. + 532.6\right) - \left(5422.21150.3\right)$$
$$\text{Lift} = \sum F_y = 2605$$

9.3



isentropic flow @ $M_1 = 4$;

$$\frac{p_{O1}}{p_1} = 151.83 = \frac{p_{O2}}{p_2}$$

$P - M$ @ $M_1 = 4$; $\nu_1 = 65.78$

$\nu_2 = \nu_1 + \theta_1 = 65.78 + 10 = 75.78$

$P - M$ @ $\nu_2 = 75.78$; $M_2 = 4.88$

isentropic flow @ $M_2 = 4.88$;

$$\frac{p_{O2}}{p_2} = 450.92$$

$M - \beta - \theta$ @ $M_2 = 4.88, \theta_2 = 10$;

$\beta = 19.65$

$M_{2n} = M_2 \sin\beta = 1.641$

normal shock @ $M_{2n} = 1.641$;

$M_{3n} = .6565, \quad \frac{p_3}{p_2} = 2.975$

$$M_3 = \frac{M_{3n}}{\sin(\beta - \theta)} = 3.917$$

isentropic flow @ $M_3 = 3.917$;

$$\frac{p_{O3}}{p_3} = 146.07$$

$$p_3 = p_1 \frac{p_3}{p_2} \frac{p_2}{p_{O2}} \frac{p_{O1}}{p_1}$$

$$p_3 = 22.975 \frac{1}{450.82} 151.83 = 1\,\text{atm}$$

$$p_{O1} - p_{O3} = 151.83 - 146.07 = 5.8\,\text{atm}$$