SYS 336: Test III

April 30, 2002

Answer Key

Name

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1) Find the transfer function $\frac{X(s)}{F(s)}$ for the system modeled by $3\ddot{x}+2\dot{x}+4x=5f(t)$

$$\boxed{\frac{5}{3s^2+2s+4}=\frac{X}{F}}$$

2) Find the transfer function $\frac{Y(s)}{F(s)}$ if the output is $y(t)=4\ddot{x}+2\dot{x}$, for the system:

$\ddot{x}+6\dot{x}+5x=f(t)+\frac{df(t)}{dt}$

$$\frac{X}{F}=\frac{s+1}{s^2+6s+5}$$

$$\frac{Y}{F}=\frac{4s^2+4s+2s+1}{s^2+6s+5}=\frac{4s^2+6s+1}{s^2+6s+5}$$

For questions (3)-(8), consider the system transfer function $\frac{Y(s)}{F(s)}=\frac{s^2+2s}{3s^2+4s+5}$

3) Find $y(t=0^+)$ immediately after a step input is applied, $f(t=0)=6$, if both $y=0$ and $f=0$ prior to the step input.

$$6\cdot\lim_{s\to\infty}\frac{Y}{F}=6\cdot\frac{1}{3}=2$$

4) Find $y(t\to\infty)$ for the same conditions as in question (3)

$$6\cdot\lim_{s\to 0}\frac{Y}{F}=0$$

5) Find the Frequency Response Function of the system

$$\text{FRF}=\frac{-\omega^2+2i\omega}{(5-3\omega^2)+i4\omega}\cdot\frac{5-3\omega^2-i4\omega}{(5-3\omega^2)-i4\omega}=\frac{[-\omega^2(5-3\omega^2)+8\omega^2]+i[2\omega(5-3\omega^2)+4\omega^3]}{(5-3\omega^2)^2+(4\omega)^2}$$

$$=\frac{(3\omega^2+3\omega^4)+i[-2\omega^3+10\omega]}{(5-3\omega^2)^2+(4\omega)^2}$$

6) Find the system's zeroes (if any)

Zeroes: $s=0,\ -2$

7) Find the system's poles (if any)

Poles: $s=\frac{-4\pm\sqrt{16-60}}{6}=-\frac{2}{3}\pm i\frac{\sqrt{46}}{6}$

8) Find the output y(t) of the system if the input is $f(t)=4\sin 5t$

*Work out: $\omega=5$ into FRF from Q5

Output $=4\cdot\text{Mag}\,\sin(5t+\phi)$ Phase $=\tan^{-1}\frac{\text{Im}}{\text{Re}}=\phi$

For questions (9)-(14), consider the system transfer function $\frac{Y(s)}{F(s)}=\frac{s+3}{s^2+2s+25}$

9) Find $y(t=0^+)$ immediately after a step input is applied, $f(t=0)=1$, if both $y=0$ and $f=0$ prior to the step input.

$$\lim_{s\to\infty} \frac{Y}{F} = 0$$

10) Find $y(t\to\infty)$ for the same conditions as in question (9)

$$\lim_{s\to 0} \frac{Y}{F} = \frac{3}{25}$$

11) Find the Frequency Response Function of the system

$$FRF = \frac{i\omega+3}{-\omega^2+2i\omega+25} \cdot \frac{(25-\omega^2)-i2\omega}{(25-\omega^2)-i2\omega} = \frac{\left[3(25-\omega^2)+2\omega^2\right]+i\left[\omega(25-\omega^2)-6\omega\right]}{(25-\omega^2)^2+(2\omega)^2}$$

$$= (\text{May be simplified})$$

12) Find the system's natural frequency

$$s^2+2\zeta\omega_n s+\omega_n^2 = s^2+2s+25 \quad \therefore \omega_n = 5$$

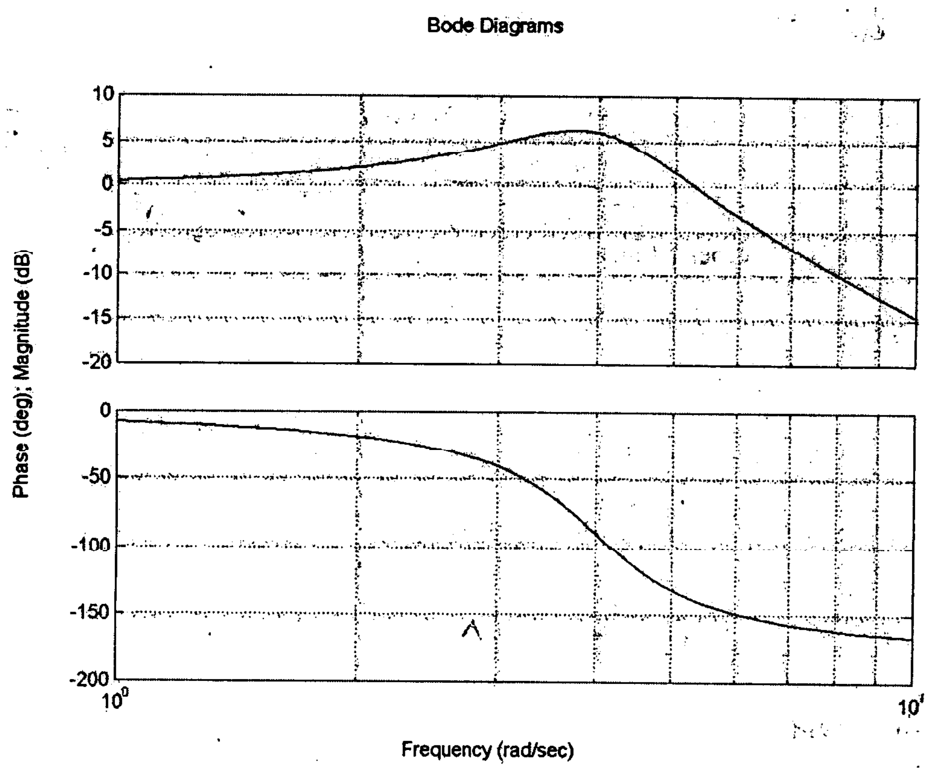
13) Find the system's damping ratio

$$2\zeta\omega_n = 2, \quad \zeta = 0.1$$

14) Find the output y(t) of the system if the input is f(t) = $4\sin 5t$

Substitute $\omega=5$ into Q11; Output $= 4\cdot \text{Mag} \sin(5t+\phi)$, $\phi = \tan^{-1}\frac{Im}{Re}$

For questions (15-21), refer to the Bode plots given below:



15) Find the steady-state expression for the output if the input is $u = 5\sin(8*t)$

At $\omega=8$, Mag $= -10$ db and Phase $\approx -165°$ $\quad 20\log_{10}\frac{y}{u} = -10$, $\frac{y}{u} = 10^{-1/2} = \frac{1}{\sqrt{10}}$

$$y(t) = \frac{1}{\sqrt{10}} \cdot 5\sin(8t - 165°)$$

16) At what input frequency(ies) (if any) is the output magnitude equal to the input magnitude? $\quad 0$ db $\Rightarrow$ $\omega \approx 5.2$ rad/sec

17) At what input frequency(ies) (if any) is the phase angle -90 degrees?

$\omega \approx 4$ rad/sec

18) Assume that the input to the system is $u = 8\sin(\omega * t)$, and that ω can vary over the range shown in the Bode plot. What is the greatest possible output magnitude?

$20\log_{10}\frac{y}{8} \cong 7$ db $\quad \therefore \quad y = 8 \cdot 10^{7/20}$

19) At what input frequency (if any) is the output magnitude approximately one-tenth the size of the greatest output magnitude?

-20 db from peak at about $\omega \cong 9.5$ rad/sec

20) This system contains no zeroes and two poles. What is the most accurate description of the system's damping ratio?

(a) $\varsigma > 1$ (b) $1 > \varsigma > 0.707$ (c) $0.707 > \varsigma > 0$ (d) $\varsigma = 0$

21) For what range of input frequencies (if any) does the system act as an amplifier?

$1 \leq \omega \leq \sim 5.2$ rad/sec

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22) Write and expression for the transfer function which has zeroes at (-6) and (-4), and poles at (-2 +/- 4i) , (-5).

$$TF = \frac{(s+6)(s+4)}{(s+2+4i)(s+2-4i)(s+5)} = \frac{(s+6)(s+4)}{(s^2+4s+20)(s+5)}$$

23) Write a differential equation model for a system whose transfer function X(s)/F(s) is

$$\frac{7}{s^2+4s+3}$$

$$\ddot{x} + 4\dot{x} + 3x = 7f(t)$$

24) You get 4 points for showing up. ✓

25) Find the transfer function matrix for the state-space system:

$$A = \begin{bmatrix} -4 & -1 \\ -3 & -8 \end{bmatrix}, \quad B = \begin{pmatrix} 2 \\ 5 \end{pmatrix}, \quad C = \begin{bmatrix} 9 & 1 \\ 3 & 6 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & \cancel{4} \\ 3 & \cancel{2} \end{bmatrix}$$

Please work out

$$C[sI-A]^{-1}B + D$$