Problem Set 6 (PS6) due Monday March. 5

6.11 6.19

6.11

From continuity equation $\frac{D\rho}{Dt} = -\rho(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z})$

(*a*) $\frac{D\rho}{Dt}$ directly: $\frac{D\rho}{Dt} = \frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} + w\frac{\partial \rho}{\partial z}$

$$= \frac{1}{\mathrm{x}} + \frac{4\mathrm{x}}{\mathrm{t}}(-\frac{\mathrm{t}}{\mathrm{x}^2}) + \frac{y^2}{t}\cdot 0 + 0$$

$$= -\frac{3}{\mathrm{x}}$$

(*b*) $\frac{D\rho}{Dt}$ from continuity equation:

$$\frac{D\rho}{Dt} = -\rho(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z})$$

$$= -(3+\frac{\mathrm{t}}{\mathrm{x}})(\frac{4}{\mathrm{t}} + \frac{2\mathrm{y}}{\mathrm{t}})$$

$$= -\frac{12}{\mathrm{t}} - \frac{6y}{t} - \frac{4}{\mathrm{x}} - \frac{2\mathrm{y}}{\mathrm{x}}$$

Answers to (a) and (b) do not degree:This flowfield is not possible.(mass is not conserved.)

6.19

$$V = -2x^2 i + 4xyj + 3k$$

$$(a)\ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = -4x + 4x + 0 = 0$$

$\therefore$ *Hence* the flowfield is incompressible.

$$\text{(b)}\ \frac{DV}{Dt} = \frac{\partial V}{\partial t} + u\frac{\partial V}{\partial x} + v\frac{\partial V}{\partial y} + w\frac{\partial V}{\partial z}$$

$$= 0 + (-2x^2)(-4xi + 4yj) + (4xy)(4xj) + (3)(0)$$

$$= i(8x^3) + j(16x^2 y - 8x^2 y) = 8x^3(i + \frac{y}{x} j)$$

$$at\ (1,3,0)\ \frac{DV}{Dt} = 8(i + 3j).$$

(c) Volume flux through area A=$\int (\mathrm{n} \cdot \mathrm{v}) dA$

$$\therefore = \int j \cdot \mathrm{v} dA = \iint 4xydxdz$$

$$= \int_0^1 4xy[\int_0^4 dz]dx$$

$$= \int_0^1 16xydx = 16y\int_0^1 xdx$$

$$= 24$$

$$(d)\ \text{dim}\,ension\ \text{of volume flux=velocity} \times area = \frac{L^3}{T}$$