Name

**MAE 340: Test III** **April 22, 2003**

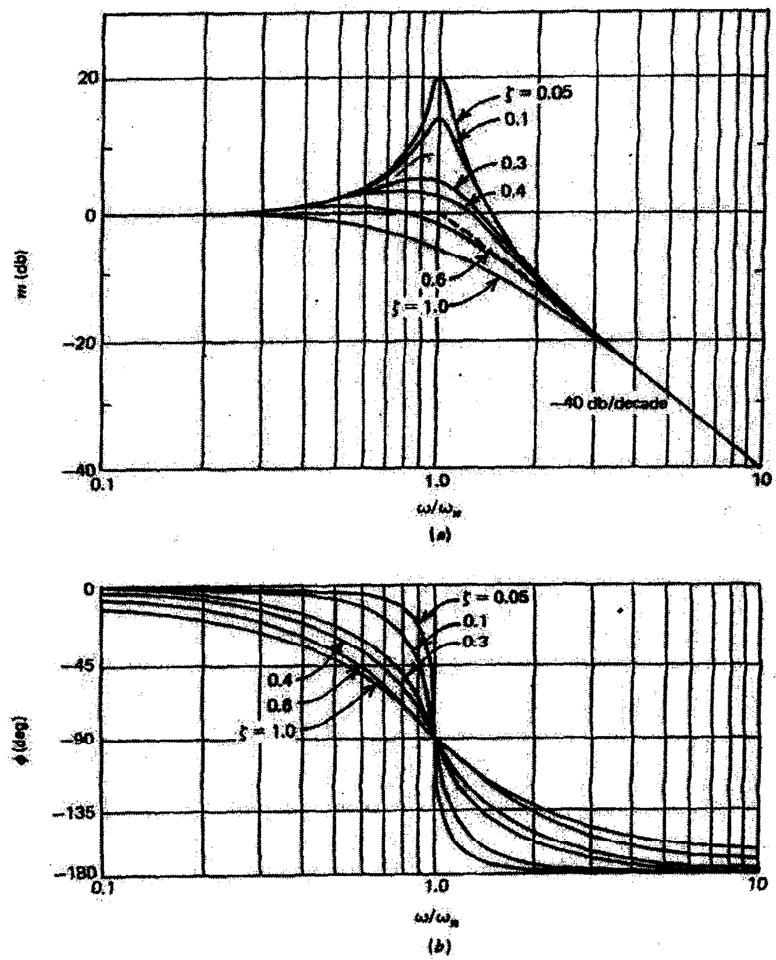
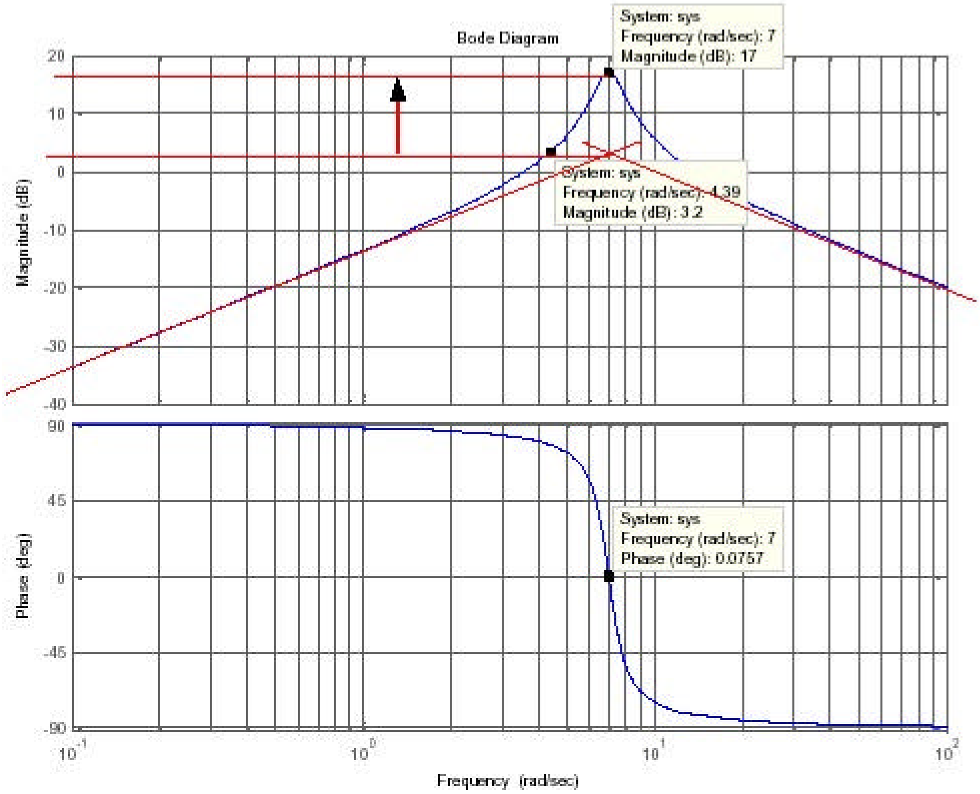



**FIGURE 4.20 Frequency response plots for the underdamped system**

$$T(s) = \frac{1}{\left(\frac{s}{\omega_n}\right)^2 + \frac{2\zeta s}{\omega_n} + 1}$$

**Questions 1-10 (worth 7 points each) refer to the Bode Plots on the next page:**

1) Estimate the system transfer function.




- Final slop is -20 (db/dec), the difference of orders (Den.-Num.) is 1.
- Phase angles: +90 → 0 → -90. We have 1 "zeros" 2 "ploes". Besides, the starting angle is +90, therefore, the "zeros" has "0" real part.
- From the peak and the arrow, we know the damping ratio is less than 0.707 and it is actually $x \cong 0.1$ (by comparing the arrow to the given reference plot).
- From 0 degree crossing ("+90" + "-90" crossing), we know $w_n \cong 7$ (rad/sec).
- The Transfer Function should have the form as: $\dfrac{K\,S}{w_n^{\,2}\left[\dfrac{S^2}{w_n^{\,2}}+\dfrac{2x\,S}{w_n}+1\right]}$.
- For $w = 0.1 << w_n$,

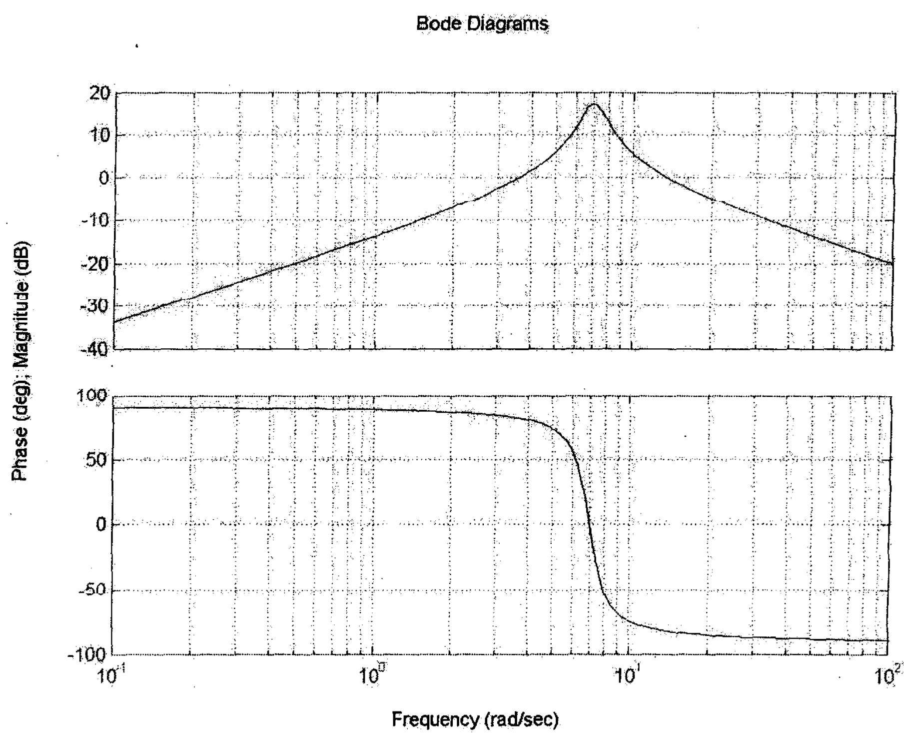
$$20\log|FRF| \equiv 20\log(\frac{K}{7^2}) + 20\log(w) - 20\log\sqrt{\left[1-\left(\frac{w}{7}\right)^2\right]^2+\left[\frac{0.2w}{7}\right]^2}$$

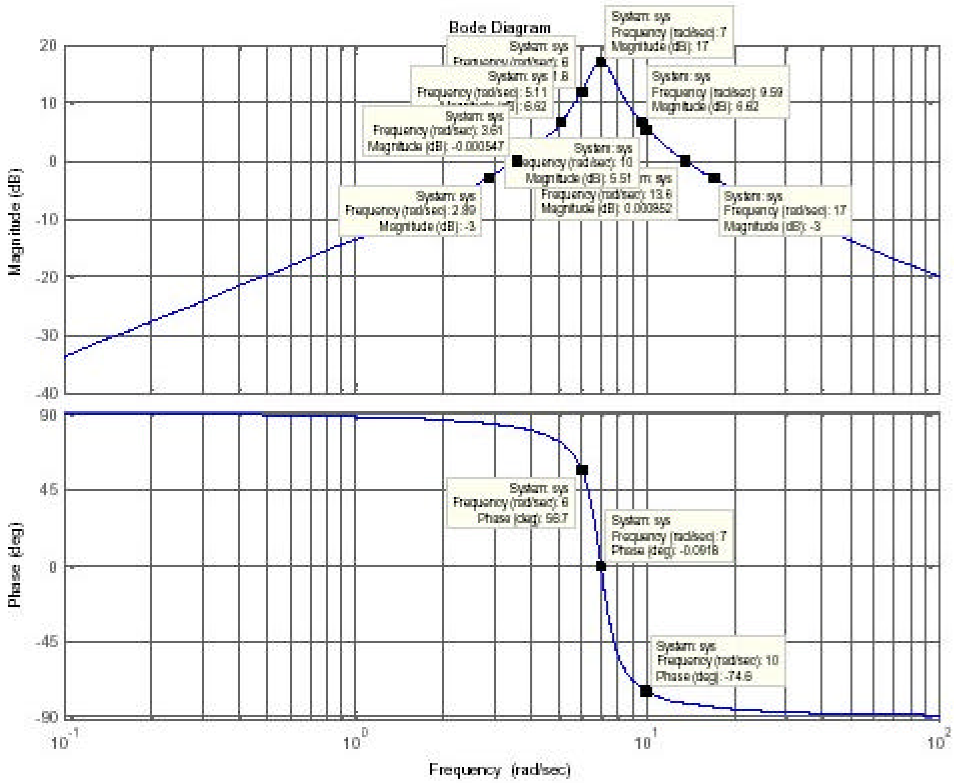
$$\cong 20\log(\frac{K}{7^2}) + 20\log(0.1) - 20\log(1) \equiv 20\log(\frac{K}{7^2}) - 20 \cong -34\,(db)$$

$$20\log(\frac{K}{7^2}) = -14\,(db), \Rightarrow K \cong 9.8. \quad \Rightarrow T.F. \cong \frac{9.8S}{S^2+1.4S+49}.$$

- Note: The actual plot was generated by $T.F. \cong \dfrac{10S}{S^2+1.4S+49}$.

2) What is the output if the input is u = 3 * sin (6t)?
3) What input will produce an output of y = 10 * sin (10t)?
4) At what frequency(ies) (if any) is the output magnitude equal to the input magnitude?
5) Assume that the input to the system is u = 7 * sin (w*t) , and w can vary over the range shown in the Bode plot. At what frequency (if any) is the magnitude of the output the greatest?
6) Assume that the input to the system is u = 7 * sin (w*t) , and w can vary over the range shown in the Bode plot. At what frequency (if any) is the magnitude of the output equal to 15?
7) Assume that the input to the system is u = 24 * sin (w*t) , and w can vary over the range shown in the Bode plot. At what frequency (if any) is the output magnitude exactly one-tenth the size of the greatest output magnitude?
8) At what frequency (if any) is the output exactly out of phase with the input?
9) At what frequency (if any) is the output exactly in phase with the input?
10) What is the approximate time constant of the system?






- ❖ $Output = 3 * 10^{\frac{11.8}{20}} \sin(6t + 56.7^{\circ}) \cong 11.67 \sin(6t + 56.7^{\circ})$

- ❖ $Input = 10^{1-\frac{5.49}{20}} \sin(10t + 74.6^{\circ}) \cong 5.315 \sin(10t + 74.6^{\circ})$

- ❖ $Find\ 0\ (db),\ w \cong 3.6\ \&\ 13.6\ (rad/\sec)$

- ❖ $Peak\ happen\ @\ w \cong 7\ (rad/\sec)$

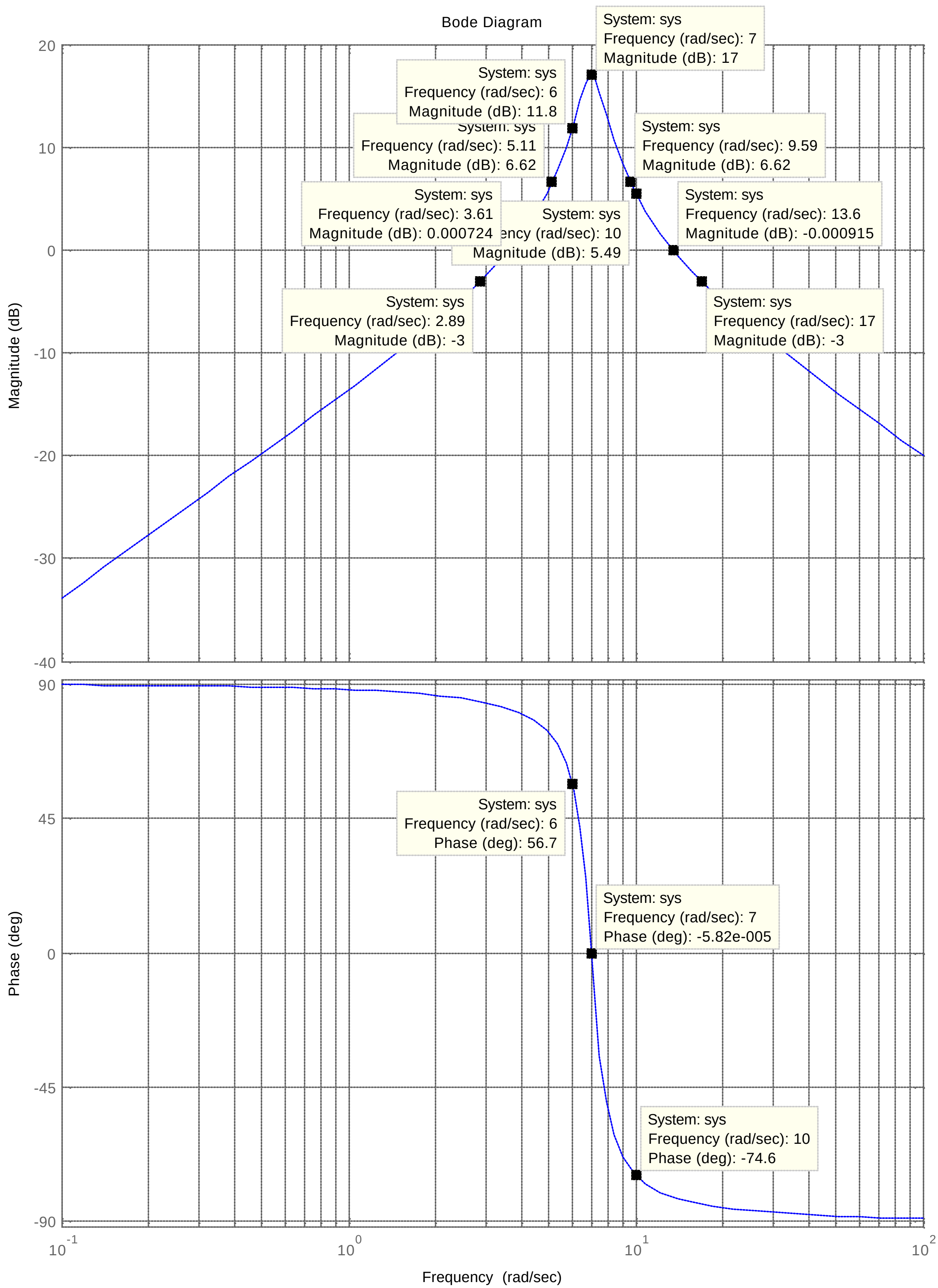
- ❖ $20\log(\frac{15}{7}) \cong 6.62\,(db), w \cong 5.1\ \&\ 9.6\,(rad/\sec)$

- ❖ $-20\ (db)\ \ from\ the\ peak, \Rightarrow 17 - 20 = -3\ (db),\ w \cong 2.9\ \&\ 17\ (rad/\sec)$

- ❖ From the plot, there are NO exactly out of phase (180 degrees difference)

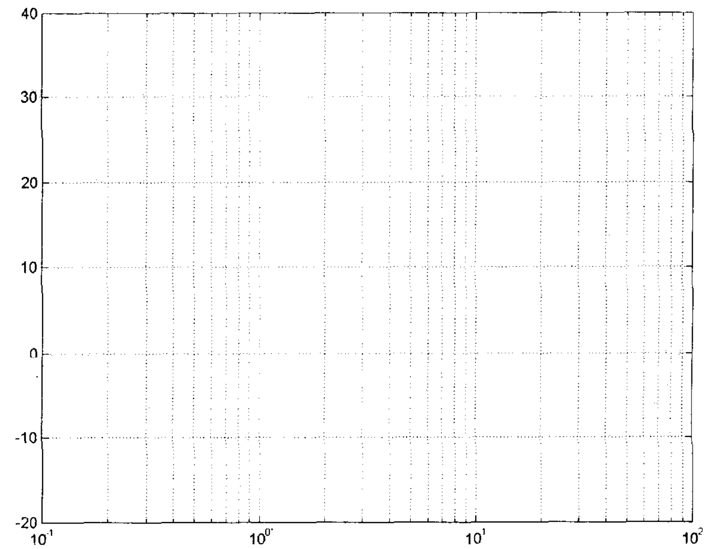
- ❖ Exactly in phase (zero degree difference), $w \cong 7\ (rad/\sec)$

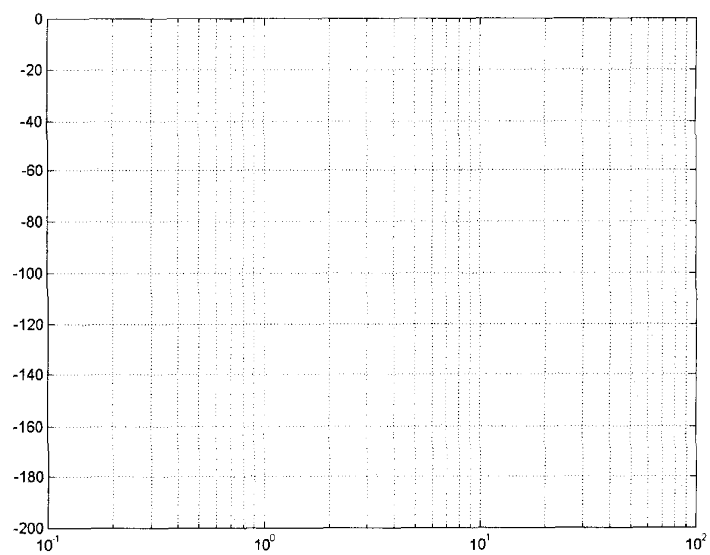
- ❖ $TimeCons\tan t\ \ t = \frac{1}{x w_n} = \frac{1}{0.1 \times 7} \cong 1.43\,(\sec)$

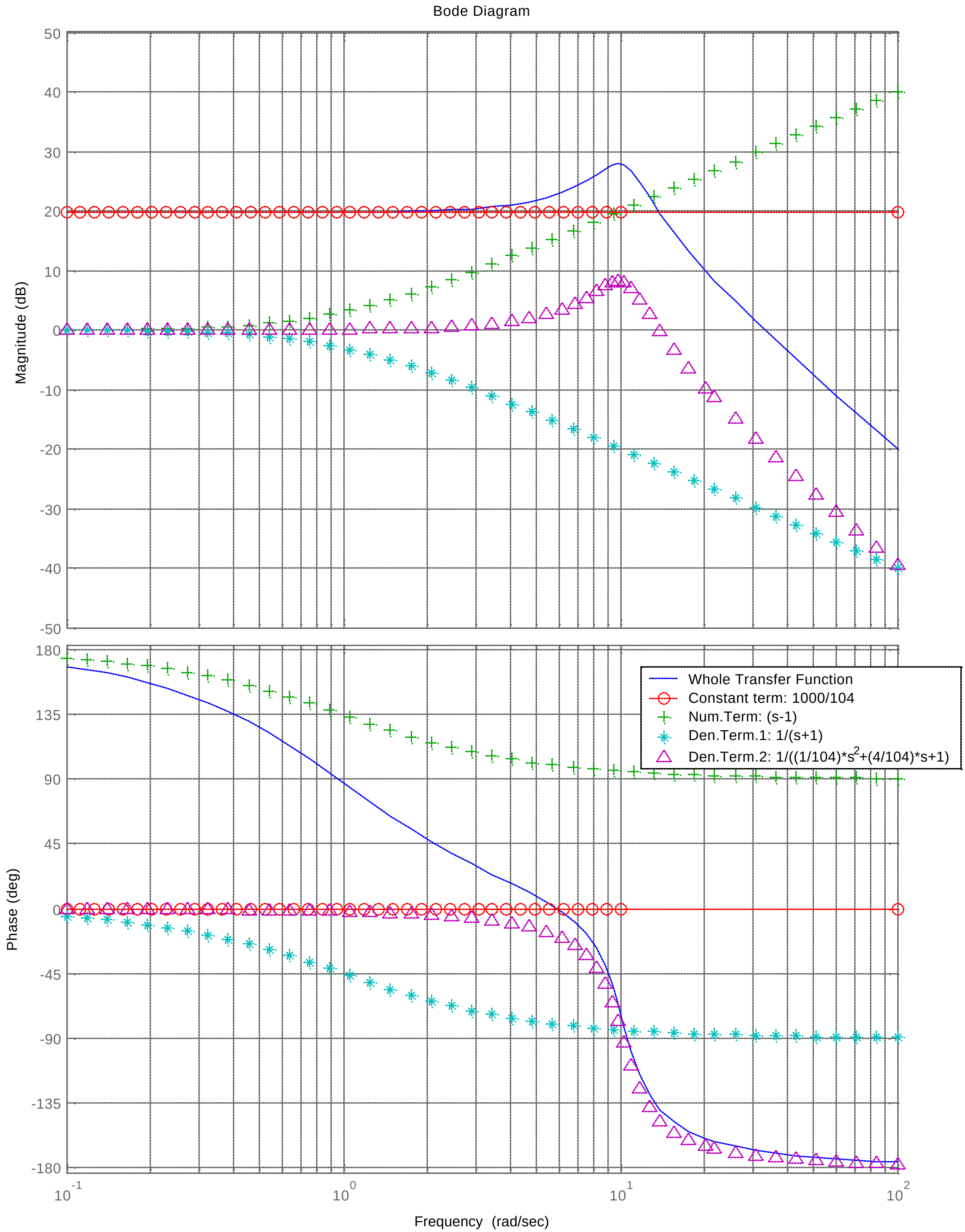


Bode Diagram
Magnitude (dB)
Phase (deg)
Frequency (rad/sec)
System: sys
Frequency (rad/sec): 7
Magnitude (dB): 17
System: sys
Frequency (rad/sec): 6
Magnitude (dB): 11.8
System: sys
Frequency (rad/sec): 5.11
Magnitude (dB): 6.62
System: sys
Frequency (rad/sec): 9.59
Magnitude (dB): 6.62
System: sys
Frequency (rad/sec): 3.61
Magnitude (dB): 0.000724
System: sys
Frequency (rad/sec): 10
Magnitude (dB): 5.49
System: sys
Frequency (rad/sec): 13.6
Magnitude (dB): -0.000915
System: sys
Frequency (rad/sec): 2.89
Magnitude (dB): -3
System: sys
Frequency (rad/sec): 17
Magnitude (dB): -3
System: sys
Frequency (rad/sec): 6
Phase (deg): 56.7
System: sys
Frequency (rad/sec): 7
Phase (deg): -5.82e-005
System: sys
Frequency (rad/sec): 10
Phase (deg): -74.6

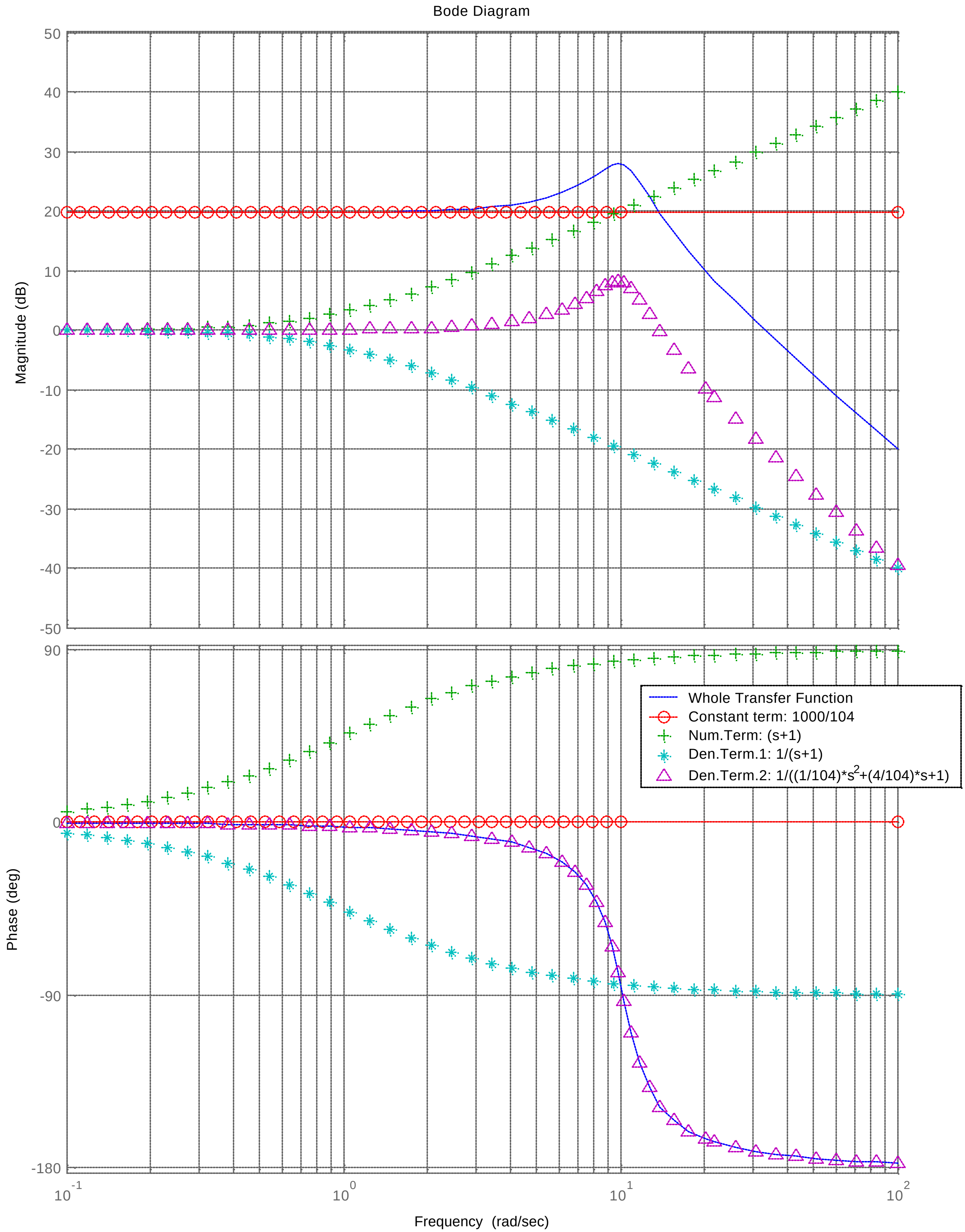
30 points: Plot (by hand) the Bode magnitude and phase angle plot for the system with a zero at $s=1$ and poles at $s=-1, \quad -2 \pm i*10$, all multiplied by a constant of 1000.



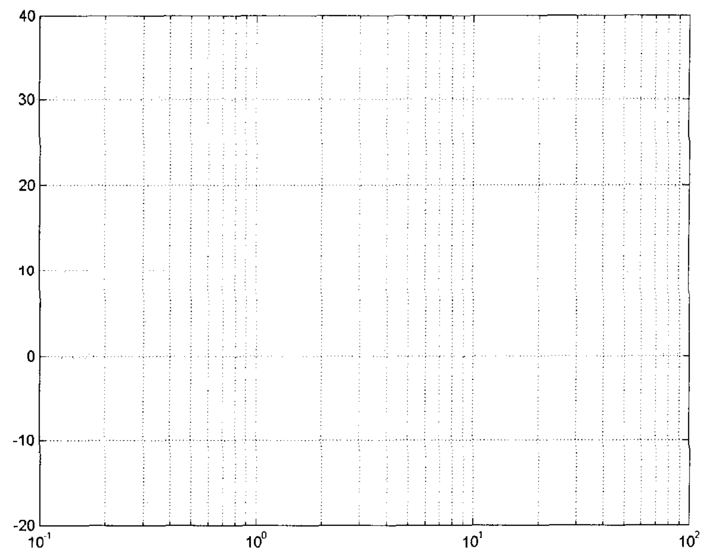



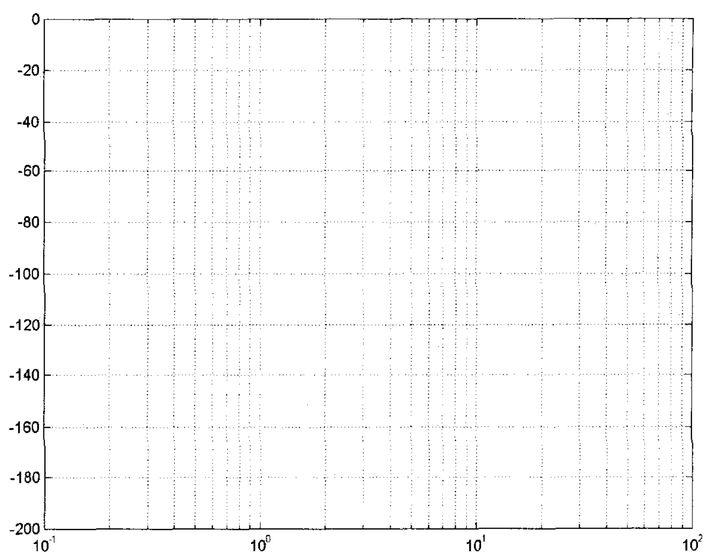


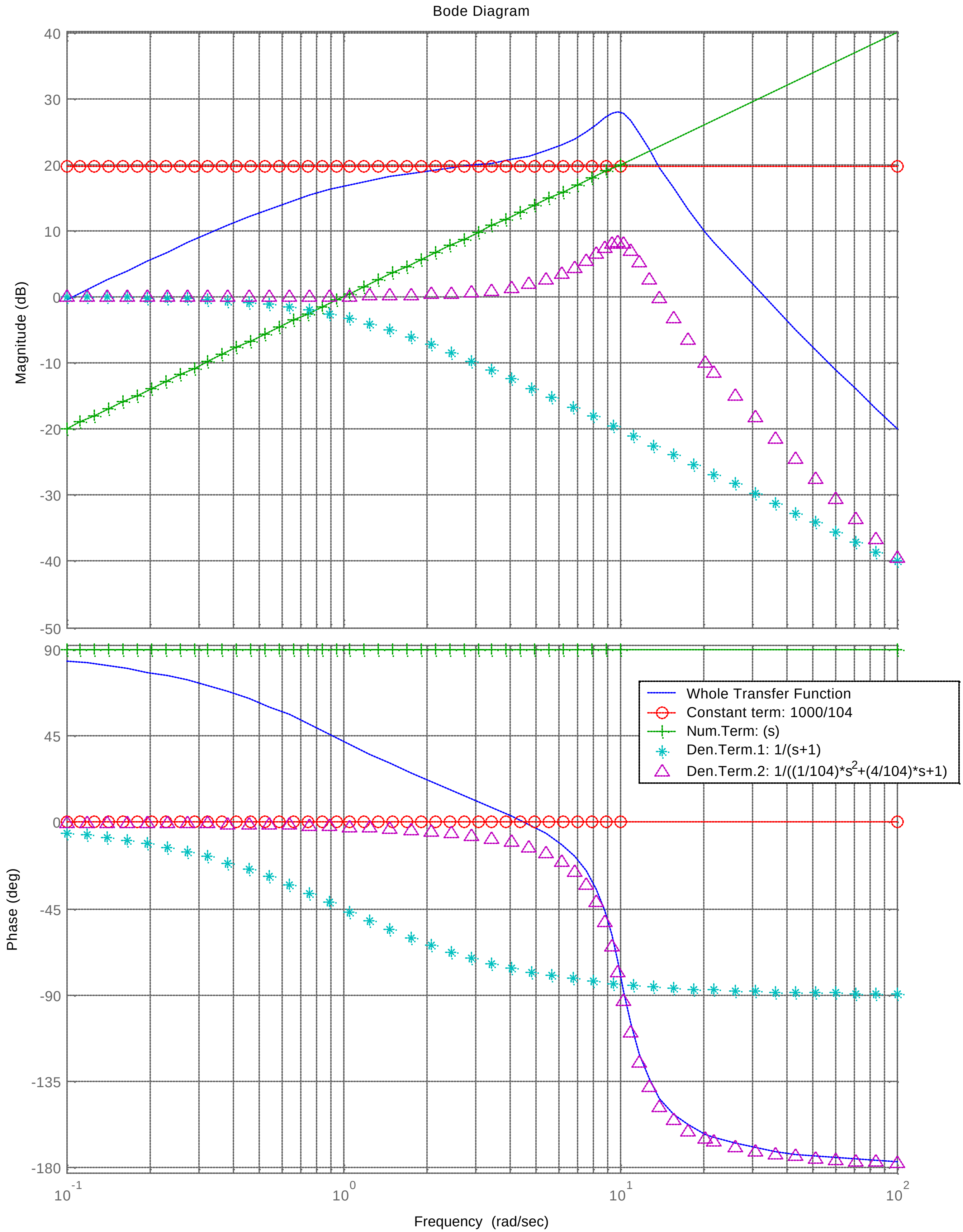
Bode Diagram
Magnitude (dB)
Phase (deg)
Frequency (rad/sec)
Whole Transfer Function
Constant term: 1000/104
Num.Term: (s-1)
Den.Term.1: 1/(s+1)
Den.Term.2: 1/((1/104)*s^2+(4/104)*s+1)


Bode Diagram
Magnitude (dB)
Phase (deg)
Frequency (rad/sec)
Whole Transfer Function
Constant term: 1000/104
Num.Term: (s+1)
Den.Term.1: 1/(s+1)
Den.Term.2: 1/((1/104)*s^2+(4/104)*s+1)

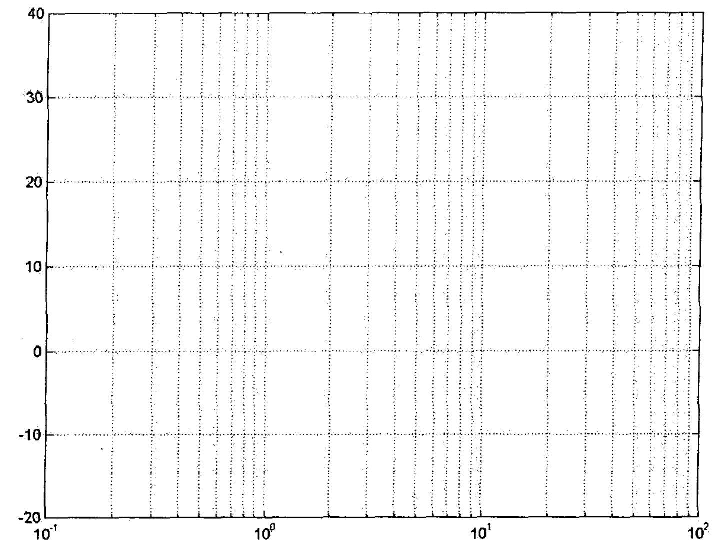
30 points: Plot (by hand) the Bode magnitude and phase angle plot for the system with a zero at $s = 0$ and poles at $s = -1, \quad -2 \pm i * 10$, all multiplied by a constant of 1000.

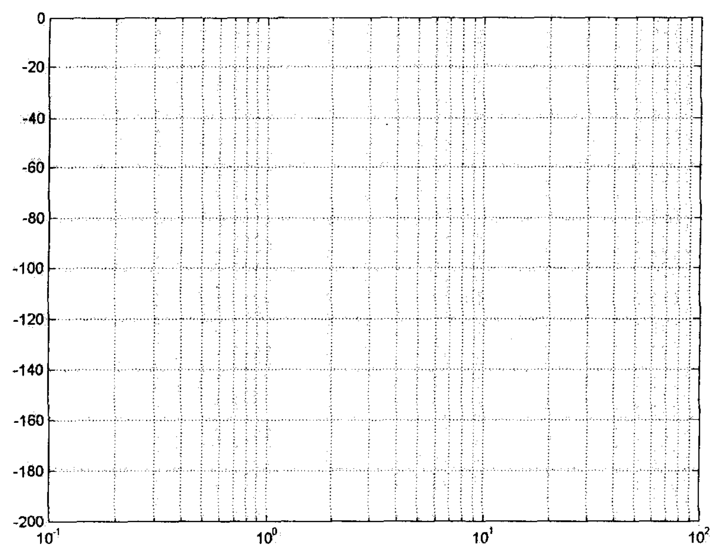


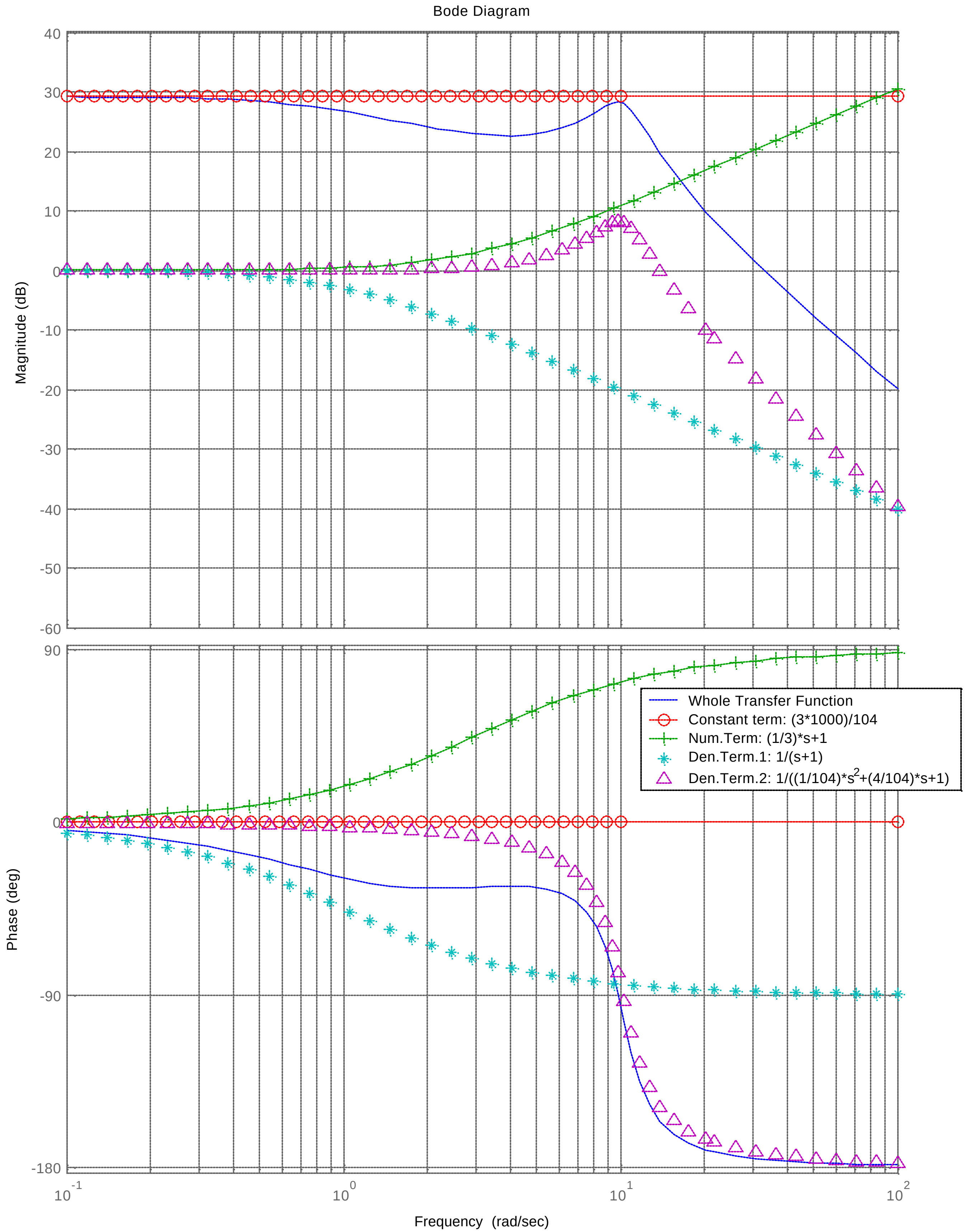




Bode Diagram
Magnitude (dB)
40
30
20
10
0
-10
-20
-30
-40
-50
Phase (deg)
90
45
0
-45
-90
-135
-180
Whole Transfer Function
Constant term: 1000/104
Num.Term: (s)
Den.Term.1: 1/(s+1)
Den.Term.2: 1/((1/104)*s^2+(4/104)*s+1)
10^-1
10^0
10^1
10^2
Frequency (rad/sec)

30 points: Plot (by hand) the Bode magnitude and phase angle plot for the system with a zero at $s=-3$ and poles at $s=-1, \quad -2 \pm i*10$, all multiplied by a constant of 1000.





Bode Diagram
Magnitude (dB)
40
30
20
10
0
-10
-20
-30
-40
-50
-60
Phase (deg)
90
0
-90
-180
Whole Transfer Function
Constant term: (3*1000)/104
Num.Term: (1/3)*s+1
Den.Term.1: 1/(s+1)
Den.Term.2: 1/((1/104)*s2+(4/104)*s+1)
10-1
100
101
102
Frequency (rad/sec)