Problem Set 9 (PS9) due Monday April. 2

7.25 8.17 8.23

$T = f(D, N, V, \rho)$

$T = f(D^a, N^b, V^c, \rho^d)$

<u>8-17</u>

unit exponents of parameters

		T	D	N	V	ρ	← Parameters
Units ↕	M	1	0	0	0	1	
	L	1	1	0	1	-3	↖ Exponents
	T	-2	0	-1	-1	0	page 237

5 parameters - 3 units

expect 2 dimensionless parameters

solve for a, c, d, in terms of b,

$\sum \text{exponents} = 0$

find : $\begin{pmatrix}\text{first}\\ \text{dimensionless}\\ \text{number}\end{pmatrix}^{\text{integer}}, \begin{pmatrix}\text{second}\\ \text{dimensionless}\\ \text{number}\end{pmatrix}^{b}$

M units : $\boxed{1 = d}$

L units : $1 = a + c - 3d$

T units : $-2 = -b - c$

from T, $\boxed{c = 2 - b}$

from L $1 = a + 2 - b - 3$

$\boxed{a = 2 + b}$

a)

$$T = f\left((D)^a (N)^b (V)^c (\rho)^d\right)$$

$$T = f(\ )^{\text{integer}}(\ )^b$$

$$T = f\left((D)^{a=(2+b)} (N)^{b=b} (V)^{c=2-b} (\rho)^{d=1}\right)$$

$$T = f(\ )^{\text{integer}}(\ )^b$$

$$T = \left(D^2 V^2 \rho\right)\left(\frac{DN}{V}\right)^b$$

$$\frac{T}{D^2 V^2 \rho} = f\left(\frac{DN}{V}\right)$$

b)

$$\frac{T_1}{V_1^2 D_1^2 \rho_1} = \frac{T_2}{V_2^2 D_2^2 \rho_2}$$

$$\left(\frac{D_2}{D_1}\right) = \left(\frac{T_2}{T_1}\right)\left(\frac{V_1}{V_2}\right)^2$$

$$V_1 = V_2, T_1 = 2T_2$$

$$\left(\frac{D_2}{D_1}\right)^2 = \left(\frac{T_2}{2T_2}\right)\left(\frac{V_2}{V_2}\right)^2$$

$$D_2 = D_1 / \sqrt{2}$$

$$\frac{N_1 D_1}{V_1} = \frac{N_2 D_2}{V_2}$$

$$\frac{N_2}{N_1} = \frac{V_2}{V_1}\frac{D_1}{D_2}$$

$$\frac{N_2}{N_1} = 1 \times \sqrt{2}$$

$$N_2 = N_1 \sqrt{2}$$

c) Power = thrust × velocity

$$= T_1 \times V_1 = 2T_2 \times V_2$$

BY INSPECTION

		T	D	N	V	ρ	Parameters
	M	1	0	0	0	1	
Units	L	1	1	0	1	-3	
	T	-2	0	-1	-1	0	

Thrust T is major variable

T $\quad \frac{ML}{T^2}$

density is the only other source of M

$\frac{T}{\rho}$ $\quad \frac{ML}{T^2}\frac{L^3}{M} = \frac{L^2}{T^2} \Rightarrow$ divide by V^2

$\frac{T}{\rho V^2}$ $\quad \frac{ML}{T^2}\frac{L^3}{M}\frac{T^2}{L^2} = L^2 \Rightarrow$ multiply by D^2

$\frac{T}{\rho V^2 D^2}$ $\quad \frac{ML}{T^2}\frac{L^3}{M}\frac{T^2}{L^2}\frac{1}{L^2} = 1$

8.17

$(a) T = f(d, N, V, \rho)$

Rank of matrix is 3. Number of

$\Pi - product = 5 - 3 = 2.$

By inspection:

$$\frac{\mathrm{T}}{\rho V^2 d^2} = f\left(\frac{Nd}{V}\right)$$

$(b) For$ dynamic similarity:

$$\frac{\mathrm{T}_1}{\rho_1 V_1^2 d_1^2} = \frac{\mathrm{T}_2}{\rho_2 V_2^2 d_2^2}$$

$$V_1 = V_2, \rho_1 = \rho_2, \mathrm{T}_1 = 2T_2$$

$$\therefore \left(\frac{d_2}{d_1}\right)^2 = \left(\frac{\mathrm{T}_2}{\mathrm{T}_1}\right)\left(\frac{V_1}{V_2}\right)^2 = \frac{1}{2}$$

$$\therefore d_2 = \frac{d_1}{\sqrt{2}}$$

$$Also, we\ require: \frac{N_1 d_1}{V_1} = \frac{N_2 d_2}{V_2}$$

$$\therefore N_2 = \left(\frac{d_1}{d_2}\right)\left(\frac{V_2}{V_1}\right) N_1 = \sqrt{2} N_1$$

$(c) Power = thrust \times velocity$

$$Power1 = \mathrm{T}_1 \times V_1$$

$$Total\ power2 = 2\mathrm{T}_2 \times V_2 = \mathrm{T}_1 \times V_1 = Power1$$

8.23

$(a) T = f(D, k, V, \rho, \omega)$

Rank of matrix is 3. Number of

$\Pi - product = 6 - 3 = 3.$

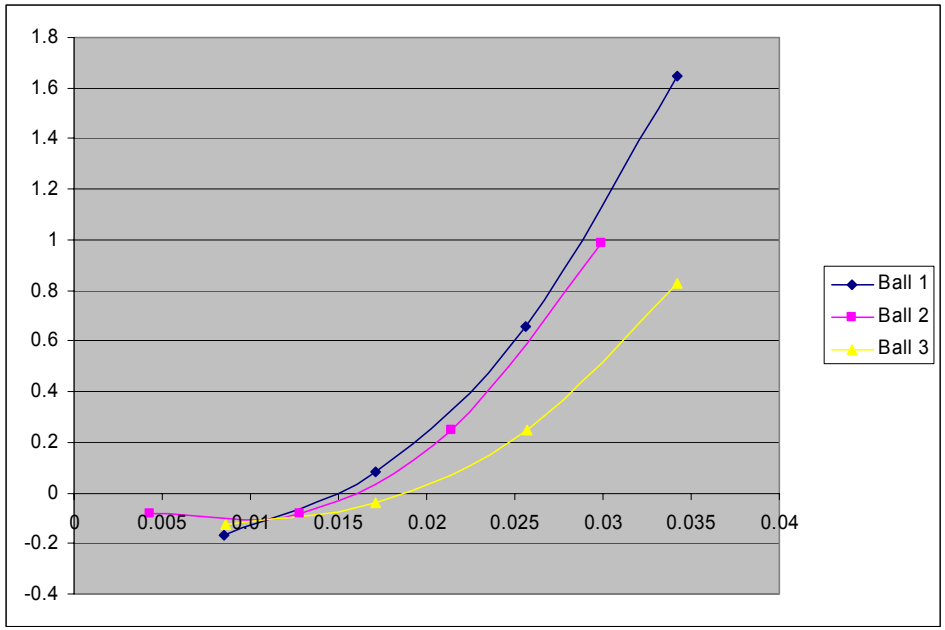
(b) *By* inspection:

$$\frac{\mathrm{L}}{\frac{1}{2}\rho V^2 D^2} = f\left(\frac{h}{D}, \frac{\omega D}{V}\right)$$

Then use data to form nondimensional groups:

Ball 1 k/D=0	$\frac{\omega D}{V}$	0.0085	0.0171	0.0256	0.0342
	$\frac{L}{\frac{1}{2}\rho V^1 D^1}$	-0.165	0.082	0.658	1.645
Ball 2 k/D=0.023	$\frac{\omega D}{V}$	0.0043	0.0128	0.0214	0.0299
	$\frac{L}{\frac{1}{2}\rho V^1 D^1}$	-0.084	-0.084	0.249	0.987
Ball 3 k/D=0.094	$\frac{\omega D}{V}$	0.0086	0.0171	0.0257	0.0342
	$\frac{L}{\frac{1}{2}\rho V^1 D^1}$	-0.124	-0.041	0.248	0.826

8.23



The principal effects of roughness seem to be to decrease the lift coefficient at high spin rates. Note also the negative lift coefficients at low spin rates. This spin resersal is seem in experiments but it is not well understood. It seems likely to be a low Reynolds number phenomenon. We have not considered Reynolds number in plotting these data, so that these effects are not known from this data.