Bernoulli's Equation
Chapter 4.1-4.6

STATIONARY FLUID – FLUID STATICS – HYDROSTATIC EQUATION

pressure and weight force balance in vertical direction

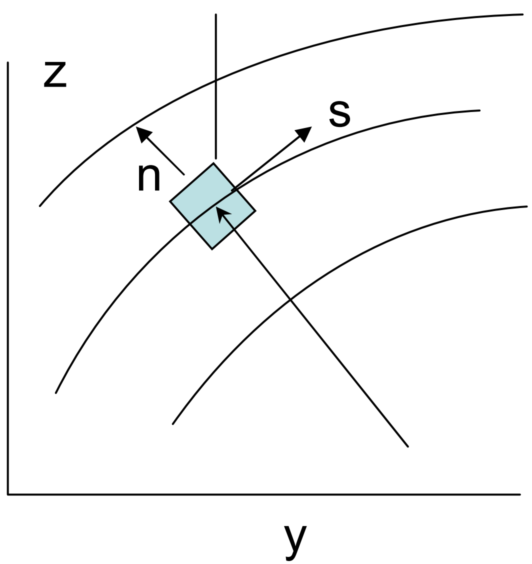
$$\frac{\partial p}{\partial z} = -\rho g, \quad p = \rho g z$$

MOVING FLUID – EULER and BERNOULLI EQUATIONS

force balance along a streamline

$$\Delta F_{momentum} = -\Delta F_{pressure} - \Delta F_{weight}$$

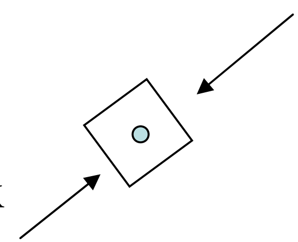
Assumptions:
- steady flow
- 1D
- inviscid
- addiabatic
- W=0
- $\rho \neq$ contant
- $\rho = f(n,s)$



$$\left[p + \frac{\partial p}{\partial s} ds\right] ds\, dx$$

$$V + \frac{\partial V}{\partial s} ds$$

$$z + \frac{\partial z}{\partial s}$$

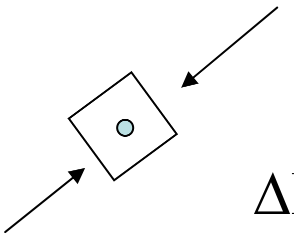


$$[p]\, ds\, dx$$

V

z

$$\left[p + \frac{\partial p}{\partial s} ds\right] ds\, dx,\ \ V + \frac{\partial V}{\partial s} ds,\ \ z + \frac{\partial z}{\partial s}$$



$[p]\,dn\,dx$

V

z

$$\Delta F_{momentum} = -\Delta F_{pressure} - \Delta F_{weight}$$

$$\Delta F_{momentum} = \Delta(\rho V) = \rho\left[\left(V + \frac{\partial V}{\partial s} ds - V\right)/dt\right] dn\, ds\, dx = \rho V \frac{\partial V}{\partial s} dn\, ds\, dx$$

$$\Delta F_{pressure} = \left[p + \frac{\partial p}{\partial s} ds - p\right] dn\, dx = \frac{\partial p}{\partial s} ds\, dn\, dx$$

$$\Delta F_{weight} = \rho\, g\, dn\, ds\, dx$$

for the s component of weight,

$$\Delta F_{weight} = \rho g \frac{dz}{ds} dn\, ds\, dx$$

$$\rho V \frac{\partial V}{\partial s} dn\, ds\, dx = -\frac{\partial p}{\partial s} ds\, dn\, dx - \rho g \frac{dz}{ds} dn\, ds\, dx$$

EULERS STREAMLINE EQUATION

$$\rho V \frac{\partial V}{\partial s} x = -\frac{\partial p}{\partial s} - \rho g \frac{dz}{ds}$$

$\rho \neq$ contant

$\rho = f(n, s)$

BERNOULLI'S EQUATION (1740)

force balance along a streamline

$$\text{EULERS STREAMLINE EQUATION} - \rho \neq \text{constant}, \rho = f(n, s)$$

$$\rho V \frac{\partial V}{\partial s} x = -\frac{\partial p}{\partial s} - \rho g \frac{dz}{ds}$$

multiplying by ds

$$\rho\, V \frac{\partial V}{\partial s} ds = -\frac{dp}{ds} ds - \rho\, g \frac{\partial z}{\partial s} ds$$

$$\frac{\partial p}{\partial s} ds = dp$$

$$\frac{\partial V}{\partial s} ds = dV$$

$$\frac{dz}{ds} dz = dz$$

$$\rho V dV = -dp - \rho\, g\, dz$$

for constant density

$$\frac{p}{\rho} + \frac{V^2}{2} + gz = \text{constant}$$

BERNOULLI'S EQUATION

- steady
- 1D – streamline
- inviscid – T= constant
- constant density
- no heat transfer
- no work

STAGAT ION PRESSURE

$$\frac{p}{\rho} + \frac{V^2}{2} + g\,z = \text{constant}$$

$$\frac{p_O}{\rho} + \frac{V_0^2}{2} + g\,z_0 = \frac{p_1}{\rho} + \frac{V_1^2}{2} + g\,z_0$$

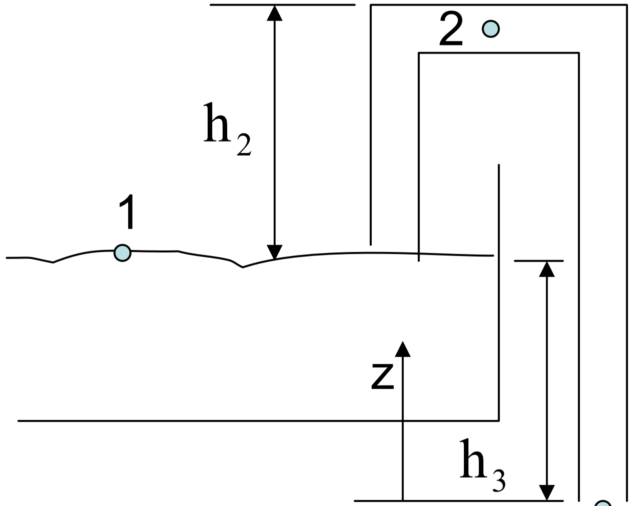
for $V_0 = 0$

$$p_O = p_1 + \rho\frac{V_1^2}{2}$$

pressure coefficient, C_p

$$C_p = \frac{p_1 - p}{\frac{1}{2}\rho\,V_\infty^2}$$

SIPHON



from point 1to point 3

$$\frac{p_1}{\rho}+\frac{V_1^2}{2}+gz_1=\frac{p_3}{\rho}+\frac{V_3^2}{2}+gz_3$$

$$V_1=0, z_1=h_3, z_2=0$$

$$\frac{p_{atm}}{\rho}+0+gh_3=\frac{p_3}{\rho}+\frac{V_3^2}{2}+g0$$

$$p_1=p_{arm}=p_3$$

$$gh_3=\frac{V_3^2}{2}$$

$$V_3=\left(2gh_3\right)^{\frac{1}{2}}$$

from point 1to point 2

$$\frac{p_1}{\rho}+\frac{V_1^2}{2}+gz_1=\frac{p_2}{\rho}+\frac{V_2^2}{2}+gz_2$$

$$V_1=0, z_1=h_3, z_2=h_2+h_3, V_2^2=V_1^2=2gh_3$$

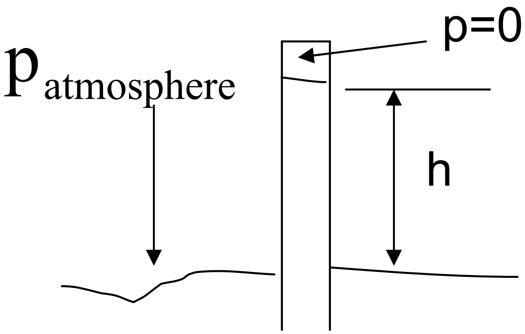
$$p_1=p_{arm}=p_3$$

$$\frac{p_{atm}}{\rho}+0+gh_3=\frac{p_2}{\rho}+\frac{2gh}{2}+g\left(h_2+h_3\right)$$

$$\frac{p_2}{\rho}=\frac{p_{atm}}{\rho}-g\left(h_2+h_3\right)$$

$$h_2+h_3=\frac{p_{atm-}p_2}{\rho g}$$

the higher h_2 the lower V_3



$$p_{atm} = \rho\, g\, h$$

$$\text{for water at 50 F,}\ \ \rho = 1.94\,\frac{\text{slugs}}{\text{ft}^3}$$

$$14.7\,\frac{\text{lb}_f}{\text{in}^2}\times 144\,\frac{\text{in}^2}{\text{ft}^2} = 1.94\,\frac{\text{slugs}}{\text{ft}^3}\times 32.2\,\frac{\text{ft}}{\text{sec}^2}\times h_{critical}\,\text{ft}$$

$$14.7\,\frac{\text{lb}_f}{\text{in}^2}\times 144\,\frac{\text{in}^2}{\text{ft}^2} = 62.4\,\frac{\text{lb}_f}{\text{ft}^3}\times h_{critical}\ \text{ft}$$

$$h_{critical} = \frac{14.7\times 144}{62.4} = 33.9\ \text{ft}$$

$$p_{atm} = \rho\, g\, h$$

$$\text{for water a t20 C,}\ \ \rho = 999.2\,\frac{\text{kg}}{\text{m}^3}$$

$$101.325\text{kPa} = 999.2\,\frac{\text{kg}}{\text{m}^3}\times 9.9\,\frac{\text{m}}{\text{sec}^2}\times \text{hm}$$

$$\text{lkPa} = \frac{1000\text{N}}{\text{m}^2} = 1000\,\frac{\text{kgm}}{\text{sec}^2\text{m}^2}$$

$$101{,}325\,\frac{\text{kgm}}{\text{sec}^2\text{m}^2} = 999.2\,\frac{\text{kg}}{\text{m}^3}\times 9.9\,\frac{\text{kgm}}{\text{m}^3\text{sec}}\times h_{critical}\ \text{m}$$

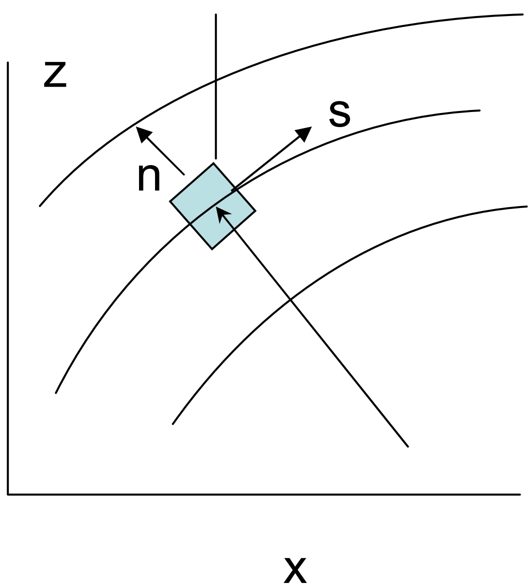
$$h_{critical} = \frac{101{,}325}{999.2\times 9.9}\,\text{m} = 10.24\ \text{m}$$

EULER NORMAL EQUATION

force balance normal to a streamline

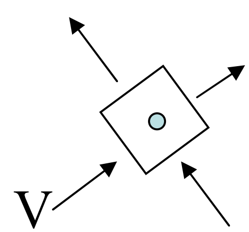
$$\Delta F_{momentum} = -\Delta F_{pressure} - \Delta F_{weight}$$

Assumptions:
- Steady flow
- 1D
- inviscid
- addiabatic
- W=0



$$\left[p + \frac{\partial p}{\partial n} dn\right] dn\ dx$$

$$z + \frac{\partial z}{\partial n}$$



$$\left[p\right] dn\ dx$$

$$z$$

EULER NORMAL EQUATION

force balance normal to a streamline

$$\Delta F_{momentum} = -\rho \frac{V^2}{R}$$

$$\Delta F_{pressure} = \left[p + \frac{\partial p}{\partial n} dn - p \right] dn\ dx = \frac{\partial p}{\partial n} ds\ dn\ dx$$

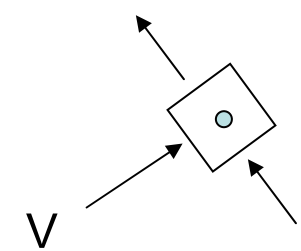
$$\Delta F_{weight} = \rho\ g \frac{\partial z}{n} dn\ ds\ dx$$

$$-\rho \frac{V^2}{R} = -\frac{\partial p}{\partial n} ds\ dn\ dx - \rho\ g \frac{\partial z}{n} dn\ ds\ dx$$

$$-\rho \frac{V^2}{R} = -\frac{\partial p}{\partial n} - \rho\ g \frac{\partial z}{n}$$

$$\left[p + \frac{\partial p}{\partial n} dn \right] dn\ dx$$

$$z + \frac{\partial z}{\partial n}$$



$$[p]\, dn\ dx$$

z

$$\frac{\partial p}{\partial n} + \rho g \frac{\partial z}{\partial n} = \rho \frac{V^2}{R}$$ EULER NORMAL EQUATION \

for $R \to \infty$, $\frac{\partial p}{\partial n} = -\rho g \frac{\partial z}{\partial n}$

where gravity has no effect, $\frac{\partial p}{\partial n} = 0$

First Law of Thermodynamics

Observations:

work can be transferred into heat

more friction = more heat

$$\sum \delta Q \propto \sum \delta W$$

Examples:

rubbing you hands together

friction in a wheel bearing

braking a wheel

Experiments:

$$\sum Q = C \sum W$$

$$\oint \delta Q = C \oint \delta W$$

$$1\,\text{BTU} = 778\ \text{ft lb}_f$$

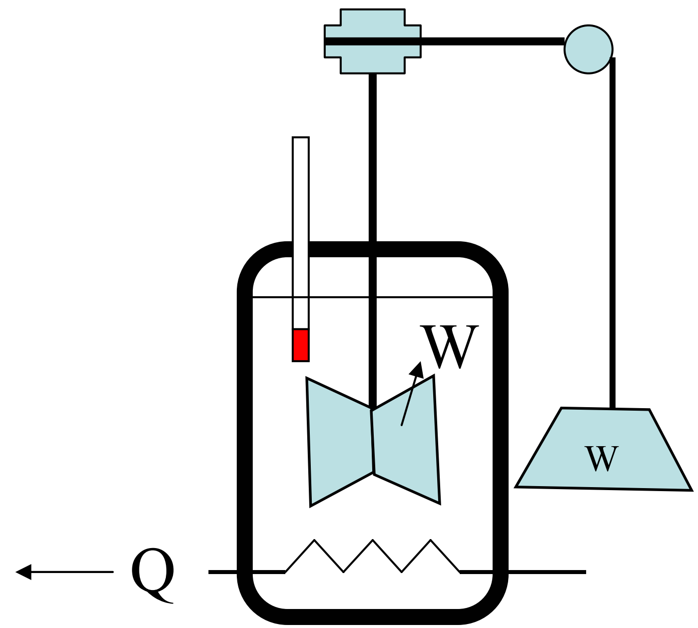
$$1\ \text{calorie} = .427\ \text{kg m}$$

First Law for a Cycle

$$\oint \delta Q = \oint \delta W$$

$$\oint (\delta Q - \delta W) = 0$$

The First Law is a fundamental observation of nature, an axiom, which can not be proved but has never been found to be violated



JOULE EXPERIMENT

First Law of Thermodynamics

$$\oint \delta Q = \oint \delta W \quad \text{First Law for a Cycle}$$

$$\oint (\delta Q - \delta W) = 0$$

Cycle $A \Rightarrow B$ $\quad {}_A\int(\delta Q - \delta W) - {}_B\int(\delta Q - \delta W) = 0$

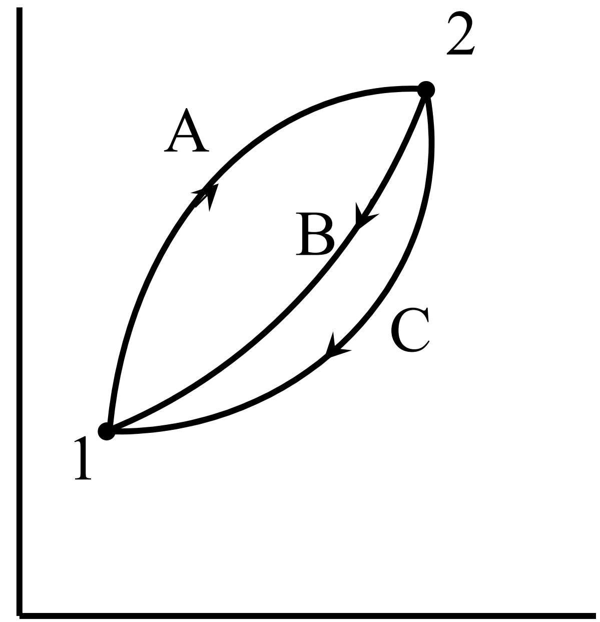
Cycle $A \Rightarrow C$ $\quad {}_A\int(\delta Q - \delta W) - {}_C\int(\delta Q - \delta W) = 0$

Processes B and C $\quad {}_B\int(\delta Q - \delta W) - {}_C\int(\delta Q - \delta W) = 0$

$${}_B\int(\delta Q - \delta W) = {}_C\int(\delta Q - \delta W)$$

$\int(\delta Q - \delta W)$ is independent of path and therefore a property. Define E as energy in all forms, KE + PE + U(T).

$$E_1 - E_2 = \int_2^1 (\delta Q - \delta W) = Q - W$$



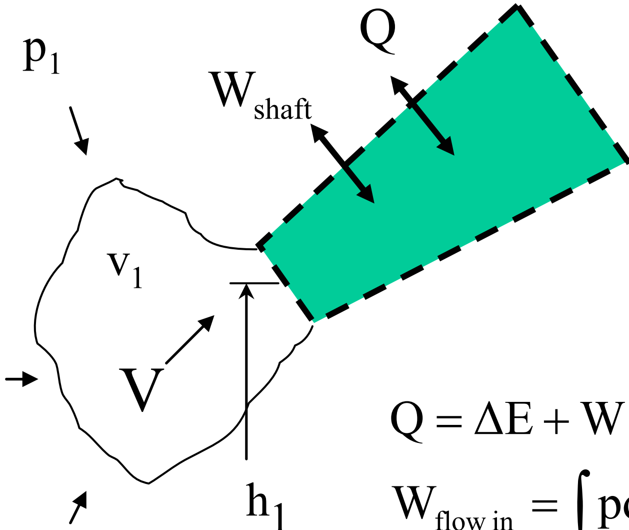
First Law for a Processes

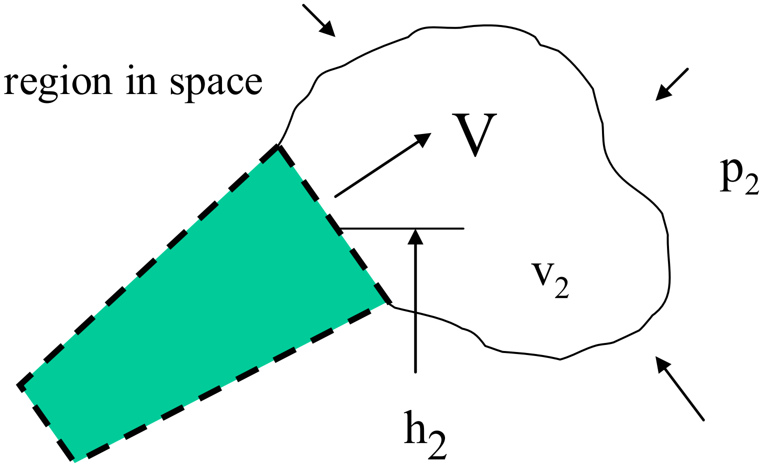
$$\mathbf{Q = \Delta E + W}$$

$$\mathbf{\delta Q = dE + \delta W}$$

Steady Flow Energy Equation

Open, steady flow thermodynamic system - a region in space





$$Q = \Delta E + W \quad \text{First Law}$$

$$W_{\text{flow in}} = \int p dV = p_1 \left(V_{1 \text{ initial}} - V_{1 \text{ final}} \right) = p_1 V_1 = m \, p_1 v_1$$

$$W_{\text{flow out}} = m \, p_2 v_2$$

$$W = W_{\text{shaft}} + W_{\text{flow in}} + W_{\text{flow out}} = W_{\text{shaft}} + m \, p_1 v_1 - m \, p_2 v_2$$

$$E = U(T) + KE + PE = U(T) + \frac{V^2}{2} + \rho g h$$

$$Q = m(u_1 + p_1 v_1 + \frac{V^2}{2} + \rho g h_1) - m(u_2 + p_2 v_2 + \frac{V^2}{2} + \rho g h_1) + W_{\text{shaft}}$$

$$Q = m\Delta(u + pv + \frac{V^2}{2} + \rho g h) + W_{\text{shaft}}$$

BERNOULLI'S EQUATION from THE FIRST LAW

$$Q = m\Delta(u + pv + \frac{V^2}{2} + \rho gz) + W_{shaft}$$

$$1D, Q = 0, W = 0$$

$$\Delta(u + pv + \frac{V^2}{2} + \rho gh)$$

$$u = c_v \Delta T = 0, \mu = 0 \Rightarrow \Delta T = 0$$

$$pv + \frac{V^2}{2} + \rho gz = 0$$

$$\frac{p}{\rho} + \frac{V^2}{2} + gz = \text{constant}$$

$$m\Delta(u + pv + \frac{V^2}{2} + \rho gz) + W_{shaft} = 0$$