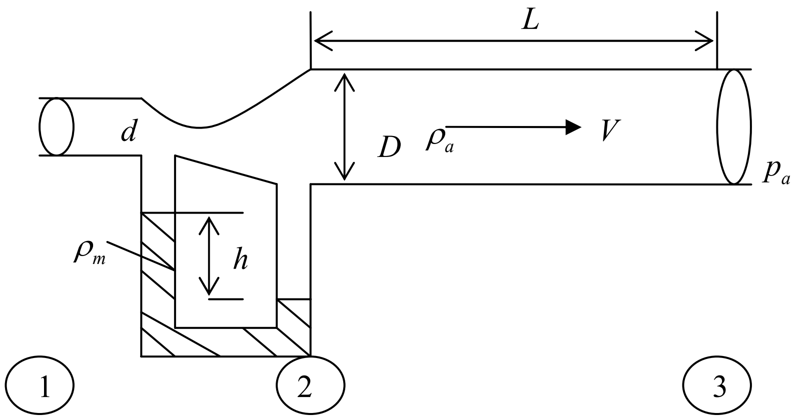
Problem Set 4 (PS4) due Monday Feb. 19

4.33 5.6 5.22

4.33



If there are no losses in the system,

$$p_2 = p_3 = p_a$$

Applying Bernoulli:

$$\frac{p_1}{\rho_a} + \frac{1}{2}V_1^2 = \frac{p_2}{\rho_a} + \frac{1}{2}V_2^2$$

$$\therefore \frac{p_1 - p_a}{\rho_a} = \frac{1}{2}(V^2 - V_1^2)$$

Continuity equation:

$$V_1^2 \frac{\pi D_1^2}{4} = V_2^2 \frac{\pi D_2^2}{4}$$

And also,

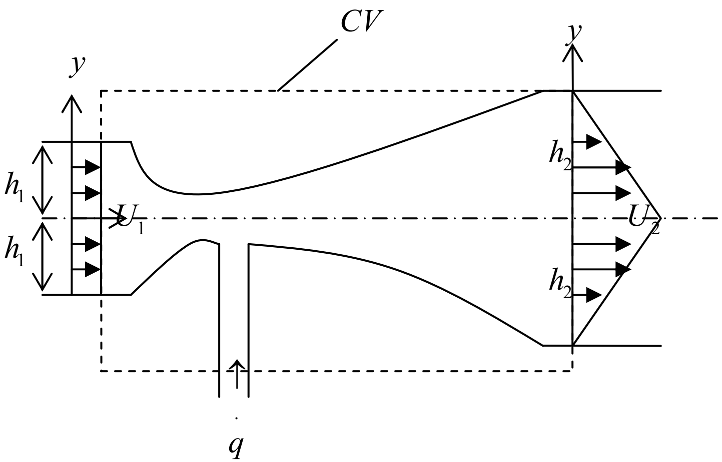
$$p_2 - p_1 = \rho_m g h$$

$$\therefore p_1 - p_a = -\rho_m g h$$

$$Hence \ -\frac{\rho_m}{\rho_a} g h = \frac{1}{2}(V^2 - V^2 (\frac{D}{d})^4)$$

$$\therefore V = [\frac{2\frac{\rho_m}{\rho_a}}{(\frac{D}{d})^4 - 1} g h]^{\frac{1}{2}}$$

<u>5.6</u>



Steady, constant density flow.

Continuity equation:

$$\int n \cdot V dA = 0$$

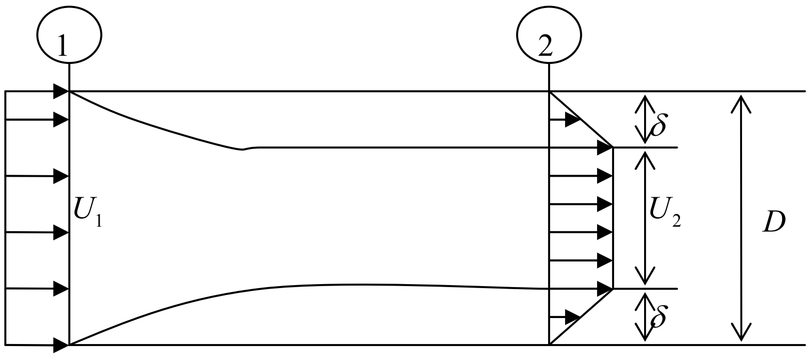
$$\int n \cdot V dA_1 + \int n \cdot V dA_2 - \dot{q} = 0$$

$$\therefore \dot{q} = -2U_1 h_1 w + 2\int_0^{h_2} U_2 (1 - \frac{y}{h_2}) w dy$$

$$= -2U_1 h_1 w + U_2 h_2 w = -2U_1 h_1 w + 2U_1 h_2 w$$

$$\therefore \dot{q} = 2U_1 w (h_2 - h_1)$$

<u>5.22</u>



(a)Constant density, steady flow:

$$\int \mathrm{n}\cdot \mathrm{V}dA=0$$

$$-U_1Dw+2(U_2\times\frac{\delta}{2})w+U_2(D-2\delta)w=0$$

$$\therefore U_1=\frac{\delta}{D}U_2+U_2(1-\frac{2\delta}{D})$$

$$\therefore \frac{U_1}{U_2}=1-\frac{\delta}{D}$$

(b)Outside the boundary layers,the flow is inviscid, and Bernoulli equation can be used:

$$\therefore \mathrm{p}_1+\frac{1}{2}\rho U_1^2=\mathrm{p}_2+\frac{1}{2}\rho U_2^2$$

$$\therefore \mathrm{p}_1-\mathrm{p}_2=\frac{1}{2}\rho(U_2^2-U_1^2)$$

$$\therefore \frac{\mathrm{p}_1-\mathrm{p}_2}{\frac{1}{2}\rho U_1^2}=\frac{U_2^2}{U_1^2}-1$$

5.22

(c)x-component momentum equation:

F_v=viscous force acting on walls,

$\therefore$ -F_v=viscous force acting on fluid.

$$\therefore -F_v + (p_1 - p_2)Dw = -\rho U_1^2 Dw +$$

$$\rho U_2^2 (D-2\delta)w + 2\rho w\int_0^\delta U^2 dy$$

$$\therefore \frac{F_v}{\frac{1}{2}\rho U_1^2 Dw} = \frac{p_1 - p_2}{\frac{1}{2}\rho U_1^2} + 2 -$$

$$\frac{2U_2^2}{U_1^2}(1-\frac{2\delta}{D}) - \frac{4}{D}\frac{U_2^2}{U_1^2}\int_0^\delta \frac{y^2}{\delta^2}dy$$

$$= 1 - \frac{U_2^2}{U_1^2}(1-\frac{8\delta}{3D})$$

$$\therefore \frac{F_v}{\frac{1}{2}\rho U_1^2 Dw} = 1 - \frac{U_2^2}{U_1^2}(1-\frac{8\delta}{3D})$$