EQUATION OF MOTION

one dimensional, steady

$$\frac{dp}{dx} = \mu \frac{d^2u}{dx^2}$$

dimensionless x, u, p,

$$U = \frac{u}{V_O} \quad u = V \times U$$

$$u\,du = V_O^2\,U\,dU, \quad d^2u = V_O\,d^2U$$

$$X = \frac{x}{L_O} \quad x = L_O \times X$$

$$dx = L_O\,dX, \quad dx^2 = L_O^2 dX^2$$

$$P = \frac{p}{P_O} \quad p = P_O \times P$$

$$dp = P_O\,dP$$

substituting for u, x, and p,

$$\left(\frac{P_o}{L_O}\right)\frac{dP}{dX} = \left(\frac{\mu V_O}{L_O^2}\right)\frac{d^2U}{dX^2}$$

divide by $\frac{\rho V_O^2}{L_O}$,

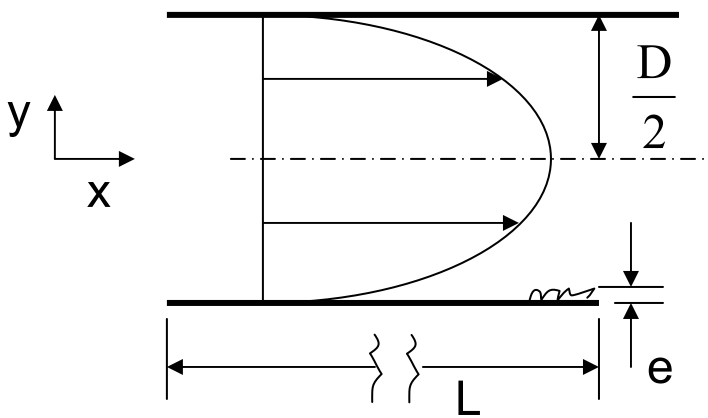
$$\left(\frac{P_o}{\rho V_O^2}\right)\frac{dP}{dX} = \left(\frac{1}{\frac{\rho V_O L_O}{\mu}}\right)\frac{d^2U}{dX^2}$$

$\frac{\rho V_O L_O}{\mu}$ is the dimensionless parameter

Reynolds Number

$$\left(\frac{P_o}{\rho V_O^2}\right)\frac{dP}{dX} = \left(\frac{1}{N_{RE}}\right)\frac{d^2U}{dX^2}$$

PIPE FLOW



$$\begin{vmatrix} a_{11}a_{12}a_{13} \\ a_{21}a_{22}a_{23} \\ a_{31}a_{32}a_{33} \end{vmatrix}$$

determinant of order 2

$$a_{11}a_{22} - a_{12}a_{21}$$

determinant of order 3

$$a_{11}(a_{23}a_{33} - a_{23}a_{32})$$
$$- a_{12}(a_{21}a_{33} - a_{23}a_{31})$$
$$+ a_{13}(a_{21}a_{32} - a_{22}a_{31})$$

hypothesis: $\Delta P = f(\rho, \mu, V, L, D, e)$

	Δp	ρ	μ	V	L	D	e
M	1	1	1	0	0	0	0
L	-1	-3	-1	1	1	1	1
T	-2	0	-1	-1	0	0	0

7 Parameters - 3 Units = 4

Expect 4 Dimensionless Groups

Exponent Matrix Rank -

order of largest positive determinant

Determinant check, select ρ, V, D

$$1((1\times 0)-(-1\times 1))+0(\ \ )+0(\ \ )=+1$$

Matrix Rank = 3

7 Parameters – 3(Matrix Rank) = 4

Expect 4 Dimensionless Groups

Balance Unit Exponents

$$\text{for M,} \quad 1 = a + b$$

$$\text{for L,} \quad -1 = -3a - b + c + d + e + f$$

$$\text{for T,} \quad -2 = -b - c$$

$$\text{from M} \quad a = 1 - b$$

$$\text{from T} \quad c = 2 - b$$

substituting into the L equation,

$$-1 = -3 + 3b - b + 2 - b + d + e + f$$

$$b = -d - e - f$$

$$c = 2 - b = 2 + d + e + f$$

$$\Delta p = f\left(\rho^a \times \mu^b \times V^c \times L^d \times D^e \times e^f\right)$$

$$\Delta p = f(\rho^{(1+d+e+f)} \times \mu^{(-d-e-f)} \times V^{(2+d+e+f)} \times L^d \times D^e \times e^f)$$

$$\Delta p = \rho V^2 \left(\frac{\rho V L}{\mu}\right)^d \left(\frac{\rho V D}{\mu}\right)^e \left(\frac{\rho V e}{\mu}\right)^f$$

PIPE FLOW

$$\text{hypothesis:} \quad \Delta P = f\left(\rho, \mu, V, L, D, e\right)$$

	Δp	ρ	μ	V	L	D	e
M	1	1	1	0	0	0	0
L	-1	-3	-1	1	1	1	1
T	-2	0	-1	-1	0	0	0

Multiplication or division of one dimensionless number by another result in a dimensionless number

$$\frac{\Delta p}{\rho V^2} = f\left(\frac{\left(\frac{\rho V L}{\mu}\right)}{\left(\frac{\rho V D}{\mu}\right)} \left(\frac{\rho V D}{\mu}\right) \frac{\left(\frac{\rho V e}{\mu}\right)}{\left(\frac{\rho V D}{\mu}\right)}\right)$$

$$\frac{\Delta p}{\rho V^2} = f\left(\frac{\rho V D}{\mu}, \frac{L}{D}, \frac{e}{D}\right)$$

Select a group of 4 parameters that Include all the units

select $\rho^a, V^b, D^c, \Delta p$

equate parameter exponents

from M: $0 = a + 1$ from L: $0 = -3a + b + c - 1$ from T: $0 = -b - c$

$a = -1, b = -2, c = 0$

$$1\text{st Number} = \frac{\Delta p}{\rho V^2}$$

Combine this group with each of the other parameters

$\rho^d V^e D^f \mu$

fromM $0 = d + 1$

fromL $0 = -3d + e + f = 1$

fromT $0 = -e - 1$

$d = -1, e = -1, f = -1$

$$2\text{nd Number} = \frac{\mu}{\rho D V}$$

$\rho^g V^h D^j L$

fromM $0 = g$

fromL $0 = -3g + h + i + 1$

fromT $0 = -h$

$g = 0, h = 0, i = -1$

$$3\text{rd Number} = \frac{L}{D}$$

$\rho^j V^k D^l e$

from M: $0 = j$

from L: $0 = -3 + j + k + l + 1$

from T: $0 = -k$

$j = 0, k = 0, l = -1$

$$4\text{thNumber} = \frac{e}{D}$$

$$\frac{\Delta p}{\rho V^2} = f\left(\frac{\rho V D}{\mu}, \frac{L}{D}, \frac{e}{D}\right)$$

DIMENSION CHECK

$$\frac{\Delta p}{\rho V^2} = f\left(\frac{\rho V D}{\mu}, \frac{L}{D}, \frac{e}{D}\right)$$

$$\frac{\Delta p}{\rho V^2}, \left[\frac{M}{LT^2}\right]\left[\frac{L^3}{M}\right]\left[\frac{T^2}{L^2}\right] = 1$$

$$\frac{\rho V D}{\mu}, \left[\frac{M}{L^3}\right]\left[\frac{L}{T}\right]\left[\frac{L}{1}\right]\left[\frac{LT}{M}\right] = 1$$

$$\frac{L}{D}, \left[\frac{L}{1}\right]\left[\frac{1}{D}\right] = 1$$

$$\frac{e}{D}, \left[\frac{L}{1}\right]\left[\frac{1}{D}\right] = 1$$

$$\frac{e}{D}$$

8.5

The thrust of a propeller is assumed to depend only on its diameter, D, the fluid density, ρ, the fluid viscosity, μ, the revolutions per unit time, and the foreward velocity, V. Determine the dimensionless numbers.

	T	D^a	ρ^b	μ^c	ω^d	V^e
M	1	0	1	1	0	0
L	1	1	-3	-1	0	1
T	-2	0	0	-1	-1	-1

balance the exponents of the units

for M $\quad 1 = a + b$

for L $\quad 1 = a - 3b - c + e$

for T $\quad -2 = -c - d - e$

from M, $\quad b = 1 - c$

from T, $\quad c = 2 - d - e$

substituting into equation from T,

$1 = a = 3 = 3c - c + e$

$a = -1 + 2d + e$

then

$b = -1 + d + e$

$$T = D^a \times \rho^b \times \mu^c \times \omega^d \times V^e$$

$$T = D^{(-1+2d+e)} \times \rho^{(-1+d+e)} \times \mu^{(2-d-e)} \times \omega^d \times V^e$$

$$T = \frac{\mu^2}{D\rho}\left(\frac{D^2\rho\,\omega}{\mu}\right)^d\left(\frac{D\,\rho V}{\mu}\right)^e$$

$$\frac{T\,D\,\rho}{\mu^2} = f\left(\frac{D^2\rho\,\omega}{\mu}\right)^d \times \left(\frac{D\rho\,V}{\mu}\right)^e$$

check

$$\text{for}\ \frac{T\,D\,\rho}{\mu^2},\left(\frac{ML}{T^2}\right)\left(\frac{L}{1}\right)\left(\frac{M}{L^3}\right)\left(\frac{L^2T^2}{M^2}\right) = 1$$

$$\text{for}\ \frac{D^2\rho\,\omega}{\mu},\left(\frac{L^2}{1}\right)\left(\frac{M}{L^3}\right)\left(\frac{1}{T}\right)\left(\frac{LT}{M}\right) = 1$$

$$\text{for}\ \frac{D\,\rho V}{\mu},\left(\frac{L}{1}\right)\left(\frac{M}{L^3}\right)\left(\frac{LT}{M}\right)\left(\frac{L}{T}\right) = 1$$

8.5

Multiplication or division of one dimensionless number by another result in a dimensionless number

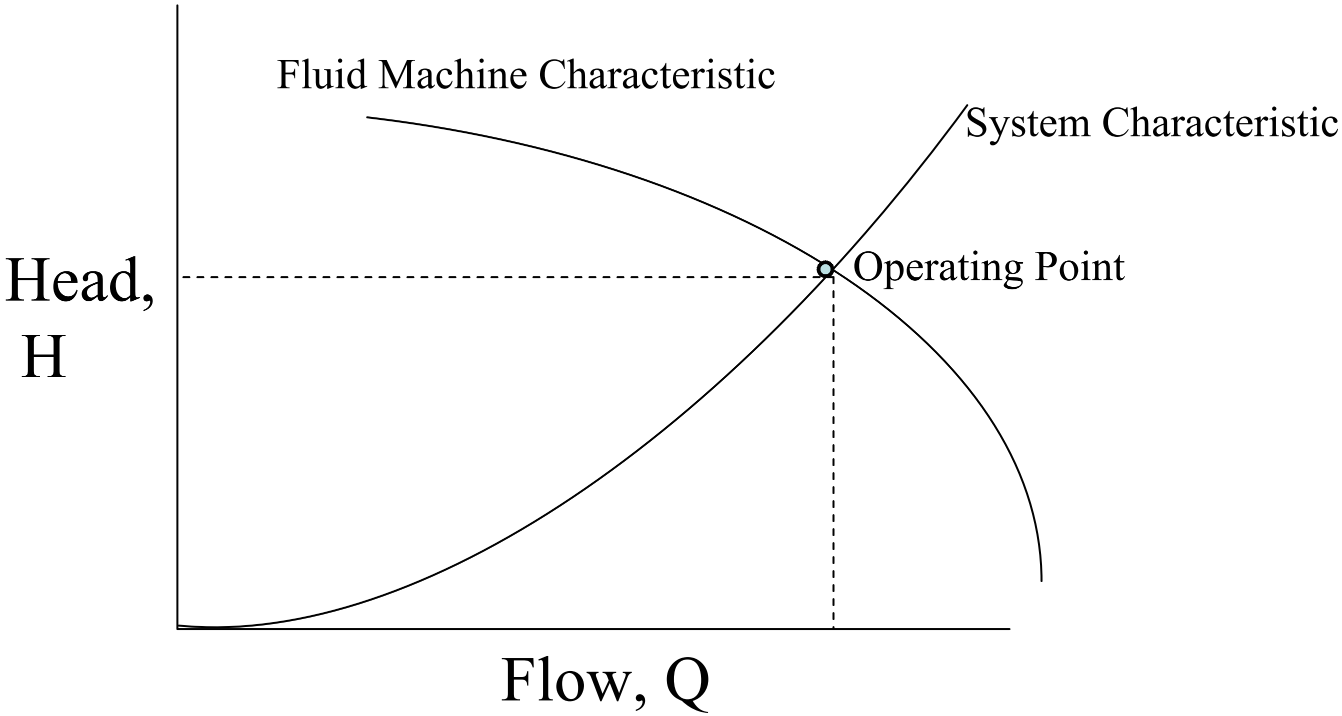
$$\frac{T\,D\,\rho}{\mu^2} = \left(\frac{D^2\rho\,\omega}{\mu}\right)^d \left(\frac{D\,\rho V}{\mu}\right)^e$$

$$\frac{D^2\rho\,\omega}{\mu} \times \frac{\mu}{D\,\rho} = \frac{D\,\omega}{V}, \qquad \left(\frac{L}{1}\right)\left(\frac{T}{L}\right)\left(\frac{1}{T}\right) = 1$$

$$\frac{TD\rho}{\mu^2} \times \left(\frac{\mu}{D\,\rho\,V}\right)^2 = \frac{T}{\rho\,D\,V^2}, \qquad \left(\frac{ML}{T^2}\right)\left(\frac{1}{L}\right)\left(\frac{L^3}{M}\right)\left(\frac{T^2}{L^2}\right) = 1$$

$$\frac{T}{\rho D V^2} = f\left(\frac{D\,\omega}{V}, \frac{D\,\rho\,V}{\mu}\right)$$

FLUID SYSTEM CHARACTERISTICS



Energy (head) is put into the system fluid by the fluid machine in the form of velocity and pressure.

Energy (head) is removed from the system fluid by friction in the piping or duct work.

FLUID MACHINE DIMENSIONLESS PARAMETERS

If the full set of equations for fluid machine,

Energy Balance - First Law
Mass balance - continuity equation
Equations of motion F=mass x acceleration

are non-dimensionalized 6 dimensionless parameters result.

If the machine variables are changes so that 5 of these dimensionless parameters remain constant, the 6th parameter will also remain constant.

In the operation of a pump or fan Mach Number, and Specific Heat Ratio remain constant and Reynolds Number changes very little.

If the Specific Speed and Specific Diameter of a fluid machine remain the same even though rotational speed, head and flow many change, the same efficiency will be achieved.

$$\text{Specific Speed } N_s = \frac{NQ^{.5}}{H^{.75}}$$

$$\text{Specific Diameter} = \frac{DH^{.25}}{Q^{.5}}$$

$$N = \text{rotauional speed}$$
$$Q = \text{volume flow}$$
$$H = \text{head}$$
$$D = \text{diameter}$$

$$\text{Mach Number } M = \frac{\text{Velocity}}{\text{Sonic Velocity}}$$

$$\text{Reynolds Number} = \frac{\mu V L}{\rho}$$

$$V = \text{velocity}$$
$$\mu = \text{viscosity}$$
$$\rho = \text{density}$$

$$\text{Specific Heat Ratio } k = \frac{c_p}{c_v}$$

Efficiency

$$\eta_{compressor} = \frac{\text{Ideal Work}}{\text{Actual Work}}$$

$$\eta_{compressor} = \frac{\text{Actual Work}}{\text{Ideal Work}}$$

PUMP SPECIFIC SPEED SPECIFIC DIAMETER DIAGRAM

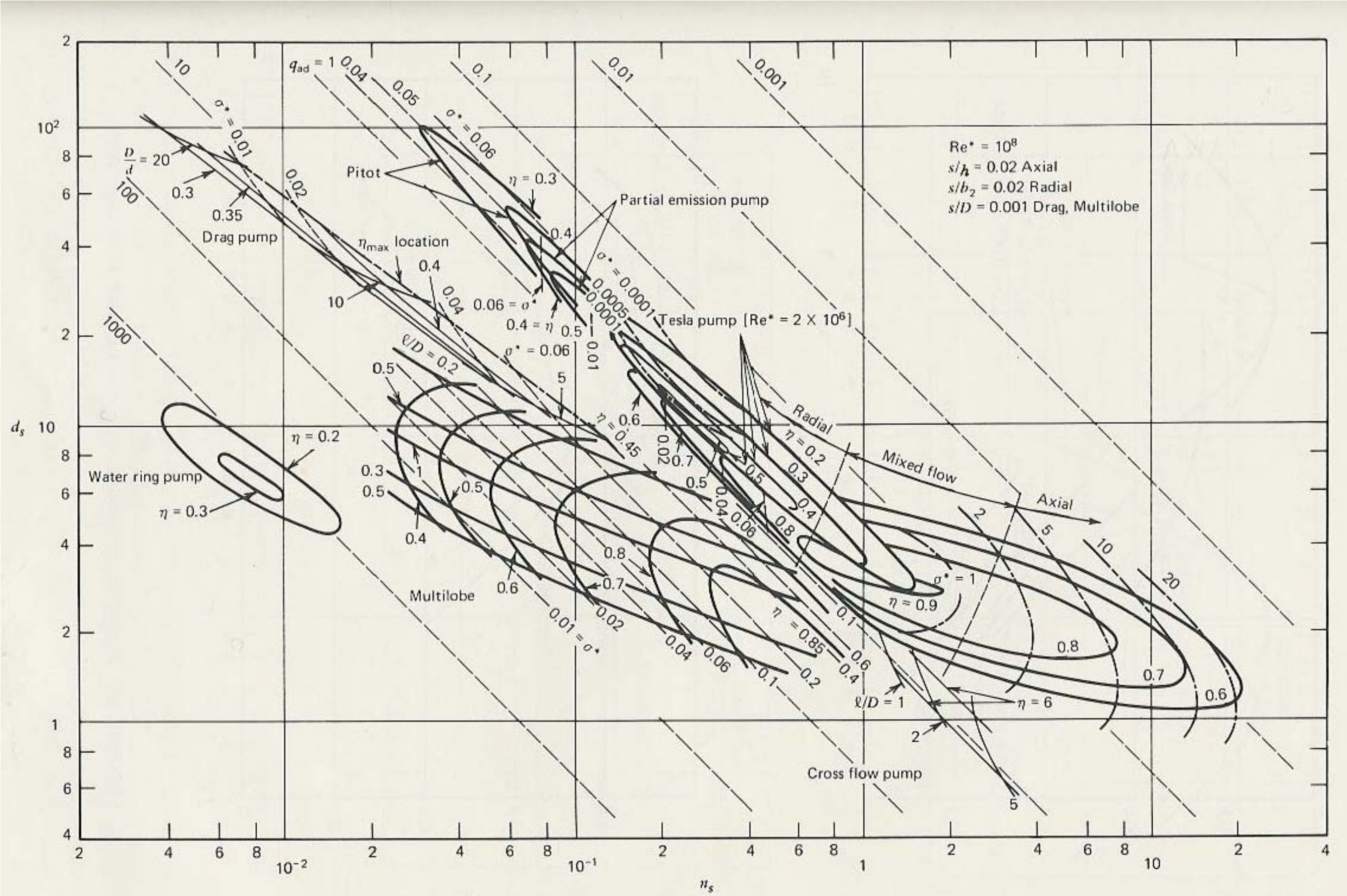


FIGURE 3.8 $n_s d_s$ diagram for single stage pumps.

1) Similar geometry+Constant Specific Speed and Specific Diameter = Same Efficiency

2) Each machine type has an optimum Specific Speed for maximum efficiency.

MACHINERY AFFINITY LAWS , (pump laws, fan laws)

$$\text{SPECIFIC DIAMETER, } D_s = \frac{DH^{.25}}{Q^{.5}} = \text{CONSTANT}$$

$$\frac{D_0 H_0^{.25}}{Q_0^{.5}} = \frac{D_1 H_1^{.25}}{Q_1^{.5}}$$

$$\left(\frac{Q_1}{Q_0}\right)^{.5} = \frac{D_{`}}{D_0}\left(\frac{H_1}{H_0}\right)^{.25}$$

$$\left(\frac{Q_1}{Q_0}\right)^{.5} = \frac{D_1}{D_0}\left(\left(\frac{N_1}{N_0}\right)^2\left(\frac{D_1}{D_0}\right)^2\right)^{.25}$$

$$\frac{Q_1}{Q_0} = \left(\frac{D_1}{D_0}\right)^3\left(\frac{N_1}{N_0}\right)$$

for the same impeller, $D_0 = D$

$$\frac{Q_1}{Q_0} = \left(\frac{N_1}{N_0}\right)$$

$$\text{POWER} = Q \times H$$

$$\frac{\text{Power}_1}{\text{Power}_2} = \left(\frac{Q_1}{Q_0}\right)\left(\frac{H_1}{H_0}\right) = \left(\frac{N_1}{N_0}\right)\left(\frac{N_1}{N_0}\right)^2$$

$$\frac{\text{Power}_1}{\text{Power}_2} = \left(\frac{N_1}{N_0}\right)^3$$

$$\text{SPECIFIC SPEED, } N_s = \frac{NQ^{.5}}{H^{.75}} = \text{CONSTANT}$$

$$\frac{N_0 Q_0^{.5}}{H_0^{.75}} = \frac{N_1 Q_1^{.25}}{H_1^{.75}}$$

$$\left(\frac{Q_1}{Q_0}\right)^{.5} = \frac{N_{0`}}{N_1}\left(\frac{H_1}{H_0}\right)^{.75}$$

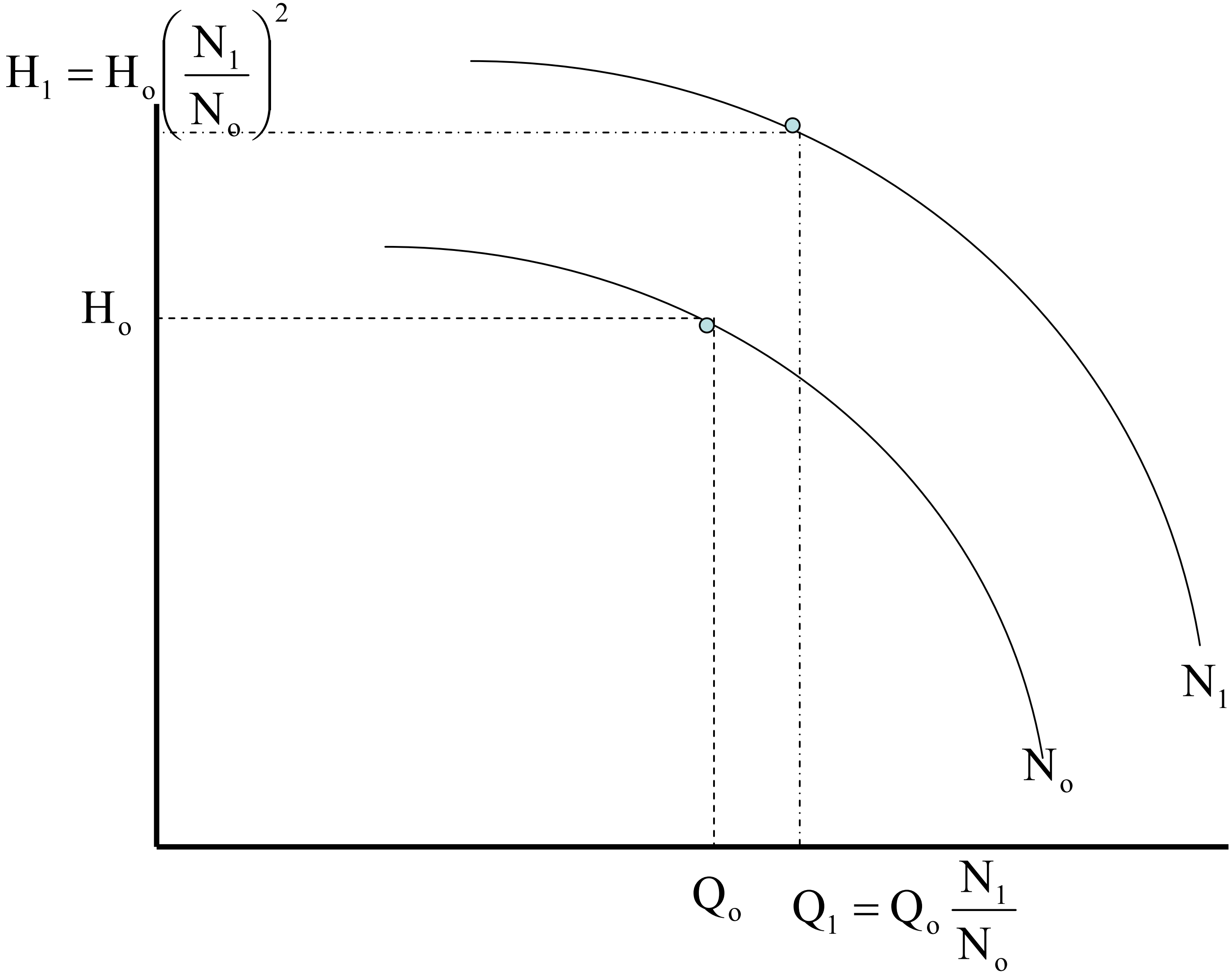
$$\frac{D_1}{D_0}\left(\frac{H_1}{H_0}\right)^{.25} = \frac{N_0}{N_1}\left(\frac{H_1}{H_0}\right)^{.75}$$

$$\left(\frac{H_1}{H_0}\right)^{.5} = \left(\frac{N_1}{N_0}\right)\left(\frac{D_1}{D_0}\right)$$

$$\left(\frac{H_1}{H_0}\right) = \left(\frac{N_1}{N_0}\right)^2\left(\frac{D_1}{D_0}\right)^2$$

for the same impeller, $D_0 = D_1$

$$\frac{H_1}{H_0} = \left(\frac{N_1}{N_0}\right)^2$$


$H_1 = H_o \left(\frac{N_1}{N_o}\right)^2$
H_o
N_1
N_o
Q_o
$Q_1 = Q_o \frac{N_1}{N_o}$