Problem Set 7 (PS7) due Monday March. 19

6.16 6.37

6.16

$$V = 2xi + 5yz^2 j - t^3 k$$

$$\nabla \cdot V = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 2 + 5z^2$$

The divergence of the velocity is impotant because when it equals to 0, the flowfield is incompressible, For the case given here

$\nabla \cdot V \neq 0$(for any value of z)

6.37

Incompressible N-S equation:

$$\rho \cdot \frac{\mathrm{DV}}{Dt} = -\nabla p + \rho g + \mu \nabla^2 V$$

Pressure is constant everywhere; ignore gravity:

$$\rho \cdot \frac{\mathrm{DV}}{Dt} = \mu \nabla^2 V$$

Now V=ui where u=u(y)

$$\therefore \frac{\mathrm{DV}}{\mathrm{Dt}} = i[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}]$$

$$= i[0 + u(0) + v\frac{\partial u}{\partial y} + 0(0)]$$

$$\textit{By}\ \text{contimuity, } \nabla \cdot \mathrm{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\textit{Since}\ \frac{\partial u}{\partial x} = 0, v = cons\tan t.$$

At y=0,v=0 $\rightarrow$ v=0 everywhere.

$$\text{Hence}\ \frac{\mathrm{DV}}{\mathrm{Dt}} = 0.$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = i[0 + \frac{\partial^2 u}{\partial y^2} + 0]$$

$$\textit{Since}\ \mathrm{u} = \mathrm{U}\frac{\mathrm{y}}{\mathrm{h}}, \frac{\partial^2 u}{\partial y^2} = 0.$$

Hence $\nabla^2 V = 0.$

Therefore, *linear* couette flow is an exact solution to the N-S equation.