6.1 $S_y = 44$ Kpsi Find n (margin of safety)

(a) $\sigma_x = 9$ Kpsi $\sigma_y = -5$ Kpsi

By observation:

$\sigma_1 = 9$ $\sigma_2 = 0$ $\sigma_3 = -5$

MSST : yielding occurs when $\tau_{max} \geq \frac{S_y}{2}$

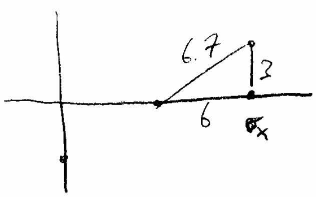
$\tau_{max} = \frac{9+5}{2} = 7$ $\frac{S_y}{2} = 22$ $n = \frac{22}{7} = \boxed{3.14}$

DE : yielding occurs when $\sigma' \geq S_y$

$$\sigma' = \left[\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2}{2}\right]^{1/2}$$

$$\sigma' = \left[\frac{9^2 + 5^2 + 14^2}{2}\right]^{1/2} = 12.29 \quad n = \frac{44}{12.29} = \boxed{3.58}$$

(b) $\sigma_x = 12$ $\tau_{xy} = 3$ cw



$$\underbrace{\begin{array}{l}\sigma_1 = 12.7 \\ \sigma_2 = 0 \\ \sigma_3 = -0.7\end{array}}_{\text{By eq. or by Mohr's circle.}}$$

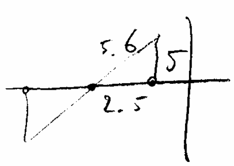
MSST : $\tau_{max} = 6.7$

$\boxed{n = 3.28}$

DE : $\sigma' = 13.06$

$\boxed{n = 3.37}$

(c) $\sigma_x = -4$ $\sigma_y = -9$ $\tau_{xy} = 5$ cw



$\sigma_1 = 0$

$\sigma_2 = -0.9$

$\sigma_3 = -12.1$

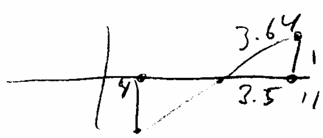
MSST : $\tau = 6.05$

$\boxed{n = 3.64}$

DE : $\sigma' = 11.68$

$\boxed{n = 3.77}$

(d) $\sigma_x = 11$ $\sigma_y = 4$ $\tau_{xy} = 1$ cw



$\sigma_1 = 11.14$

$\sigma_2 = 3.86$

$\sigma_3 = 0$

MSST : $\tau = 5.57$

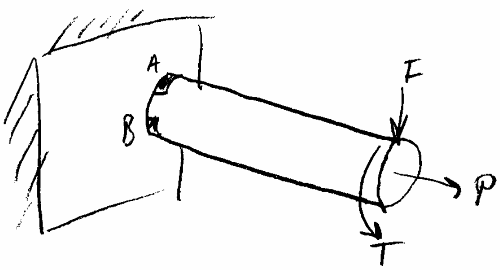
$\boxed{n = 3.95}$

DE : $\sigma' = 9.8$

$\boxed{n = 4.49}$

6.6 AISI 1006 Cold-drawn steel

$F = 0.55\ kN \quad P = 8.0\ kN \quad T = 30\ Nm$



$l = 100\ mm \quad A = \pi\left(\frac{20}{2}\right)^2 = 314.16\ mm^2$

$d = 20\ mm \quad I/c = \frac{\pi}{32} d^3 = 785.4\ mm^3$

A: $\sigma_x = \frac{P}{A} + \frac{M}{I/c} = \frac{8}{314.16} + \frac{.55 \cdot 100}{785.4} = 95.5\ MPa$

$\tau_{yz} = \frac{T}{J/r} = \frac{30}{\frac{\pi}{16} d^3} = 19.1\ MPa$

$\sigma_{1,2} = \frac{95.5}{2} \pm \left[\left(\frac{95.5}{2}\right)^2 + (19.1)^2\right]^{1/2} = 99.18, -3.68$

$\sigma' = 101\ MPa \quad S_y = 330\ MPa$

$\boxed{n = 3.27}$

B: $\sigma_x = \frac{P}{A} = \frac{8}{314.16} = 25.5\ MPa$

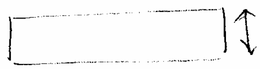
$\tau_{xy} = \frac{T}{J/r} + \frac{4V}{3A} = 19.1 + \underbrace{\frac{4(0.55)}{3(314.16)}}_{2.3\ MPa} = 21.4\ MPa$

$\sigma_{1,2} = \frac{25.5}{2} \pm \sqrt{\left(\frac{25.5}{2}\right)^2 + (21.4)^2} = 37.66, -12.16$

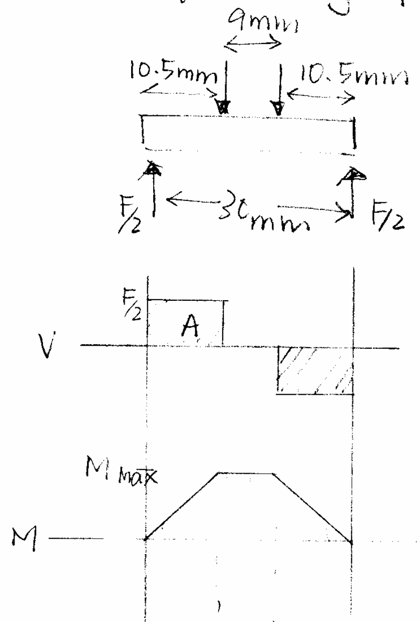
$\sigma' = 45 \quad S_y = 330$

$\boxed{n = 7.33}$

6-16/



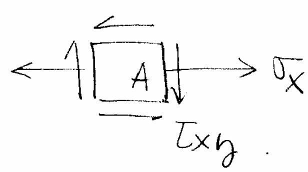
case (1) loading of C $\qquad$ F = 4.4 kN (max)



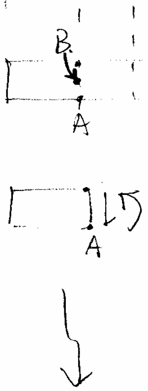
M_{max} will be the total area A in shear diagram,

$$M_{max} = \tfrac{1}{2} F (10.5\text{mm})$$
$$= \tfrac{1}{2}(4.4\times10^{3}\,\text{N})(10.5\times10^{-3}\,\text{m})$$
$$= 23.1\ \text{Nm}$$

For stress element at A



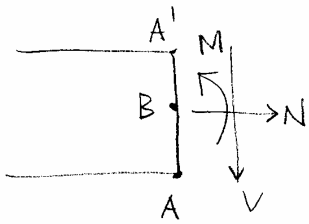
$$\sigma_x = \frac{Mc}{I}$$



For circular area,

$$I = \tfrac{1}{4}\pi r^4 \qquad (r = c)$$

$$\sigma_x = \frac{Mc}{\frac{1}{4}\pi c^4}$$
$$= \frac{4M}{\pi c^3} = \frac{4(23.1\,\text{Nm})}{\pi(6\times10^{-3}\,\text{m})^3} = 136.16\ \text{MPa} = \sigma_{max}$$



For this question, we only consider point A & B. where point A' will have the same stress as A, with opposite direction. we consider point A because this is the point where max stress cause by moment M occur (stress cause by shear is zero). Also, at point B, stress cause by shear V is max but so at point A, using equation in class note,

MSST: $n = \dfrac{S_y}{\underset{\parallel\atop 0}{\sigma_a - \sigma_c}} = \dfrac{S_y}{\sigma_{max}} = \dfrac{220}{136.2} = 1.62$ //

At point B,

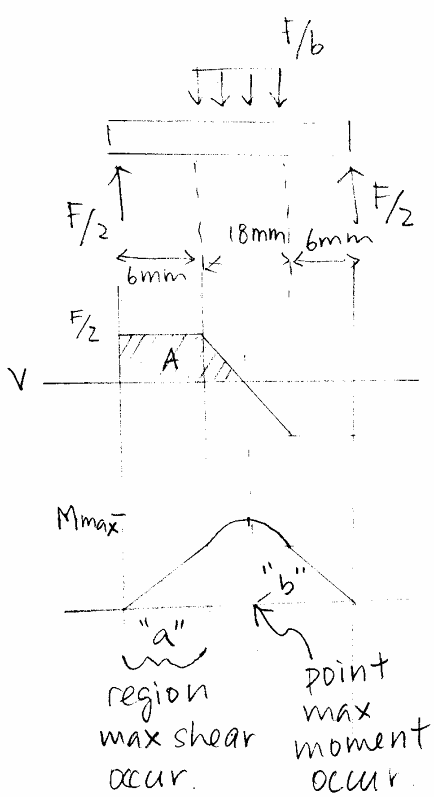
$$\tau_{xy} = \frac{4V}{3A} \quad \text{(Table 2-4, pg 52)}$$

$$= \frac{4}{3}\frac{F/2}{\pi r^2} = \frac{4}{3}\frac{(2.2\,kN)}{\pi(6\times10^{-3}m)^2} = 25.94 \text{ MPa} = \tau_{max}$$

MSST: $n = \dfrac{S_{sy}}{\tau_{max}} = \dfrac{(S_y/2)}{\tau_{max}} = \dfrac{110 \text{ MPa}}{25.94 \text{ MPa}}$

$= 4.24$ //

case (2) loading of d.



First draw shear & Moment diagram as shown

$V_{max} = F/2$

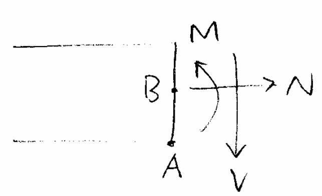
M_{max} = shaded area "A" in shear diagram.

$= \frac{F}{2}(6mm) + \frac{1}{2}(F/2)(9mm)$

$= (2.2\times10^3N)(6\times10^{-3}m) + \frac{1}{2}(2.2\times10^3N)(9\times10^{-3}m)$

$= 23.1$ Nm.

The region/point max shear/moment occur is shown in the graph.



∴ determine σ at point A at "b"

& determine τ_{xy} at point B at "a" point A,

$$\sigma_x = \frac{4M}{\pi c^3} = \frac{4(231.1\ Nm)}{\pi(6\times10^{-3}m)^3} = 136.16\ MPa$$

$$MSST:\ n = \frac{S_y}{\sigma_{max}} = \frac{220}{136.2} = 1.62,$$

point B,

$$\tau_{xy} = \frac{4}{3}\frac{V}{A} = \frac{4}{3}\frac{(2.2\,kN)}{\pi(6\times10^{-3}m)^2} = 25.94\ MPa.$$

$$MSST:\ n = \frac{S_{sy}}{\tau_{max}} = 4.24\ //$$

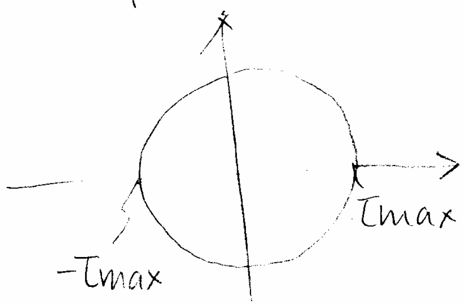
∴ The two cases has the same factor of safety.

#4 show "shear strength" - S_{sy} of a material is $0.5\,S_y$ for MSST & $0.577\,S_y$ for Distortion Energy.

By MSST,

$$n = \frac{S_y}{\sigma_a - \sigma_c}$$ class notes

In pure shear,



$\sigma_a = \tau_{max}$

$\sigma_c = -\tau_{max}$

$$\therefore n = \frac{S_y}{\tau_{max} - (-\tau_{max})} = \frac{S_y}{2\tau_{max}}$$

define shear strength S_{sy} by

$$n = \frac{S_{sy}}{\tau_{max}}$$

$$\Rightarrow \frac{S_y}{2\tau_{max}} = \frac{S_{sy}}{\tau_{max}}$$

$$\Rightarrow \boxed{S_{sy} = 0.5\,S_y} //$$

By Distortion Energy theorem,

$n = S_y / \sigma'$

For pure shear, $\sigma_1 = \tau_{max}$, $\sigma_2 = -\tau_{max}$

von-Mises stress

$$\sigma' = \sqrt{\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2}$$

$$= \sqrt{\tau_{max}^2 + \tau_{max}^2 + \tau_{max}^2}$$

$$= \sqrt{3}\,\tau_{max}$$

$$\therefore \quad n = \frac{S_y}{\sqrt{3}\,\tau_{max}} \quad \& \quad n = \frac{S_{sy}}{\tau_{max}}$$

$$\Rightarrow \frac{S_y}{\sqrt{3}\,\tau_{max}} = \frac{S_{sy}}{\tau_{max}}$$

$$S_{sy} = \frac{1}{\sqrt{3}} S_y$$

$$\boxed{S_{sy} = 0.577\, S_y} \,//$$