Problem Set 8 (PS8) due Monday March. 25

7.11 7.14 7.21 7.22

7.11

$$\phi = x^2 + y^2$$

$$u = \frac{\partial \phi}{\partial x} = 2x = \frac{\partial \varphi}{\partial y}$$

$$v = \frac{\partial \phi}{\partial y} = -2y = -\frac{\partial \varphi}{\partial x}$$

$$\therefore \varphi = 2xy + f(x)$$

$$and \;\; \varphi = 2xy + f'(y)$$

$$general \;\; solution \;\; \varphi = 2xy$$

7.14

$$(a)u = hy = \frac{\partial \varphi}{\partial y}$$

$$v = 0 = -\frac{\partial \varphi}{\partial x}$$

$$\therefore \varphi = \frac{1}{2}hy^2 + g(x)$$

$$and \ \ \varphi = f(y)$$

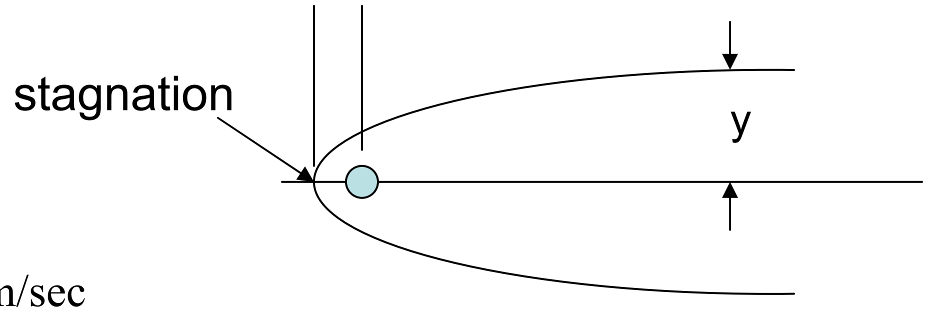
$$General \ solution: \varphi = \frac{1}{2}hy^2$$

$$(b)\nabla \times V = (\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y})h$$

$$= (0 - h)h \neq 0$$

$$\therefore Flowfield \ is \ rotational.$$

7-21



Uniform flow $U_\infty = 2\text{m/sec}$

Source Flow $q = 4\text{m}^2/\text{sec}$

$$\Psi = \Psi_{\substack{\text{uniform}\\\text{flow}}} + \Psi_{\text{source}}$$

a) $$\Psi = U_\infty r \sin\theta + \frac{q}{2\pi}\theta$$

$$u_r = \frac{1}{r}\frac{\partial \Psi}{\partial \theta} = U_\infty \cos\theta + \frac{q}{2\pi r}$$

$$u_\theta = -\frac{\partial \Psi}{\partial r} = -U_\infty r \sin\theta$$

at stagnation $u_r = 0,\ u_\theta = 0$

$u_\theta = 0$ where $\theta = 0$ and π

$$u_r = 0 \text{ where } r = \frac{-Q}{2\pi U_\infty \cos\theta}$$

$$r = \frac{-4\text{m}^2/\text{sec}}{2\pi\pi 2\text{m/seccs}(0)} = -\frac{1}{\pi} = -.318\text{m}$$

stagnation at $\theta = 0$, $r = -.318$ m from source center

b) Ψ_{surface} passes thru the stagnation point

$\Psi_{\text{surface}} = \Psi$ at $\theta = 0$

$$\Psi = U_\infty r \sin\theta + \frac{q}{2\pi}\theta$$

$$\Psi_{\text{surface}} = \frac{q}{2\pi}\pi = \frac{q}{2}$$

$$U_\infty r \sin\theta + \frac{q}{2\pi}\theta = \frac{q}{2}$$

since, $y = r\sin\theta$

$$U_\infty y = \frac{q}{2} - \frac{q}{2\pi}\theta$$

at $\theta \Rightarrow 0$

$$y = \frac{q}{2U_\infty} = \frac{4\text{m}^2/\text{sec}}{2\times 2\text{ m/sec}} = 1\text{m}$$

the body thickness is $2\,y = 2\text{m}$

d)Bernoulli'sEquation

maximum pressure

$$p_\infty + \frac{\rho U_\infty^2}{2} = p \ \ + \frac{\rho U^2}{2} = p \ \ + \frac{\rho\left(u_r^2 + u_\theta^2\right)}{2}$$

$$p - p_\infty = \frac{\rho}{2}\left(U_\infty^2 - \left(u_r^2 + u_\theta^2\right)\right)$$

$$p - p_\infty = \max \text{at}\left(u_r^2 + u_\theta^2\right) = 0, \text{stagnation point}$$

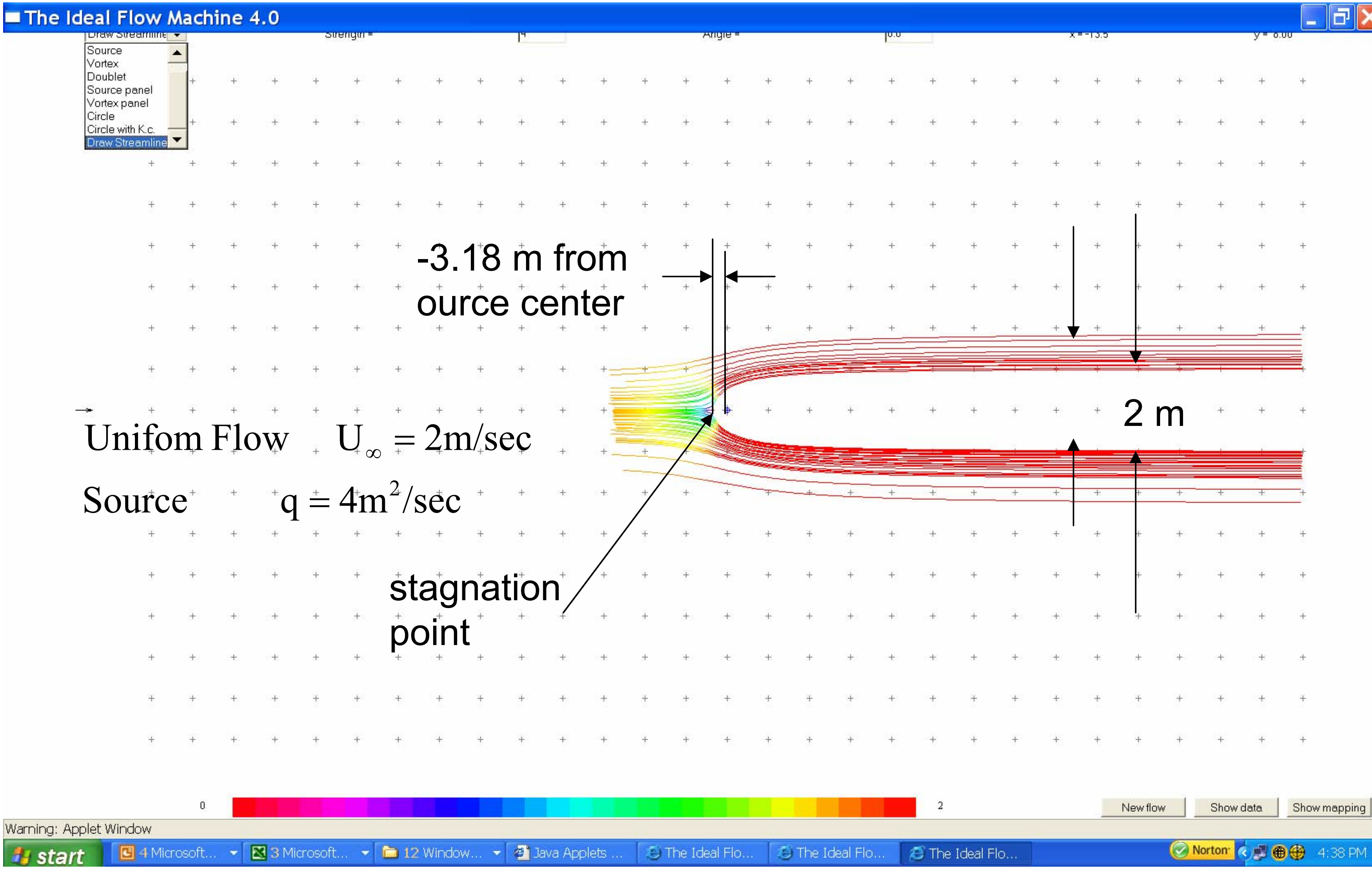
$$p - p_\infty = \frac{1.204}{2} \times (2\text{m}/\text{sec})^2 = 2.\infty 208\text{Pa}$$

minimum pressure

along the surface $u_r = 0$

$$u_\theta = -U_\infty \sin\theta$$

$$u_\theta^2 = U_\infty^2 \sin^2\theta \ \ \text{maximum at} \ \ \theta = \frac{\pi}{2}, p_{min} = p_\infty$$


The Ideal Flow Machine 4.0
Source
Vortex
Doublet
Source panel
Vortex panel
Circle
Circle with K.c.
Draw Streamline
-3.18 m from
ource center
2 m
Unifom Flow $U_\infty = 2$ m/sec
Source $q = 4$ m^2/sec
stagnation
point
0
2
New flow
Show data
Show mapping
Warning: Applet Window
start
4:38 PM