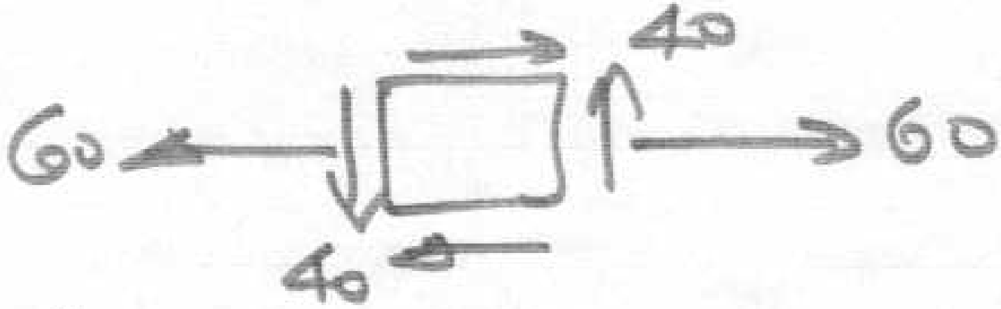


from eq. 2-8

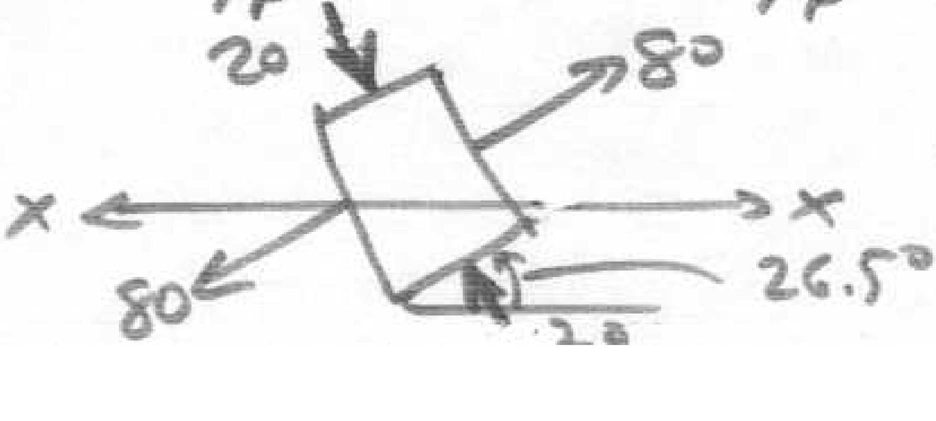
$$\sigma_1, \sigma_2 = \frac{60-0}{2} \pm \sqrt{\left(\frac{60-0}{2}\right)^2 + 40^2}$$

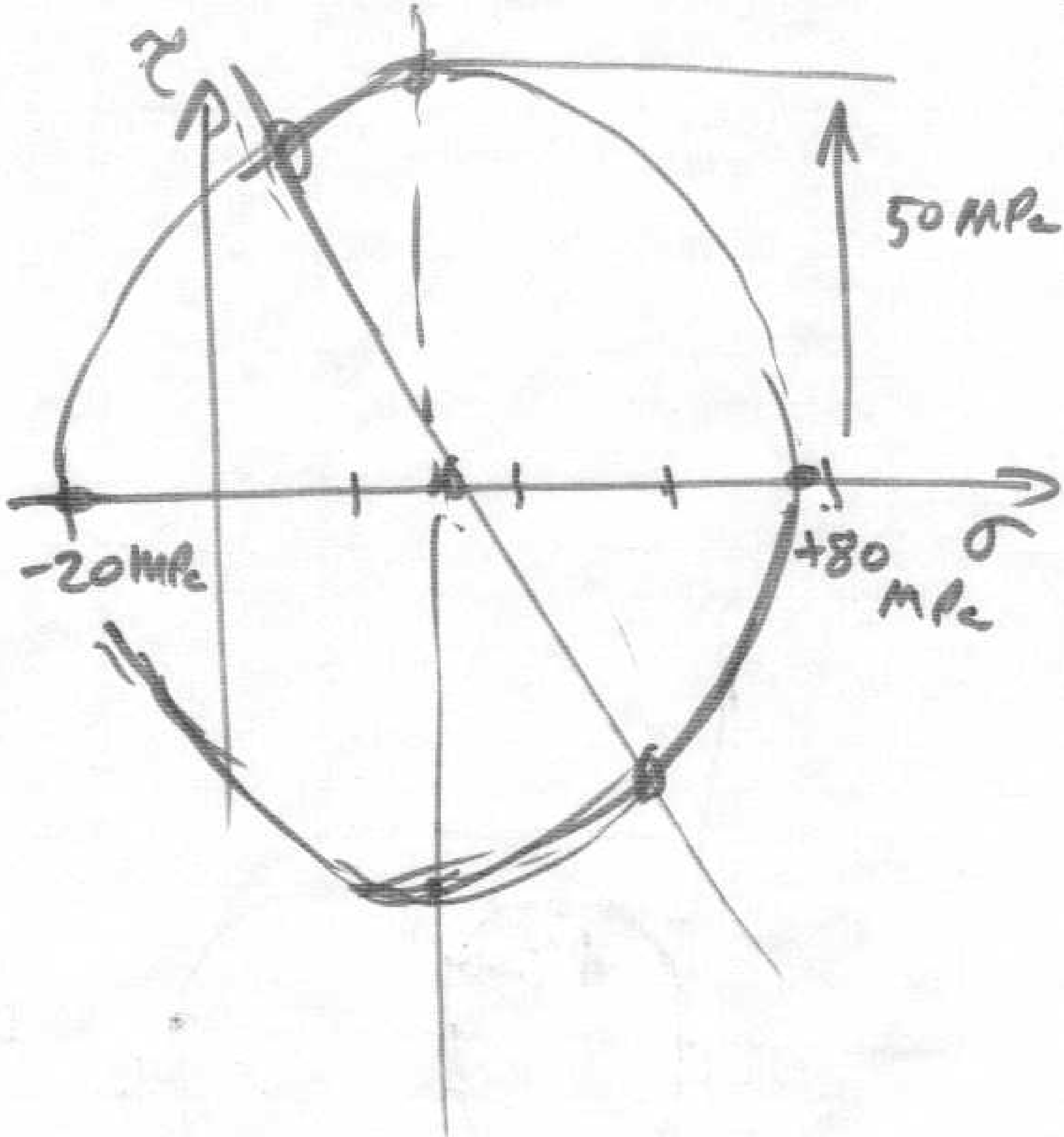
$$\boxed{\sigma_1, \sigma_2 = 80, -20 \text{ MPa}}$$

$$\boxed{\begin{aligned}\tau_{max} &= |80-(-20)|/2 \\ &= 50 \text{ MPa}\end{aligned}}$$

$$\tan 2\phi_p = \frac{2(40)}{60-0} = \frac{4}{3}$$

$$2\phi_p = 53^\circ \qquad \phi_p = 26.5^\circ$$




τ
50 MPa
-20 MPa
+80
MPa
σ

ALSO LOOK at EXAMPLES

2-4 , 2-5 , 2-6

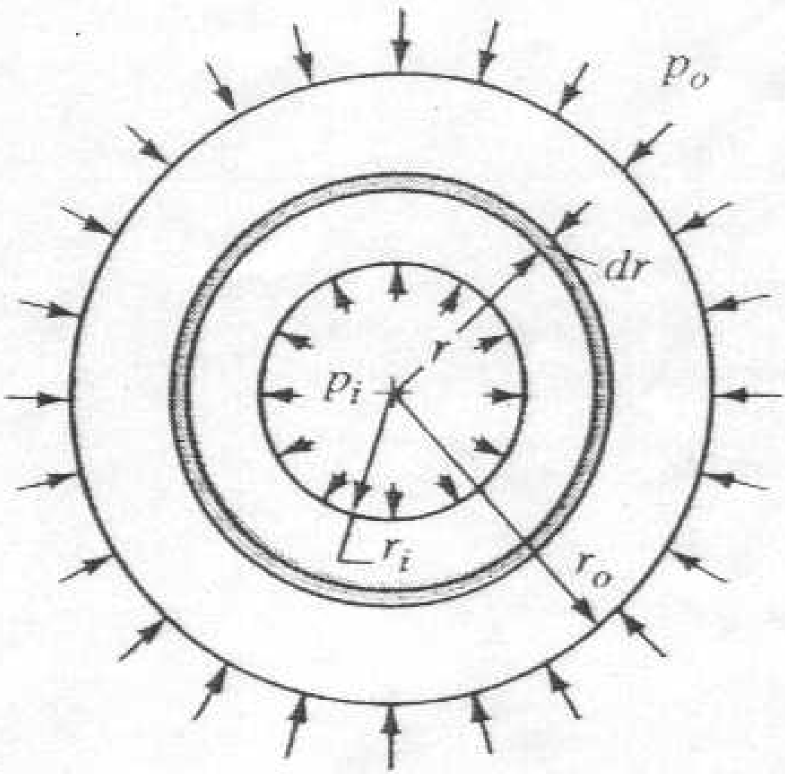
in text.

- No need to formally use vectors but you may if you are used to it

FIG 2-23

THICK-WALLED CYLINDERS

inner & outer pressures p_i & p_o



$$\sigma_t = \frac{p_i r_i^2 - p_o r_o^2 - r_i^2 r_o^2 (p_o - p_i)/r^2}{r_o^2 - r_i^2}$$

$$\sigma_r = \frac{p_i r_i^2 - p_o r_o^2 + r_i^2 r_o^2 (p_o - p_i)/r^2}{r_o^2 - r_i^2}$$

NOTE: if $r_i = 0$ (solid cylinder)

$\sigma_{r_o} = -p_o$ and $\sigma_{t_{r_o}} = 0$

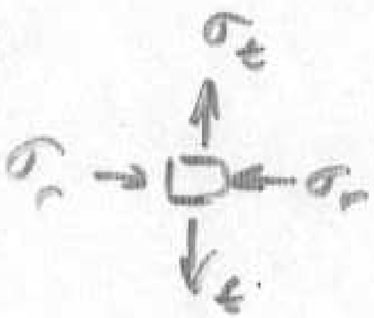
Special case, $p_o = 0$

As usual, positive values indicate tension and negative values, compression.
The special case of $p_o = 0$ gives

$$\sigma_t = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r^2}\right)$$

$$\sigma_r = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 - \frac{r_o^2}{r^2}\right) \qquad (2\text{-}51)$$

The equations of set (2-51) are plotted in Fig. 2-24 to show the distribution of stresses over the wall thickness.



Tangential

Radial

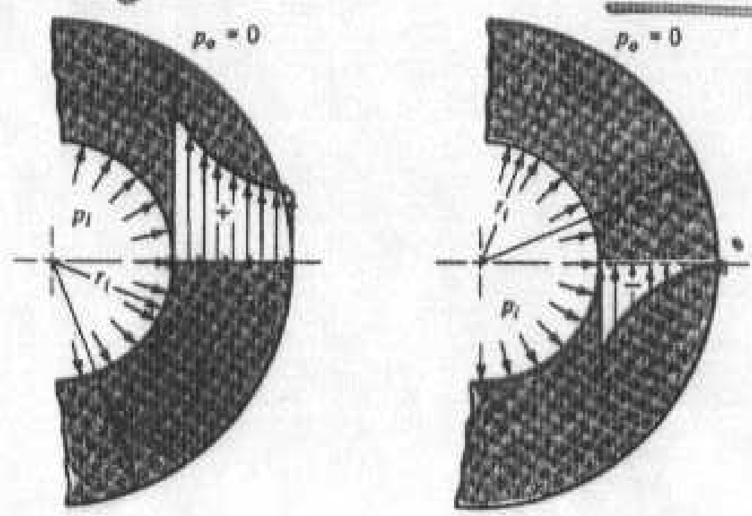


FIGURE 2-24

Distribution of stresses in a thick-walled cylinder subjected to internal pressure.

(a) Tangential stress distribution

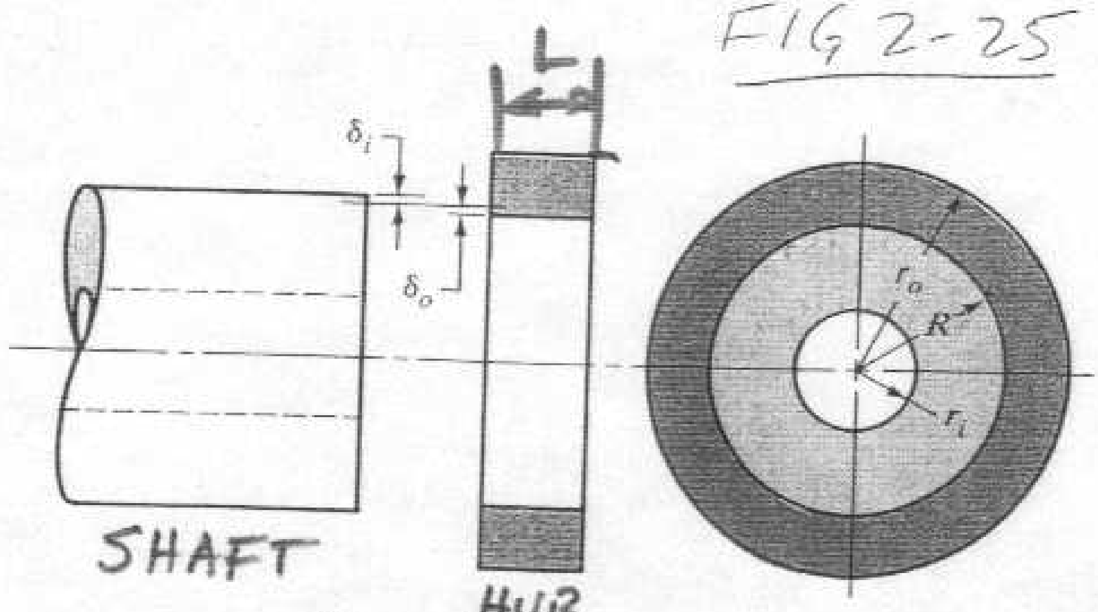
(b) Radial stress distribution

tension

Compresion

$\delta = |\delta_i| - |\delta_o|$

FIG 2-25



(a) (b)

PRESS FIT

GENERAL SITUATION:

SHAFT > nominal size by δ_i

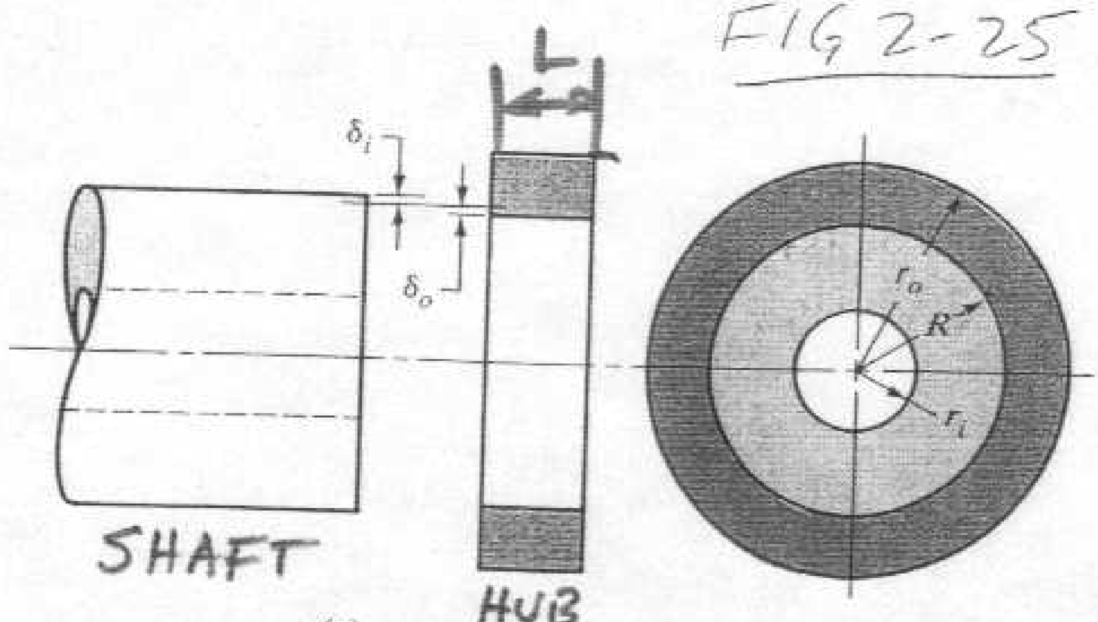
HUB < nominal size by δ_o

INTERFERENCE $\delta = |\delta_i| + |\delta_o|$

NOTE r_i & r_o have different meaning from single thick-walled cylinder eq's.

$$\delta = |\delta_i| - |\delta_o|$$

FIG 2-25



(a) (b)

PRESS FIT

GENERAL SITUATION:

SHAFT > nominal size by δ_i

HUB < nominal size by δ_o

INTERFERENCE $\delta = |\delta_i| + |\delta_o|$

NOTE r_i & r_o have different meaning from single thick-walled cylinder eq's.

STRESSES AT INTERFACE

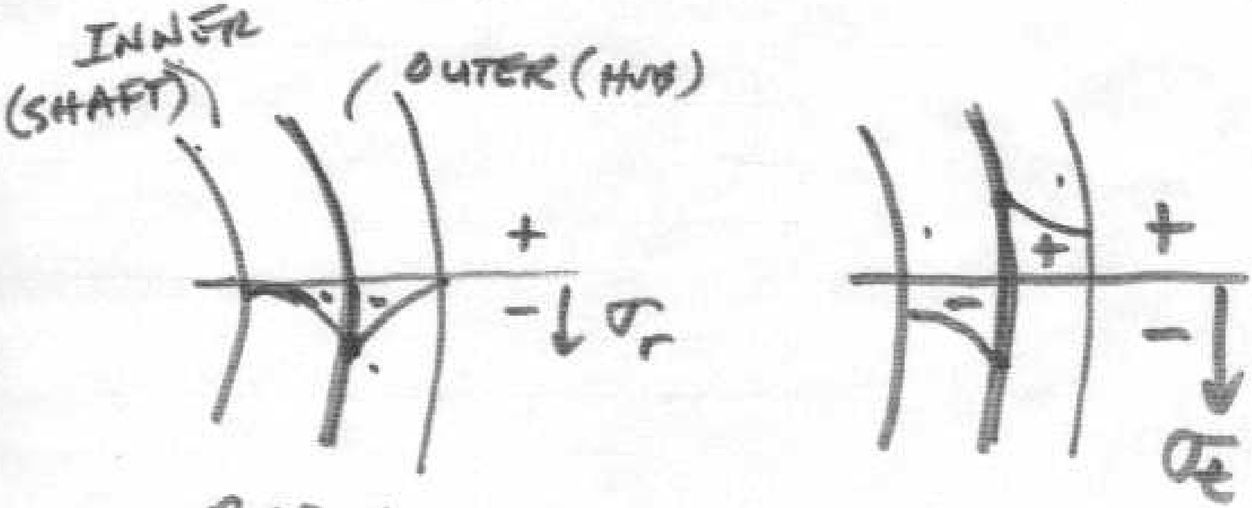
TANGENTIAL

HUB IN TENSION

SHAFT IN COMPRESSION

RADIAL

BOTH IN COMPRESSION



RADIAL

APPLYING THICK-WALLED CYLINDER EQ FOR SHAFT & HUB

SHAFT @ $\underline{\underline{R}}$

$$\sigma_{it} = -p \frac{R^2 + r_i^2}{R^2 - r_i^2} \quad \ldots 2.57$$

PRESSURE @ INTERFACE

HUB

$$\sigma_{ot} = -p \frac{r_o^2 + R^2}{r_o^2 - R^2} \quad \ldots 2.58$$

NEED TO FIND p @ INTERFACE

i.e.

$$\sigma_r = -p$$

THEN WE HAVE σ_t & σ_r values

The tangential (circumferential) strain ϵ_{ot} can be related to both the interference:

$$\epsilon_{ot} = \frac{\delta_o}{R}$$

and the stress

$$\epsilon_{ot} = \frac{\sigma_{ot}}{E_o} - \frac{\nu \sigma_{or}}{E_o}$$

So that (using 2-57 & 2-58) we have

$$\delta_o = \frac{pR}{E_o}\left(\frac{r_o^2 + R^2}{r_o^2 - R^2} + \nu_o\right)$$

Similarly

$$\delta_i = \frac{pR}{E_i}\left(\frac{R^2 + r_i^2}{R^2 - r_i^2} - \nu_o\right)$$

recall That $\delta = \delta_i + \delta_o$, we can solve previous 2 eqs. for p. If one matl E, ν

$$p = \frac{E\delta}{R}\left(\frac{(r_o^2 - R^2)(R^2 - r_i^2)}{2R(r_o^2 - r_i^2)}\right) \quad ..2.60$$

note $r_o > R$ and $R > r_i$

$\therefore$ p is positive number but The radial stress at interface $\sigma_r = -p$ (i.e. compressive)

Steel shaft & hub

e.g. $r_i = 0$, $R = 1''$ $r_o = 2''$

$\delta = 0.002''$ $E = 30 \times 10^6$ psi

from eq. 2-60

$$p = \frac{30\times10^6 \times 2 \times 10^{-3}}{1}\left(\frac{(2^2-1^2)1^2}{2(1)(2^2)}\right)$$

$$= 30\times10^6 \times 2 \times 10^{-3} \times \frac{3}{8}$$

$$\boxed{p = 22.5\times10^3 \text{ psi}}$$

1 inch length has area, A

$\pi d(1) = 2\pi R(1) =$

$= 12.56 \text{ in}^2$

$N = 282{,}000$ lb

$\mu N = 0.3(282{,}000) = 84{,}600$ lb

SHRINK FIT

THE HUB IS SIZED SMALLE
THAN THE SHAFT FOR PROPE
INTERFERENCE.

HUB IS HEATED TO INCREAS
ITS SIZE

COEF. OF THERMAL EXPANSION

$$\epsilon_T = \alpha \, \Delta T$$

$$\frac{\delta}{R} = \frac{\Delta R}{R} = \alpha \Delta T$$

IT'S SLIPPED ON AND
ALLOWED TO COOL.

$\left(\alpha_{steel} \simeq 10^{-5} / {}^\circ C \right)$ (200°C for previous example)

CONTACT STRESSES

- When convex-convex contacts or convex-concave contacts form localized contact stresses occur
- Can cause surface failure in ball bearings, roller bearings, gears, ball joints etc.
- General form of contact patch is ellipse
- we will look at circular and line contacts

91a

"Point Contacts"

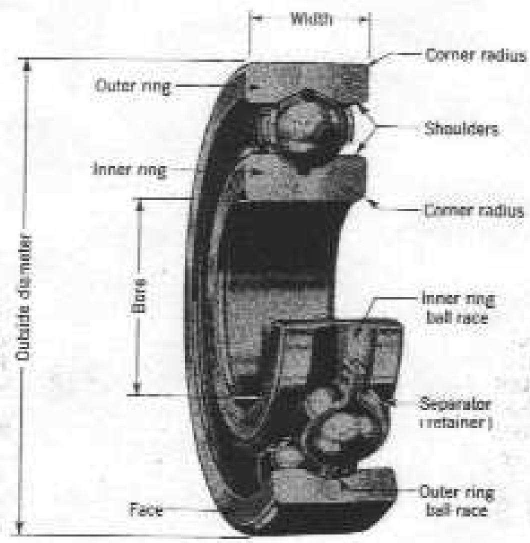
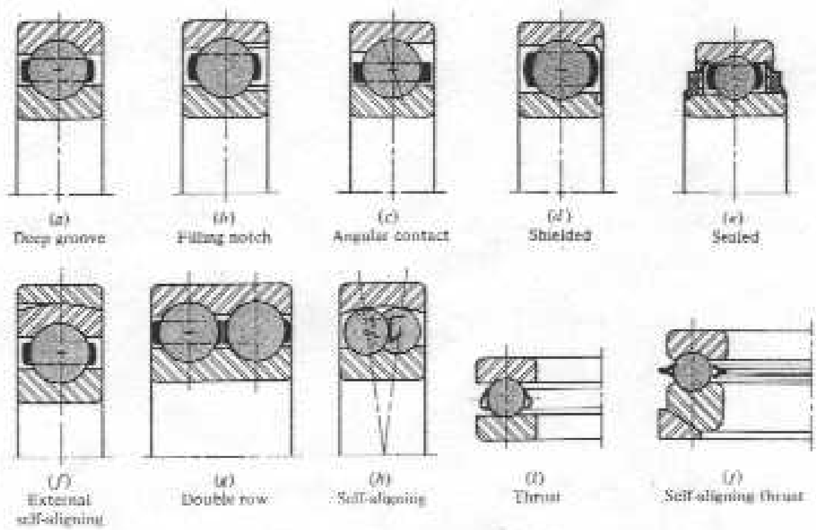


FIGURE 11-1
Nomenclature of a ball bearing. *(Courtesy of New Departure–Hyatt Division, General Motors Corporation.)*



"LINE CONTACTS"

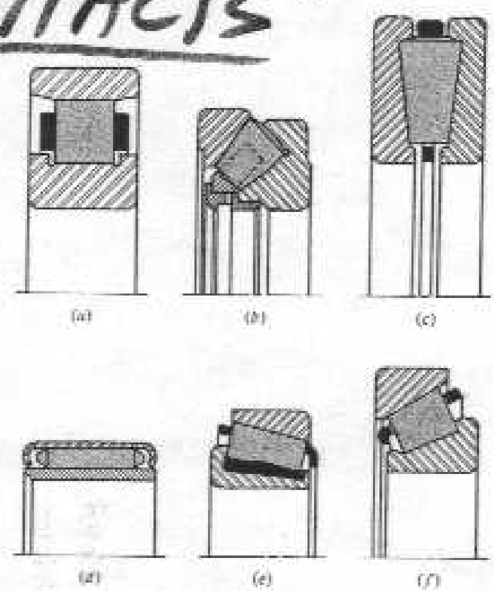
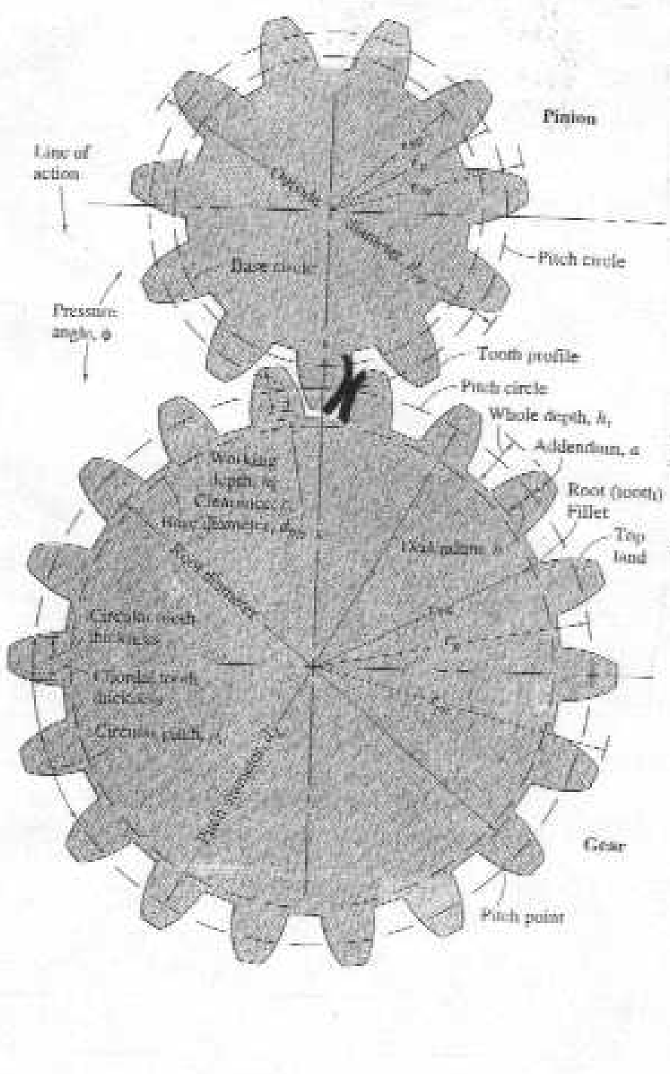
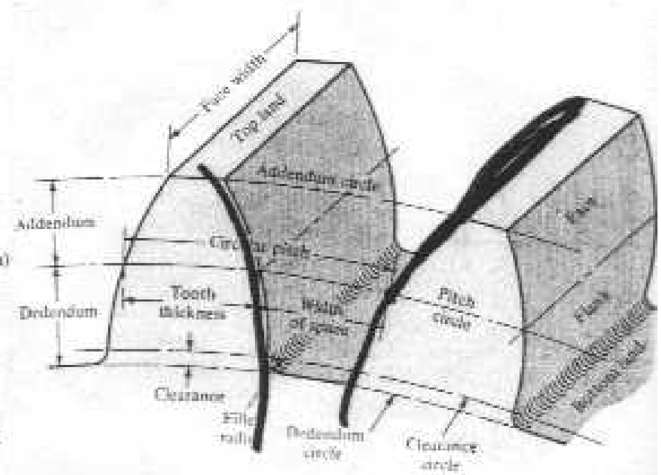


FIGURE 11-3

Types of roller bearings: (*a*) straight roller; (*b*) spherical roller thrust; (*c*) tapered roller thrust; (*d*) needle; (*e*) tapered roller; (*f*) steep-angle tapered roller. (*Courtesy of The Timken Company.*)





CONTACT STRESSES

SPHERES

Avg contact pressure

$$p_a = \frac{F}{A} = \frac{F}{\pi a^2}$$

$$\boxed{p_{max} = \frac{3}{2} p_a}$$

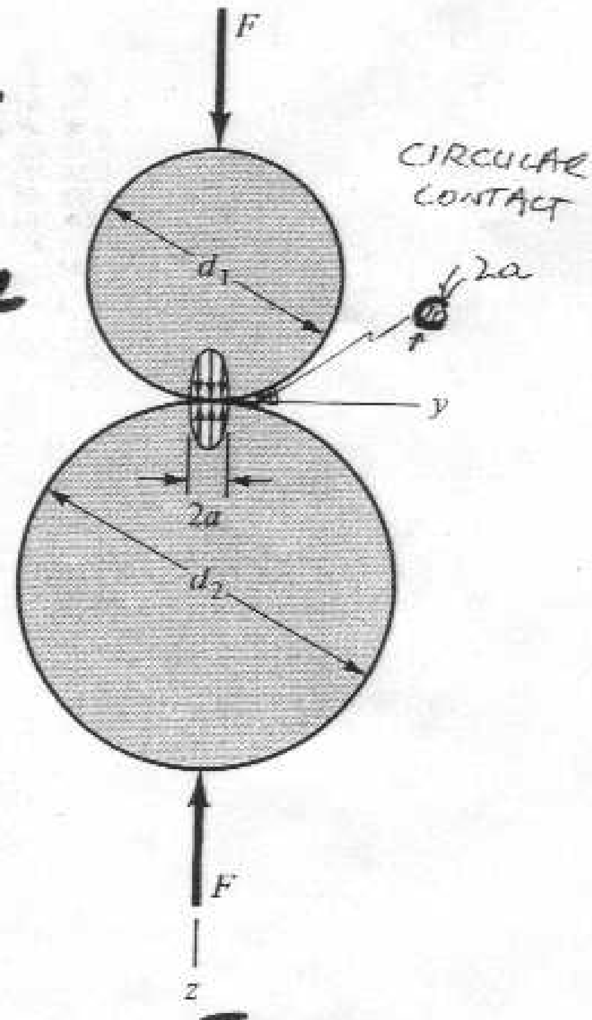


FIGURE 2-32

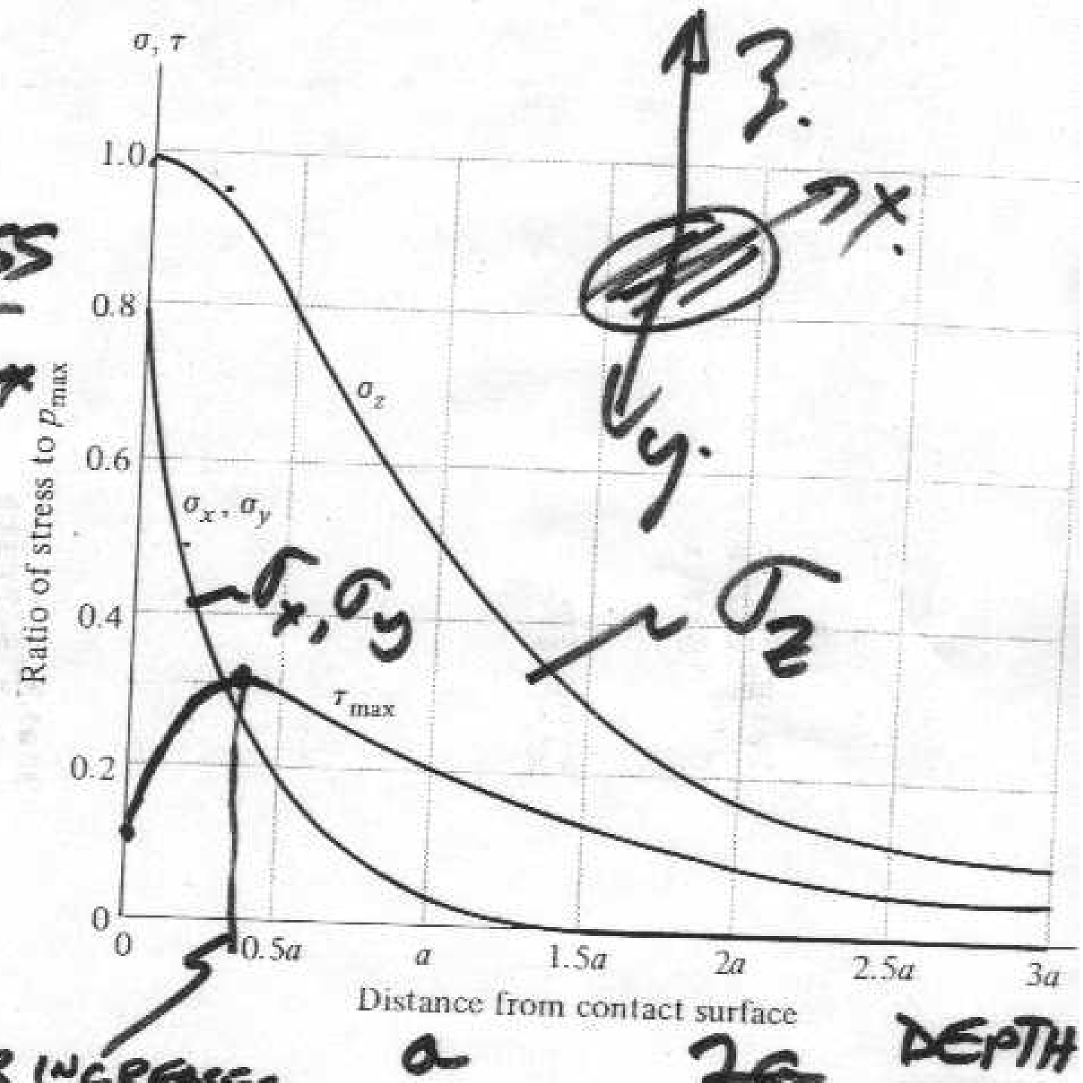
Two spheres held in contact by force F. Contact stress has an elliptical distribution at face of contact of width $2a$.

$$a = \sqrt[3]{\frac{3F}{8} \frac{(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2}{1/d_1 + 1/d_2}}$$

FROM HERTZ THEORY

d positive for convex surface
d negative for concave surface

STRESS / p_{max}



DEPTH BELOW SURFACE

- MAX SHEAR INCREASES WITH DEPTH UPTO 0.4 a.
- OFTEN FAILURES START BELOW SURFACE

FIGURE 2-33

Magnitude of the stress components below the surface as a function of the maximum pressure for contacting spheres. Note that the maximum shear stress is slightly below the surface and is approximately $0.3p_{max}$. The chart is based on a Poisson's ratio of 0.30. Note that the normal stresses are all compressive stresses.

Example

TWO STEEL SPHERES $d_1 = d_2 = d = 10\,mm$

$E_1 = E_2 = E = 207\ GPa$

$\nu_1 = \nu_2 = \nu = 0.3$

$F = 100\ N$

FIND: Contact radius "a"

p_{max}

Location of max shear

$$a = \sqrt[3]{\frac{3F}{8}\,\frac{(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2}{1/d_1 + 1/d_2}}$$

$$= \sqrt[3]{\frac{3F}{8}\,\frac{\cancel{(2)}(1-\nu^2)}{E}\cdot\frac{d}{\cancel{(2)}}}$$

$$a = \sqrt[3]{\frac{3F}{8}\frac{(1-\mu^2)d}{E}}$$

$$= \sqrt[3]{\frac{300}{8}\frac{(0.91)(0.01\,m)}{207\times10^9\,Pa}}$$

$$\boxed{a = 0.118\times10^{-3}\,m = \underline{0.118\,mm}}$$

$$p_{max} = \frac{3F}{2\pi a^2} \quad @\,0.4a = .05\,mm$$
$$= \underline{50\,\mu m}$$

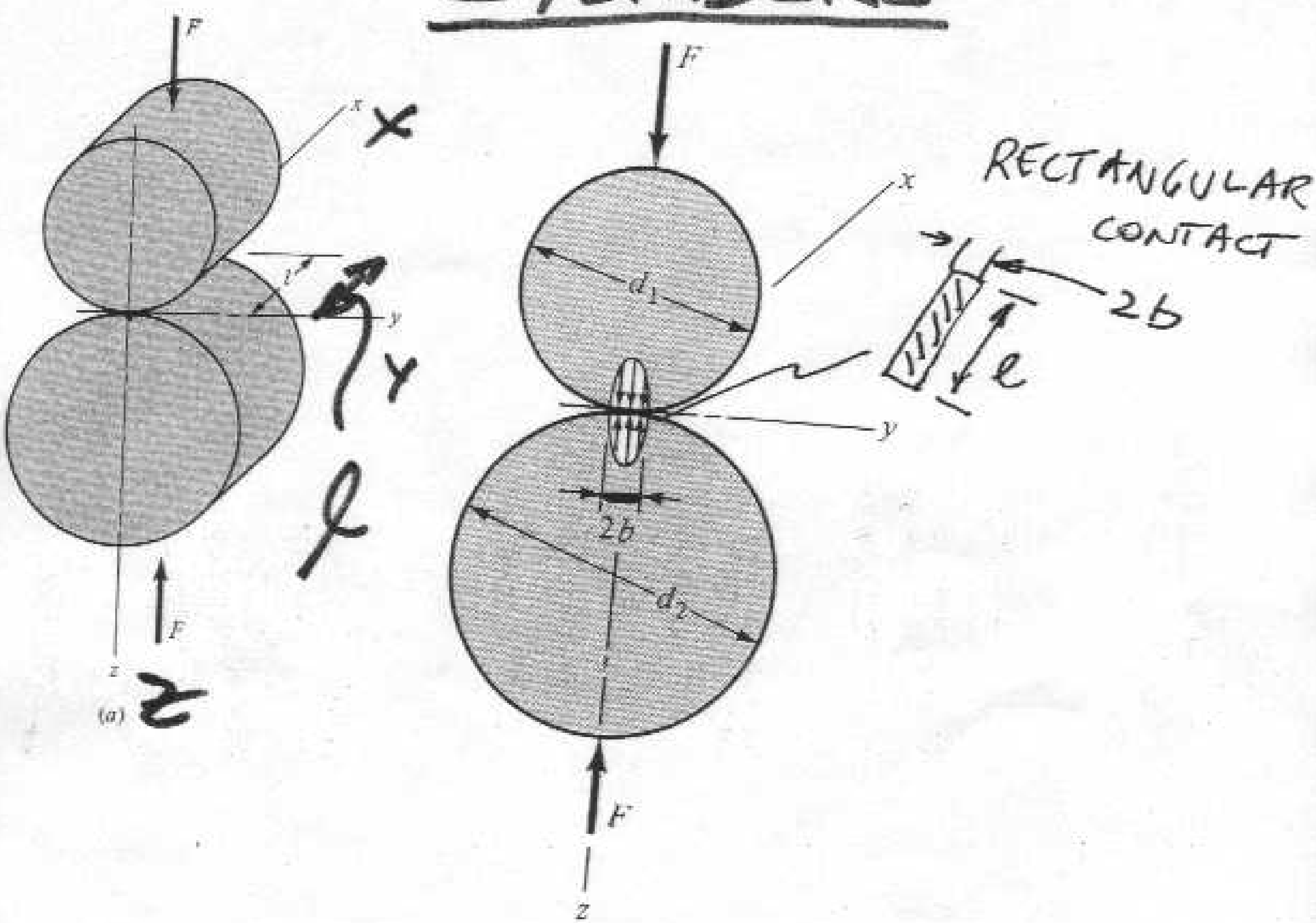
$$\boxed{p_{max} = 3.429\ GPa}$$

$$= \underline{480{,}000\ psi}\ !$$

VERY HIGH ....

Expect plastic deformation

CYLINDERS



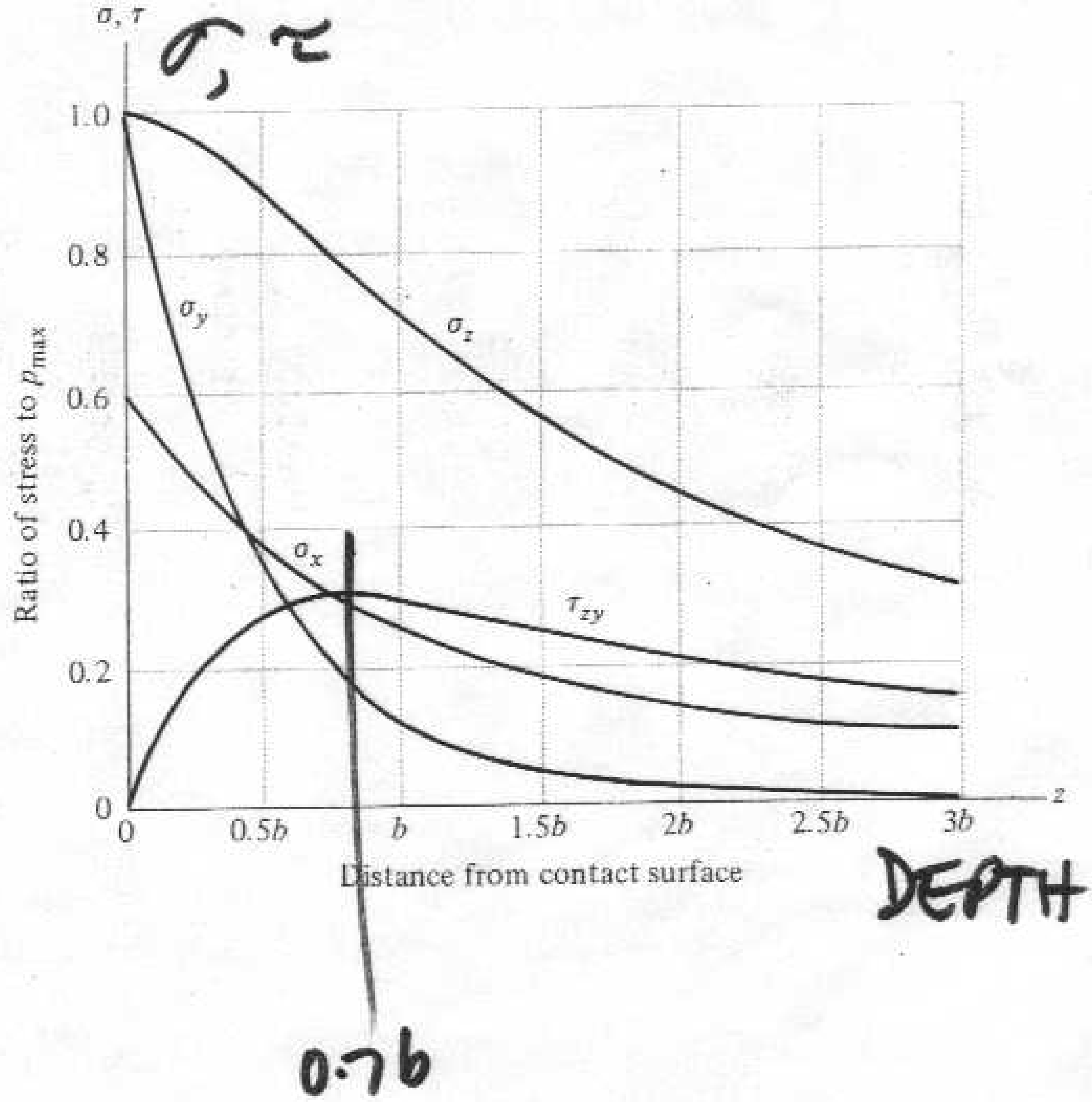
$$b = \sqrt{\frac{2F}{\pi l}\,\frac{(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2}{1/d_1 + 1/d_2}}$$

$$p_{max} = \frac{2F}{\pi b l}$$

FIGURE 2-34

(*a*) Two cylinders held in contact by force *F* uniformly distributed along cylinder length *l*. (*b*) Contact stress has an elliptical distribution at face of contact of width 2*b*.

CYLINDERS



σ, τ
σ, τ
1.0
0.8
0.6
0.4
0.2
0
σ_y
σ_z
σ_x
τ_{zy}
Ratio of stress to p_{max}
0
0.5b
b
1.5b
2b
2.5b
3b
z
Distance from contact surface
DEPTH
0.7b

Example: 2 Steel Cylinders

$d_1 = d_2 = 10\ mm = 0.01\ m = d$

$\nu_1 = \nu_2 = 0.03 = \nu$

$E_1 = E_2 = E = 207\ GPa$

$F = 100 N \quad l = 10\ mm = 0.01\ m$

Find: b, p_{max}, location of p_{max}

$$b = \sqrt{\frac{2F}{\pi l} \frac{(1-\nu^2)}{E} \cdot d}$$

$$= \sqrt{\frac{2(100)(0.91)(.01)}{\pi(0.01)\ 207\times10^9}}$$

$b = 0.0167\ mm = 1.67\times10^{-5}\ m$