

NAME: MASTER

MAE 311 Machines and Mechanisms I
February 23, 2003
Time 50 minutes

Test 1

One 8 1/2 x 11" sheet of formulas and a calculator are permitted
Some information that will be needed is provided on the last page

1. The shaft shown below is subjected to a fully reversed bending moment, M , along its entire length. At what location is the shaft likely to fail in fatigue? Justify your answer in quantitative terms, i.e., by comparing factors of safety. Consider the effects of modifying factors that could apply. (Hint: You don't have to calculate most of the modifying factors but do need to calculate one or more.) (35 pts)



5 $n = \frac{S_e}{\sigma_a} = \frac{k_a k_b k_c k_d k_e S_e}{\sigma_a}$ at A Notch sensitivity at A = 0.70
at B, C = 0.75
At all critical locations, A, B, C.

12 Smallest $k_e = 1/k_f$ or largest k_f determines most likely failure location

at A $r/d = 1/6$ $\gamma_A = 9/4$ $K_t = 1.85$ $K_f = 1 + 0.70(1.85 - 1) = 1.60$
B $r/d = 1/8$ $\gamma_B = 5/4$ $K_t = 1.75$ $K_f = 1 + 0.75(1.75 - 1) = 1.56$
C $r/d = 1/9$ $\gamma_C = 7/4$ $K_t = 1.55$ $K_f = 1 + 0.75(1.55 - 1) = 1.41$

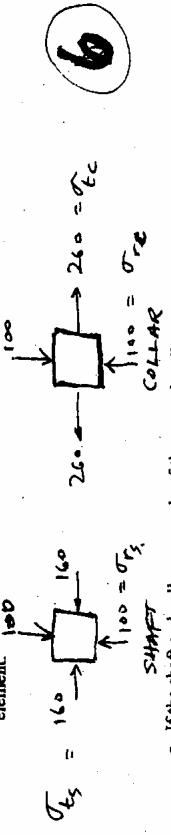
Failure at A (largest K_f)

2. A hollow shaft with inner and outer radii of 10 and 20 mm is assembled with a collar having inner and outer radii of 20 and 30 mm. There is an interference fit between the shaft and collar that results in a contact pressure of 100 MPa at the interface. (40 pts)

a. Calculate the stresses acting on the shaft and collar at the interface (They are not the same).

SHAFT $\sigma_{r3} = -100 \text{ MPa}$
COLLAR $\sigma_{r2} = -100 \text{ MPa}$
 $\sigma_{t3} = -P(R^2 + r_i^2) / (R^2 - r_i^2) = -100(30^2 + 10^2) / (30^2 - 10^2) = -166 \text{ MPa}$
 $\sigma_{t2} = P(r_o^2 + r_i^2) / (r_o^2 - r_i^2) = 100(20^2 + 30^2) / (20^2 - 30^2) = 260 \text{ MPa}$

b. Indicate the stresses and their direction (tension or compression) on a plane stress element.



c. If the shaft and collar are made of the same ductile material, having a Yield Strength of 800 MPa, calculate the factors of safety at the interface for the both the shaft and collar using an appropriate failure theory. Will it be the shaft or collar that fails first?

Stresses in 2, a, b ARE principal stresses

by MSST

Shaft: $n_s = \frac{S_y}{\sigma_a - \sigma_c} = \frac{800}{0 - (-166)} = 5.0 = n_s$
Collar: $n_c = \frac{800}{260 - (-100)} = 2.22 = n_c$

by DFT

$n_s = \frac{S_y}{\sigma_1} = \frac{800}{\sqrt{100^2 - (-100)(-166)} + 166} = 5.76 = n_s$
 $n_c = \frac{800}{\sigma_1} = \frac{800}{\sqrt{260^2 - (260)(-100)} + 100} = 2.49 = n_c$

COLLAR FAILS

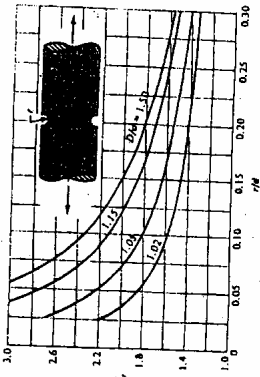


FIGURE A-15-13
Grooved round bar in tension. $\sigma_0 = F/A$, where $A = \pi r^2 h$.

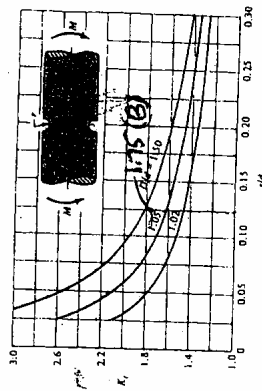


FIGURE A-15-14
Grooved round bar in bending. $\sigma_0 = Mc/I$, where $c = 0.75$ and $I = \pi r^2 h^3/4$.

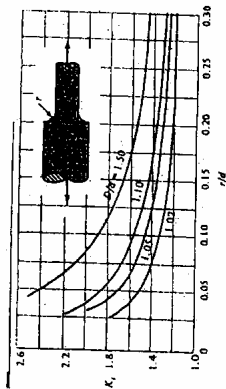


FIGURE A-15-7
Round shaft with shoulder fillet in tension. $\sigma_0 = F/A$, where $A = \pi r^2 h$.

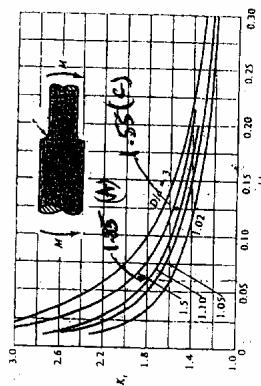


FIGURE A-15-9
Round shaft with shoulder fillet in bending. $\sigma_0 = Mc/I$, where $c = 0.75$ and $I = \pi r^2 h^3/4$.

3. A component is subjected to an alternating stress of 250 MPa. The material has an ultimate tensile strength of 900 MPa and a fully modified endurance limit, S_e , of 300 MPa. Determine the life (in cycles) of the part if a safety factor of 2 is required. (25 pts)

WITH $n=2$ need life for $\sigma_a = 2(250) = 500$ (4)

$\sigma_a = \left(\frac{\sigma_a}{\sigma_u}\right)^{1/b} = 500$ (4)

$a = \left(\frac{(0.9)(900)}{300}\right)^2 = \frac{810^2}{300} = 2187$ MPa (5)

$b = -\frac{1}{3} \log_{10} \left(\frac{0.9 S_{ut}}{S_e}\right) = -\frac{1}{3} \log_{10} \left(\frac{810}{300}\right) = -0.144$ (5)

$N = \left(\frac{500}{2187}\right)^{1/b} = (0.228)^{-1/0.144}$

$N = 28645$ cycles (7)