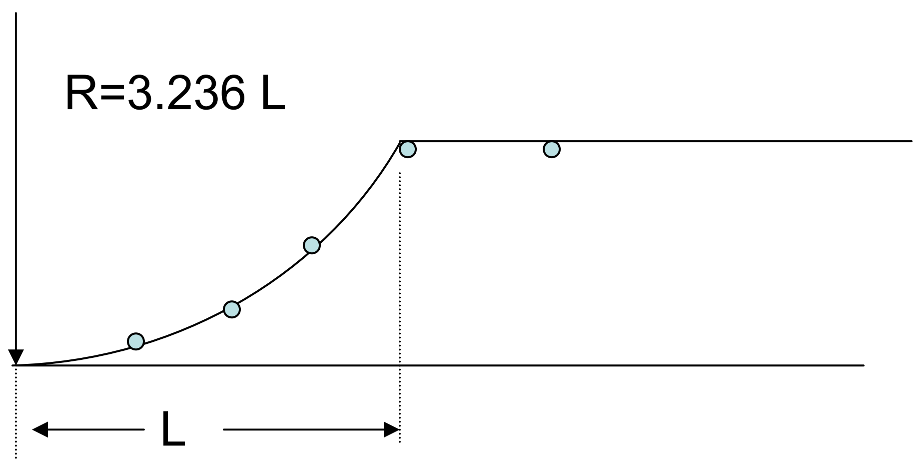
Problem Set 8 (PS 8) Due Tuesday March 21

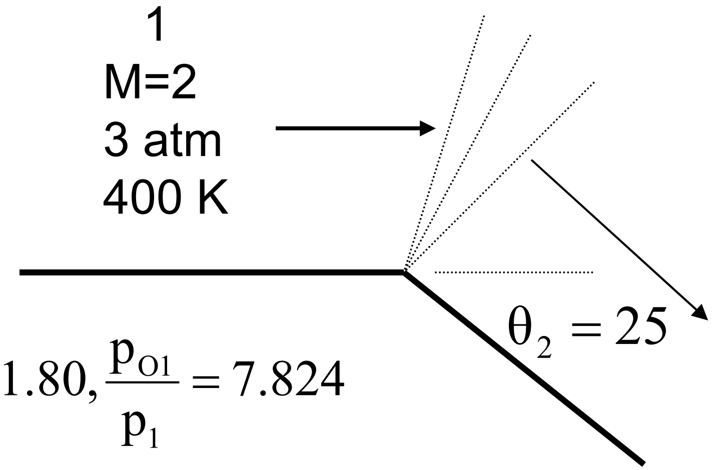
8.1 An air flow is deflected 25 degrees in passing over an expansion corner as sketched in Figure 4.32 in the text. Upstream conditions are M=2, 3 atm and 400 K. Calculate the Mach Number, temperature, pressure, stagnation pressure and stagnation temperature after the expansion.

8.2 Make a plot of the Mach Number after an expansion corner vs the deflection of the flow for deflections from 0 degrees to the maximum deflection for an approaching Mach number of 2. What does this curve represent ?

8.3 The upper surface of a 2 D wedge is a concave curve with a radius of 3.236 L centered on the leading edge of the wedge. The surface abruptly becomes horizontal, the orientation of the inlet flow, at a length of L. The Mach Number of the approaching flow is 2. For 4 points on the concave surface and one point to the horizontal surface calculate the Mach number and orientation of the wave (oblique shock wave on the concave surface and Mach wave on the horizontal surface) at each location. Sketch the streamline.



8.1



$$\text{isentropic Table A.1} @ M_1 = 2; \frac{T_{O1}}{T_1} = 1.80, \frac{p_{O1}}{p_1} = 7.824$$

$$T_{O1} = 400 \times 1.8 = 720\text{K}$$

$$p_{O1} = 7.8204 \times 3 = 23.47\text{atm}$$

$$\text{P} - \text{Table A.5} @ M_1 = 2.; \nu_1 = 26.38$$

$$\nu_2 = \nu_1 + \theta_2 = 26.38 + 25. = 51.38$$

$$\text{P} - \text{Table A.5} @ \nu_2 = 51.38; M_2 = 3.085$$

$$\text{isentropic Table A.1} @ M_2 = 3.0; \frac{T_{O2}}{T_2} = 2.904,$$

$$T_{O2} = T_{O1} = 720\text{K}, T_2 = \frac{T_{O1}}{2.904} = 247.9\text{K}$$

$$P_{O2} = p_{O1} = 23.47\text{atm}, p_2 = \frac{P_{O2}}{p_2} = \frac{23.47}{41.70} = .563\text{atm}$$

$\text{P} - \text{M}$ Table A.5 @ $M_1 = 2; \quad \nu_1 = 26.38$

ν_2	M_2	$\theta_2 = \nu_2 - \nu_1$	$M^* = \dfrac{1.2M^2}{1+.2M^2}$
46.78	2.85	20.4	1.927
66.44	4.05	40.06	2.144
86.29	6.20	54.91	2.304
106.9	12.0	80.52	2.408
124.7	50.0	98.32	2.447

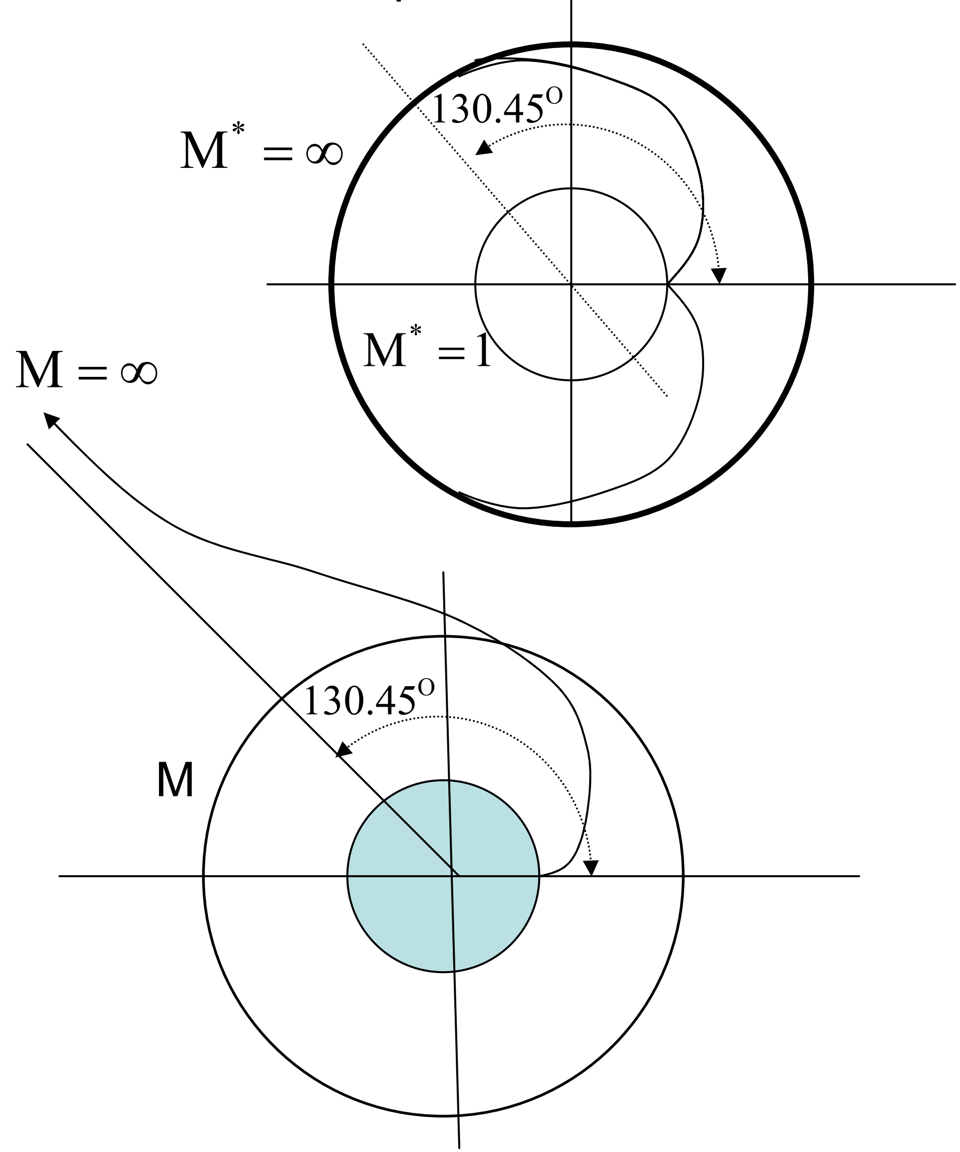
$$\nu_{max} = 90 \times \left[\sqrt{\frac{\gamma+1}{\gamma-1}} - 1 \right] = 130.45^O$$

$$\theta_{max} = 130.45 - \nu_1$$

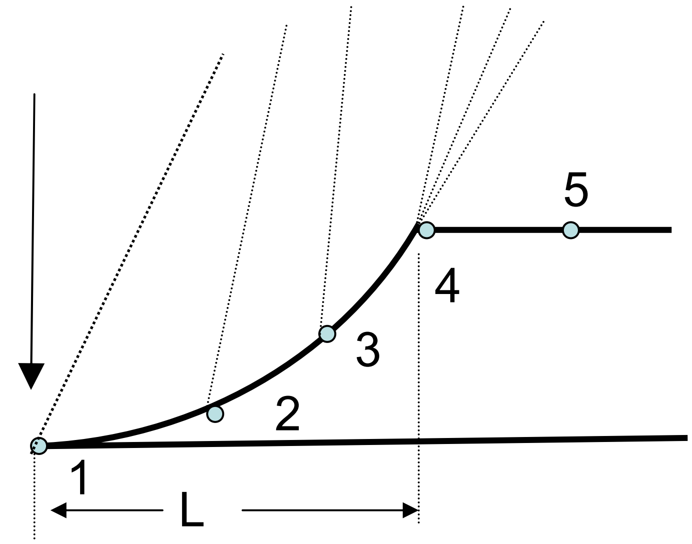
M_1	ν_1	θ_{max}
1	0.0	130.45
2	26.38	104.7
3	49.76	80.69
4	65.78	64.67
5	76.92	53.53

8.2

Plot in polar coordinates is a hodograph, analogous to a shock polar.



8.3



$$\alpha = \sin^{-1}\frac{L}{3.236L} = 18^{O}$$

3 equal deflections of 6 ^{O}each

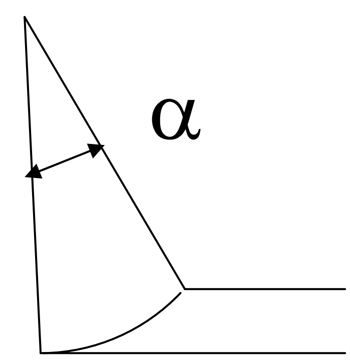
P – M Table A.5

P - M compression waves,

OR oblique shock waves with $\theta \Rightarrow 0$.

Compression waves, $\nu_{i+1} = \nu_i + \theta_i$

i	M	θ_i	μ_i	ν_i	ν_{i+1}	M_{i+1}	μ_{i+1}
1	2.	+6	30.	26.38	20.38	1.788	34.
2	1.788	+6	34.	20.38	14.38	1.584	39.15
3	1.584	+6	39.15	14.38	8.38	1.379	46.48



Expansion waves, $\nu_{i+1} = \nu_i + \theta_i$

i	M	θ_i	μ_i	ν_i	ν_{i+1i}	M_{i+1}	μ_{i+1}
4	1.379	-18	46.84	8.38	26.38	2.0	30