Assignment #6

Due Mon. March 24

8-11 Use $\frac{k_b}{k_b + k_m} = C = 0.213$

8-21 Use $C = 0.213$

8-24

8-30

8-37 (Assume bracket pivots about lower edge)

8-39

Miscellaneous elements from Ch 8

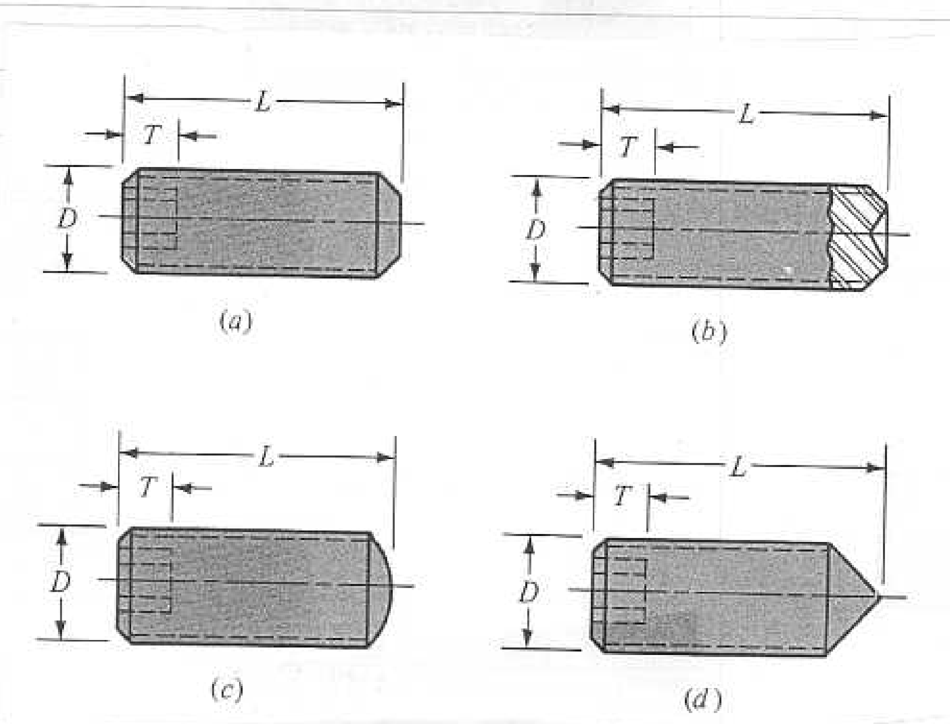
Set Screws ... rely on friction
NOT GOOD

Pins & Keys ... shear loading assumed

Retaining rings ... shear loading assumed

Used to connect components to shafts or to hold assemblies together

SET SCREWS



.. Vibrate loose ... **NEGATIVE**

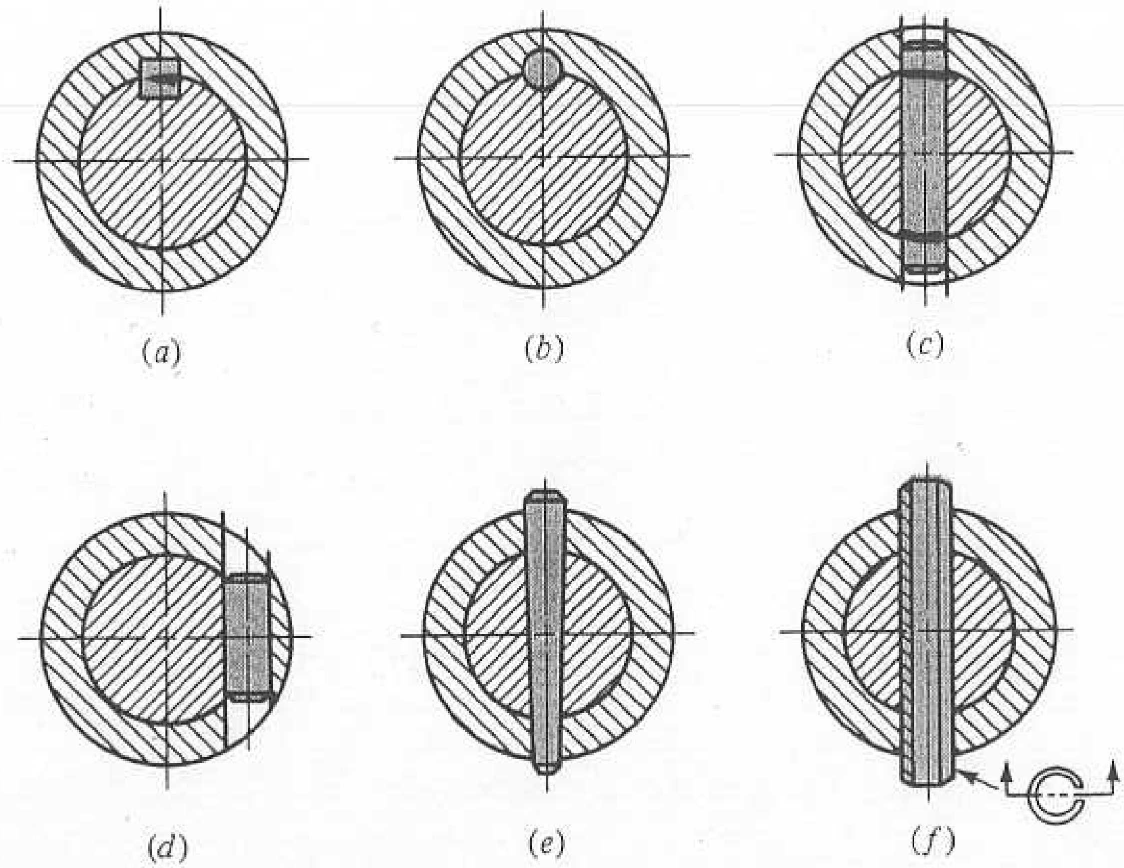


FIGURE 8-27

(*a*) Square key; (*b*) round key; (*c* and *d*) round pins; (*e*) taper pin; (*f*) split tubular spring pin. The pins in parts (*e*) and (*f*) are shown longer than necessary, to illustrate the chamfer on the ends, but their lengths should be kept smaller than the hub diameters to prevent injuries due to projections on rotating parts.

BETTER METHODS

KEY
PIN > SHEAR ACROSS INTERFACE
@ SHAFT/COLLAR"

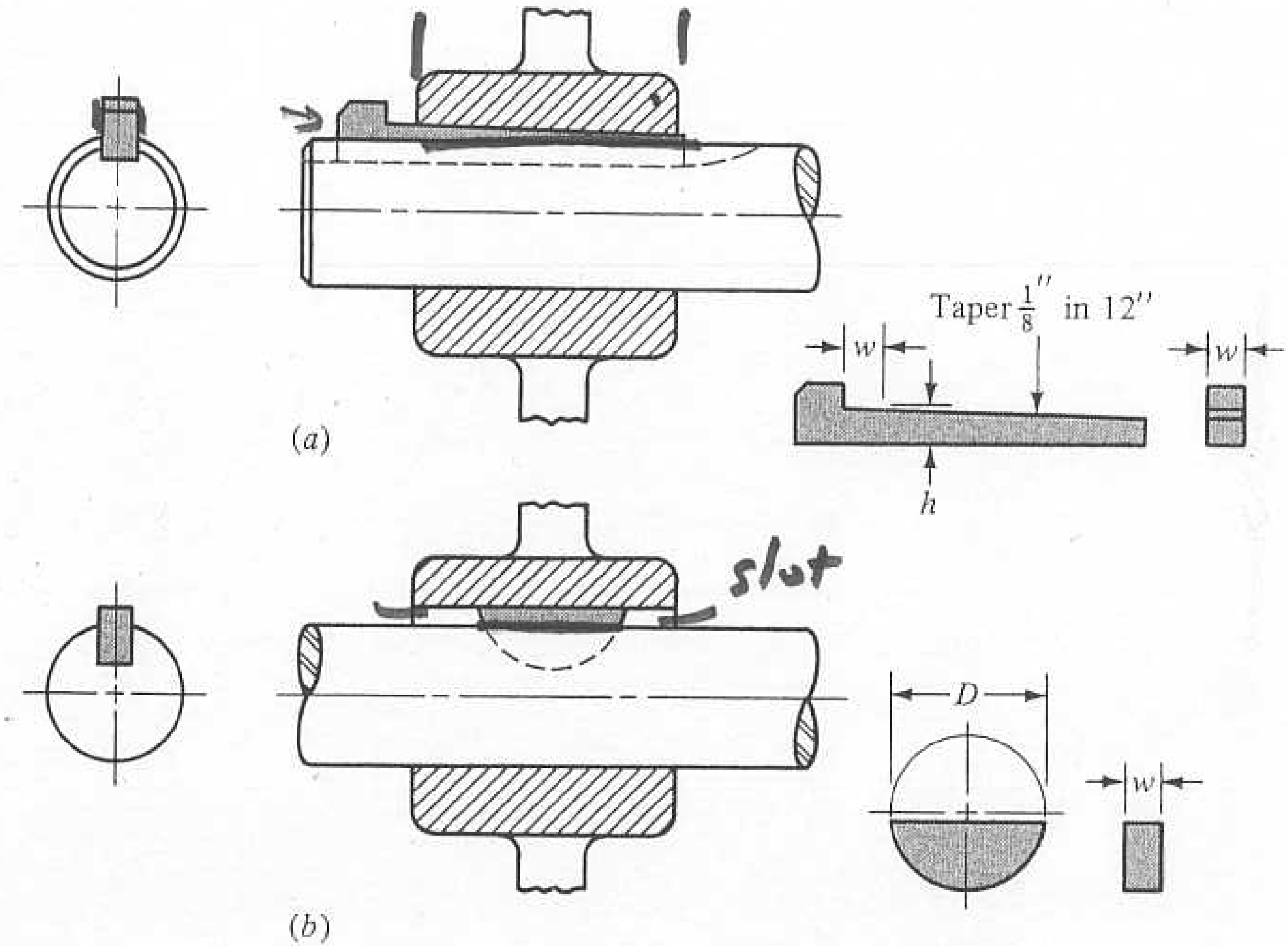
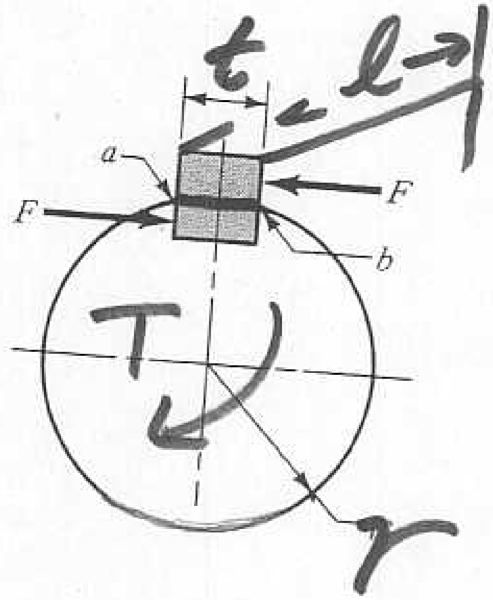


FIGURE 8-28

(a) Gib-head key; (b) Woodruff key.

KEYS & KEYWAYS



$$F = \frac{\text{Torque}}{r_{\leftarrow \text{Shaft}}}$$

$$\boxed{\tau = \frac{F}{tl}} \leftarrow \text{Area}$$

(usually ignore bending)

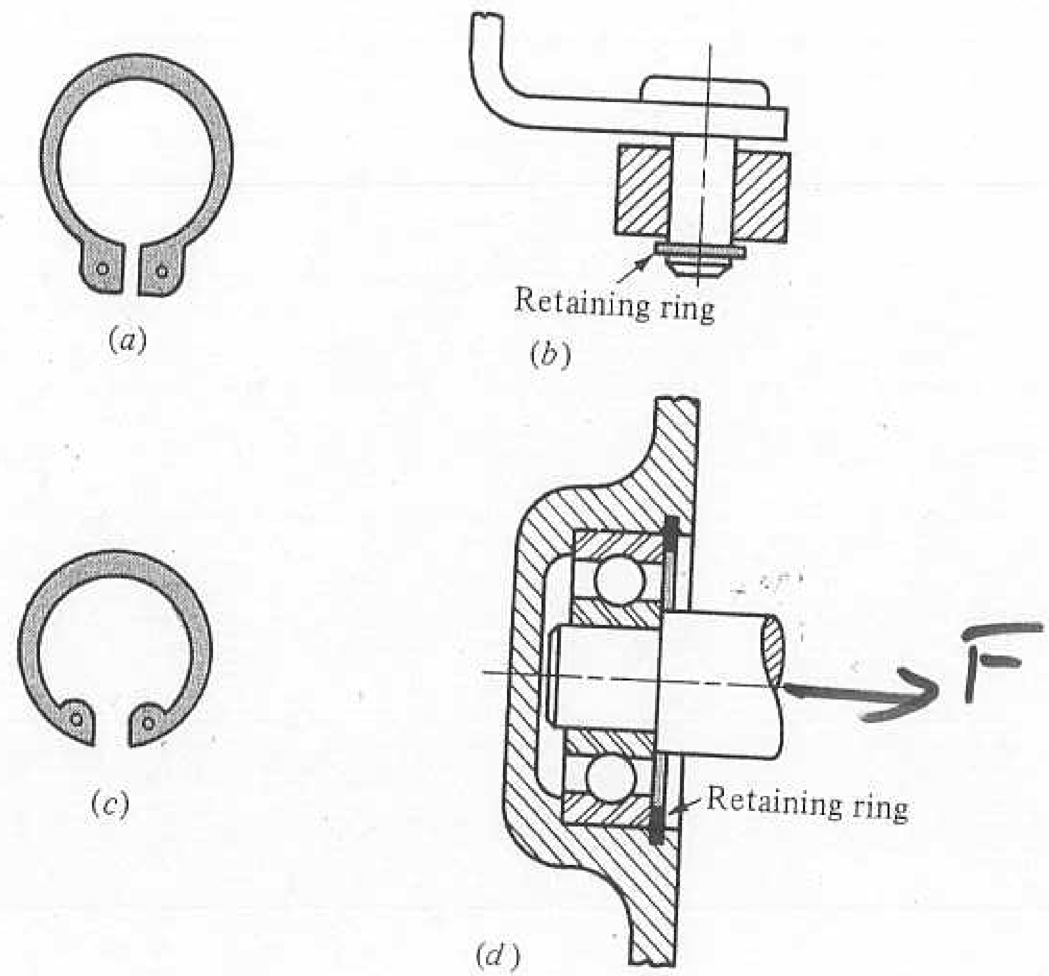


FIGURE 8-29

Typical uses for retaining rings. (a) External ring and (b) its application; (c) internal ring and (d) its application.

- Retaing (snap) rings fit into grooves
- Assumed to fail in shear

Plans for rest of semester

Wk of	
	#9 — Notes only + simple probs
	#10
3/17	Welds + Springs
3/24	Journal bearings #12
3/31	Journal bearings + Review Test
4/7	Mon Test (4/7) Rolling Bearing #11
4/14	Gears #14
4/21	Clutches & brakes #16

Welds Ch 9

÷ We will <u>not</u> cover in detail ÷

A type of <u>permanent</u> connection

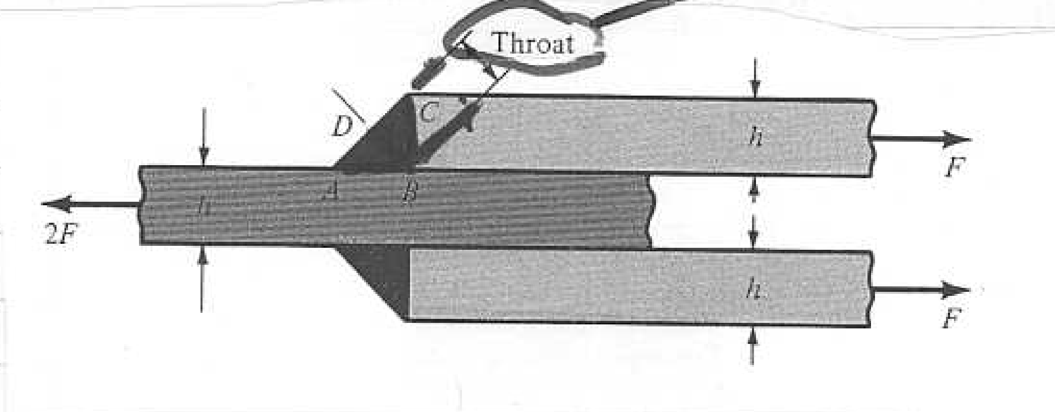
<u>Throat area</u> of weld resists applied

- ÷ TENSION/COMPRESSION
- ÷ SHEAR **

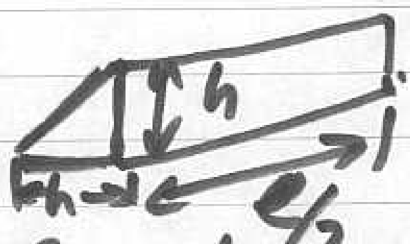
TYPES OF WELDS

Fillet

THROAT

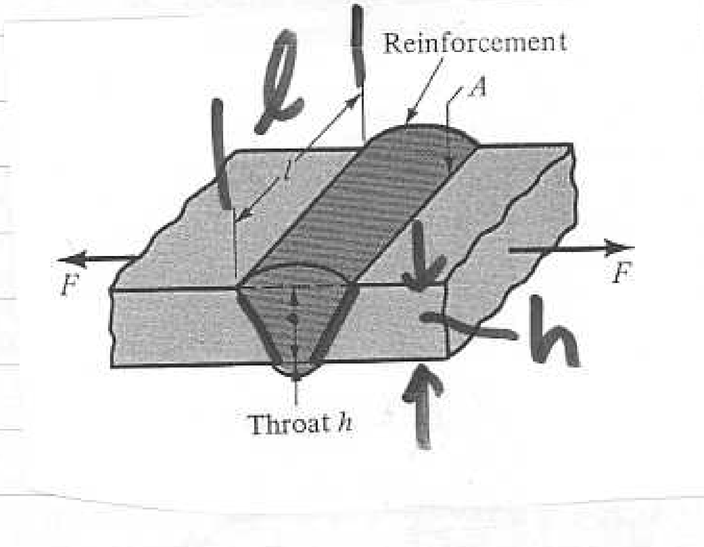


THROAT $= \underline{0.707h}$



h - size, "leg"
l - length

Butt



THROAT $= h$

FILLET WELDS

Use shear stress

$$\tau = \frac{F}{\text{Throat Area}}$$

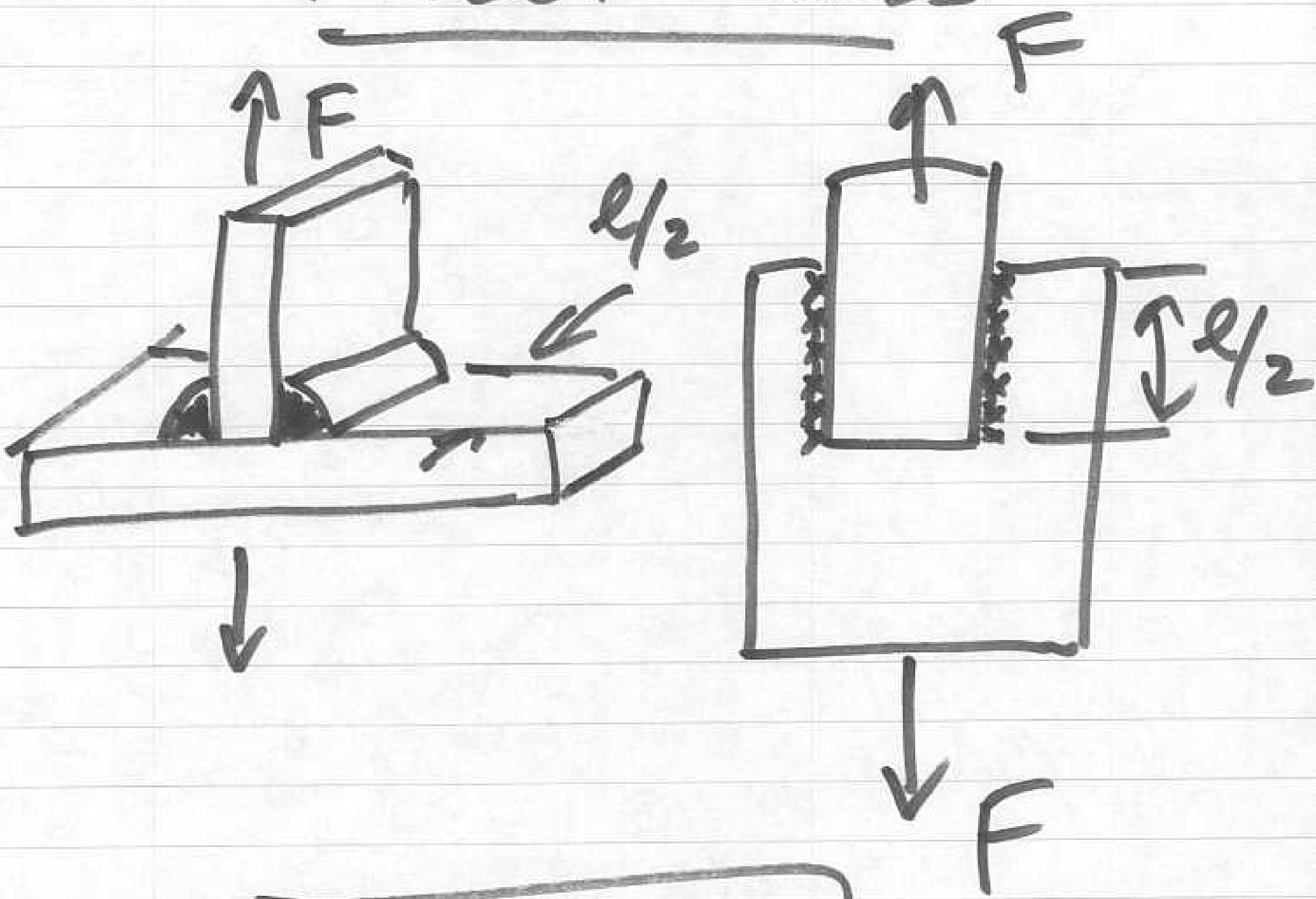
$$= \frac{F}{0.707 h l}$$

$$\boxed{\tau = \frac{1.41 F}{h l}}$$

BUTT WELDS

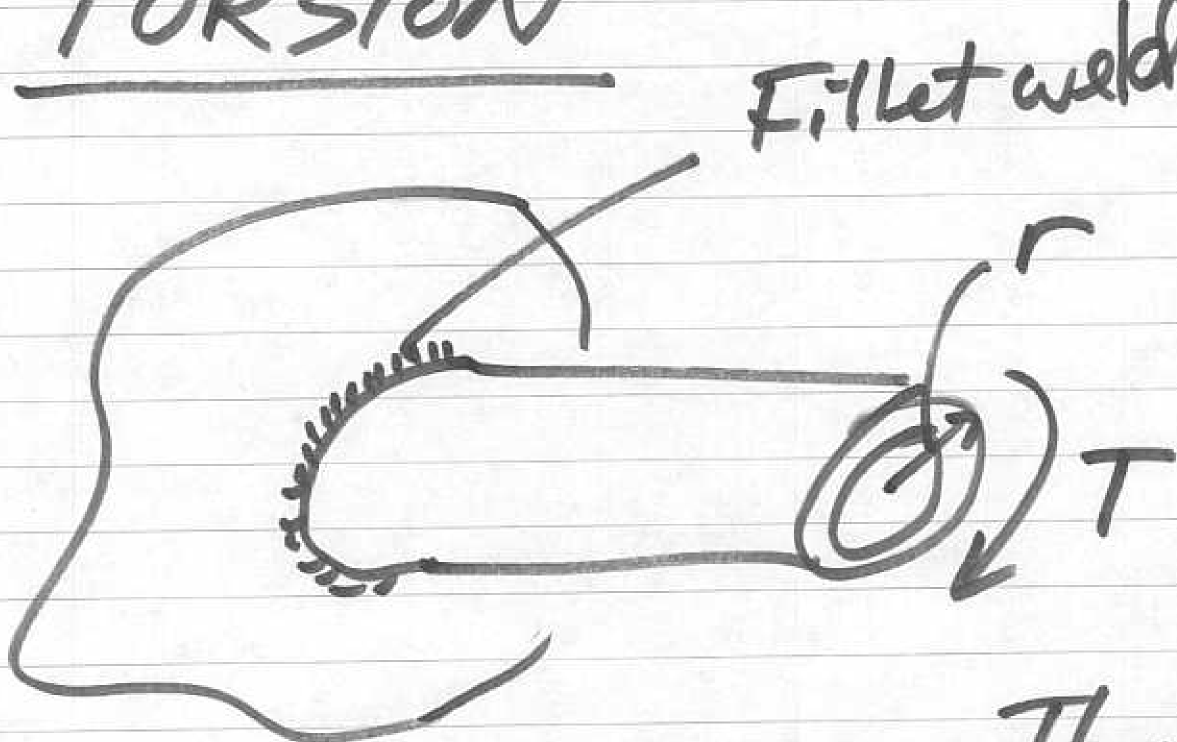
$\div$ become integral with the parts welded

DIRECT LOADING OF FILLET WELDS



$$\boxed{\tau = \frac{1.41F}{hl}}$$

TORSION



Throat

$$2\pi r \times (0.707\,h) = \text{Area}$$

Circum   through

"F"

$$\tau = \left(\frac{T}{r}\right) \Big/ (2\pi r)(0.707h)$$

Area

$$\boxed{\tau = \frac{T}{1.41\pi r^2 h}}$$

WELD MTL PROPERTIES

TABLE 9-4
Minimum Weld-Metal Properties

AWS ELECTRODE NUMBER*	TENSILE STRENGTH, kpsi (MPa)	YIELD STRENGTH, kpsi (MPa)
E60xx	62 (427)	50 (345)
E70xx	70 (482)	57 (393)
E80xx	80 (551)	67 (462)
E90xx	90 (620)	77 (531)
E100xx	100 (689)	87 (600)
E120xx	120 (827)	107 (737)

*The American Welding Society (AWS) specification code numbering system for electrodes. Thi to a four- or five-digit numbering system in which the first two or three digits designate the ap The last digit indicates variables in the welding technique, such as current supply. The nex welding position, as, for example, flat, or vertical, or overhead. The complete set of specifica the AWS upon request.

- WELD MTL IS USUALLY STRONGER THAN JOINED MATERIALS

- USE JOINED MTL PROP

Static Failure

Compare with allowable stresses: (Table 9-5)

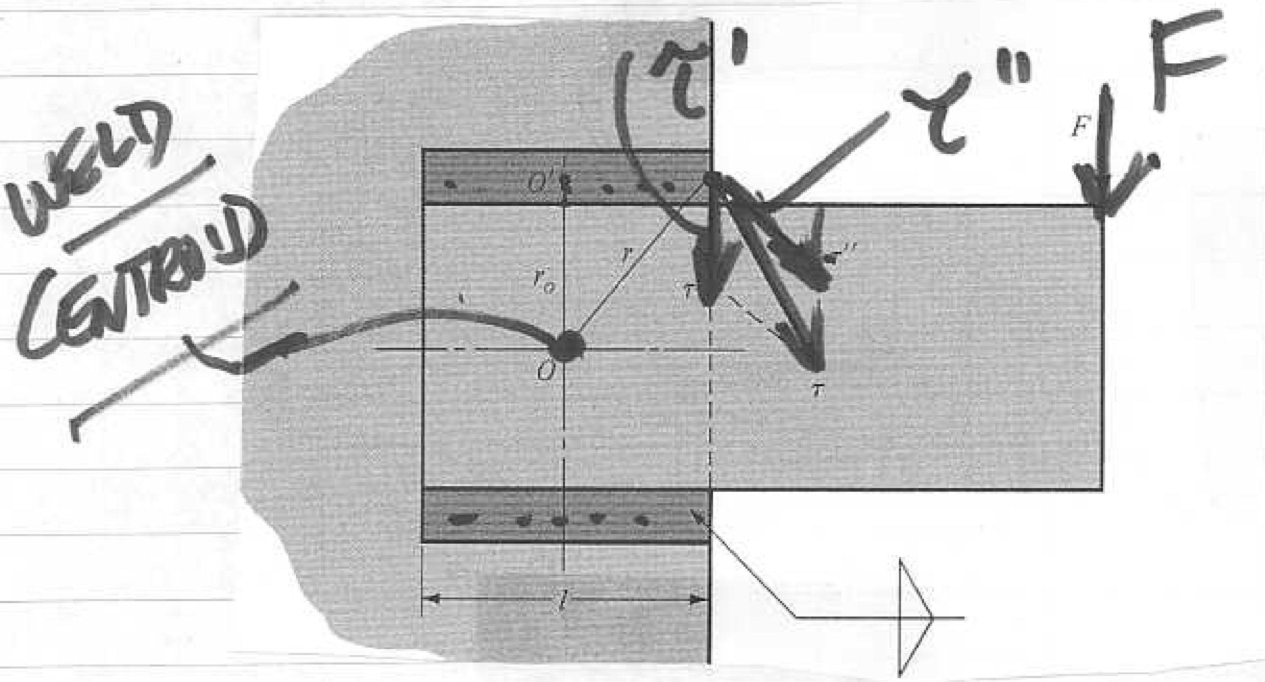
~~**~~ Shear $\tau_{all} = 0.40\, S_y$

Tension/Comp $\sigma_{all} = 0.60\, S_y$

Fatigue Failure

- "welds not really good"? for fatigue but used anyway
- Stress concentration $K_f = \frac{1}{k_e}$ & other modifying factors (Ex 9-2)

ECCENTRIC LOADING



Primary & secondary shear

$$\tau' = \frac{V}{A} \qquad \tau'' = \frac{Mr}{J}$$

POLAR MOMENT OF WELD

COMBINE VECTORIALLY

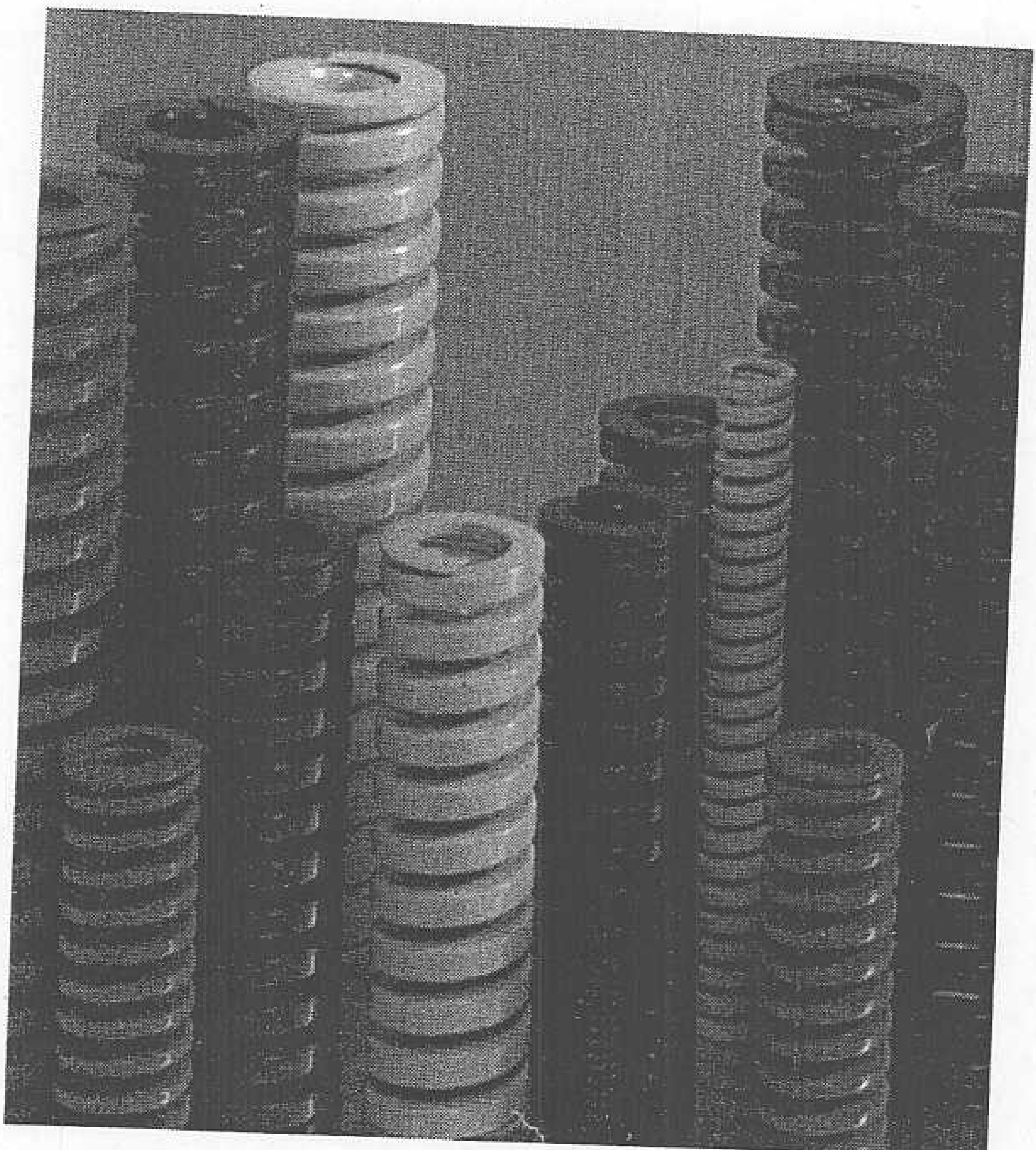
Other methods

- spot welds
- resistance welds
- brazing & soldering
- adhesives & cements

.... generally involve specialized methods of analysis ... rely on manufacturer & own tests.

SPRINGS

Ch 10



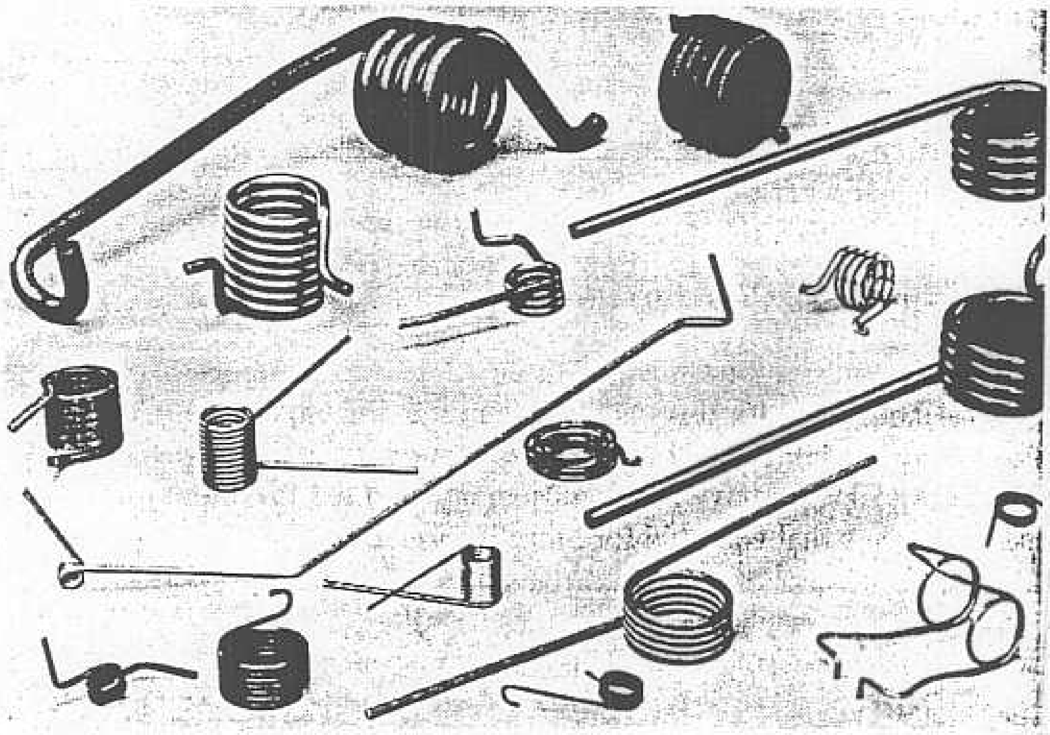
A collection of helical compression springs. (Courtesy of Danly Die)

Mechanical Springs

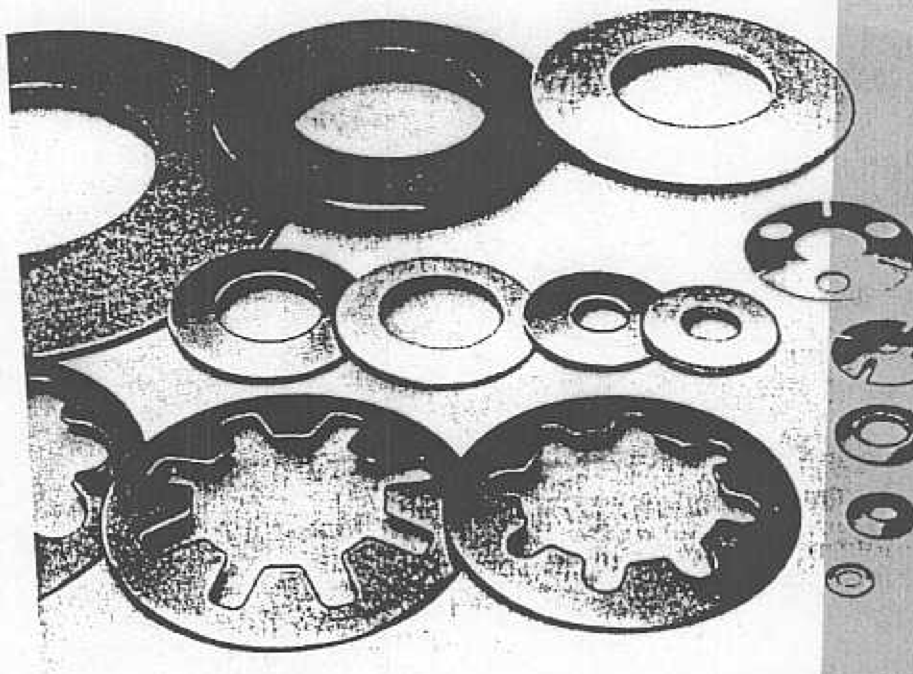
- Provide flexible connections
- Exert force
- Store energy
- Part of energy absorbing systems (along with dampers) e.g. machinery mounts

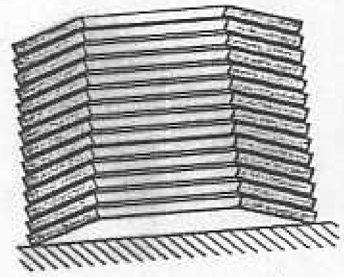
TYPES

Compression, Extension, Torsion, Other

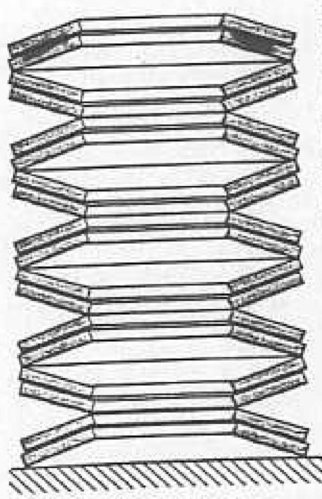


1P

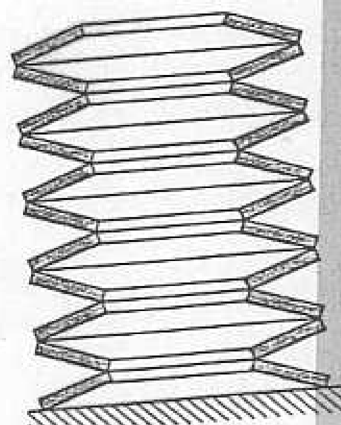




(a) Parallel stack



(c) Series-parallel stack



(b) Series stack

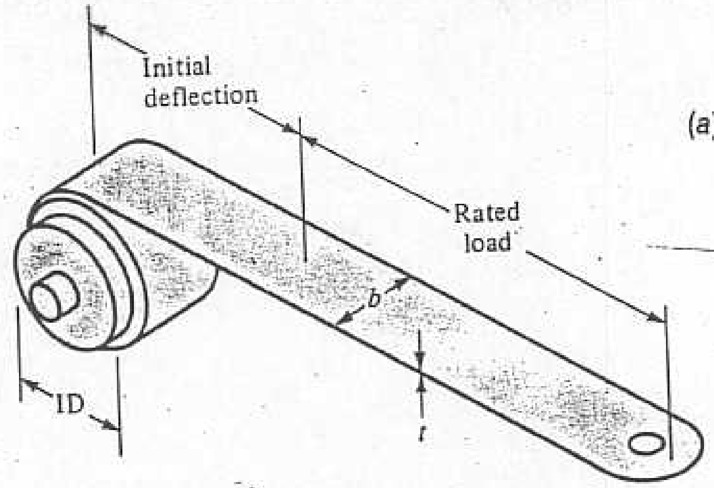


FIGURE 10-12

Constant-force spring. *(Courtesy of Vulcan Spring & Mfg. Co., Huntingdon Valley, Pa.)*

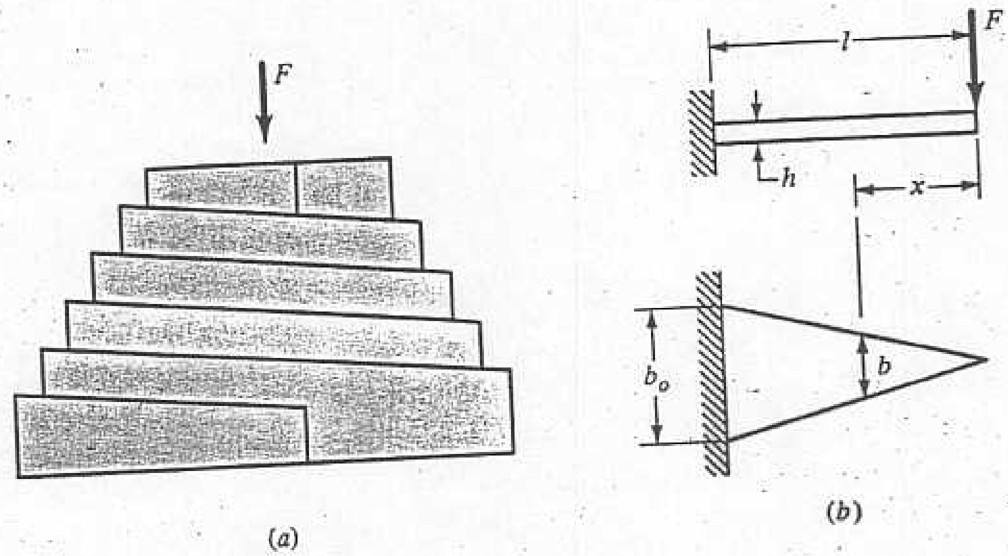


FIGURE 10-13

(*a*) A volute spring; (*b*) a flat triangular spring.

Stresses in Helical Springs

Free Length $F=0$

Solid Length

Deflection y

Spring Constant F/y

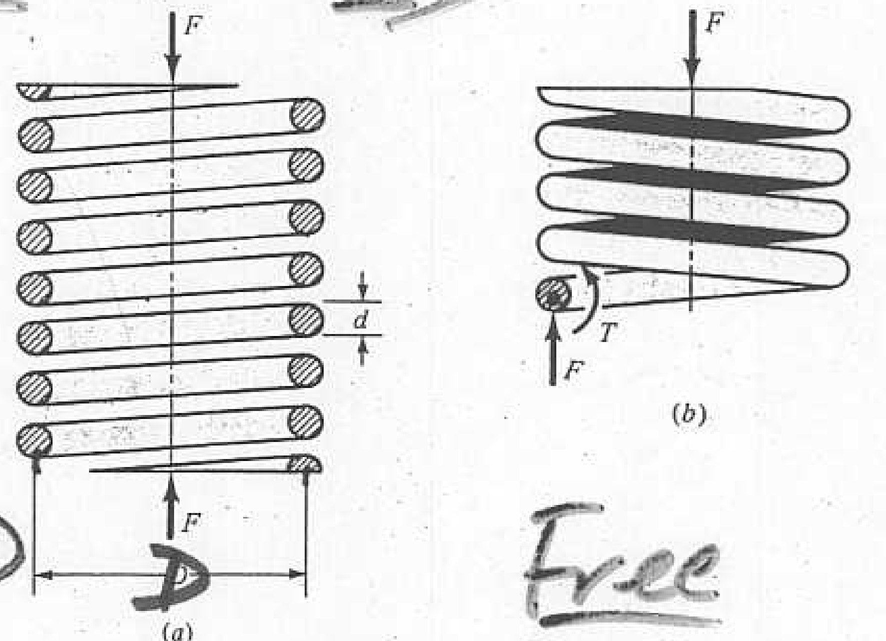


FIGURE 10-1

(*a*) Axially loaded helical spring; (*b*) free-body diagram showing that the wire is subjected to a direct shear and a torsional shear.

Mean Diameter D

"Active" Coils N

Wire Diameter, d

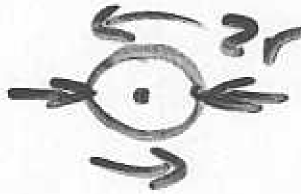
Spring Index, $C = D/d$

Free Body Diagram

$$T = \frac{FD}{2}$$

Primary Shear τ'

$$\tau' = \frac{F}{A} \Leftarrow \frac{\pi d^2}{4}$$



Secondary Shear

$$\tau'' = \frac{Tr}{J} = \frac{FD}{2J}\left(\frac{d}{2}\right)$$

but $J = \frac{2\pi d^4}{32}$

$$\tau'' = \frac{8FD}{\pi d^3}$$

Total Shear

$$\tau' + \tau'' = \frac{4F}{\pi d^2} + \frac{8FD}{\pi d^3}$$

$$\tau = \frac{4F}{\pi d^2}\left(\frac{2}{2}\,\frac{D}{D}\,\frac{d}{d}\right) + \frac{8FD}{\pi d^3}$$

$$= \frac{8FD}{\pi d^3}\left(\frac{1}{2\left(\frac{D}{d}\right)}\right) + \frac{8FD}{\pi d^3}$$

$$= \left(\frac{0.5}{C} + 1\right)\frac{8FD}{\pi d^3}$$

$\frac{D}{d} = C$ usually $6 < C < 12$ Mostly Torsion

$$\tau = K_s \frac{8FD}{\pi d^3}$$

$$K_s = \frac{2C+1}{2C}$$ Shear stress Correction Factor

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e.g. $C = 9$ $\therefore K_s = \frac{18+1}{18} = \frac{19}{18} = \underline{1.06}$

Effect of Curvature =

- like a stress concentration =
- replace K_S by K_B

$$K_B = \frac{4C+2}{4C-3} \quad \text{Bergsträsser Factor}$$

- Curvature factor K_c is

$$K_c = \frac{K_B}{K_S} = \frac{2C(4C+2)}{(4C-3)(2C+1)}$$

- use K_c as stress concentration for **fatigue**
- in place of K_f

DEFLECTION OF HELICAL SPRINGS

- Complex geometry
- Use energy method
  (Castigliano's Theorem)

$$U = \frac{T^2 l}{2GJ} + \frac{F^2 l}{2AG}$$

Potential Energy

(1) (2)

$T = \frac{FD}{2}$ $l = \pi D N$ N ~ # of turns

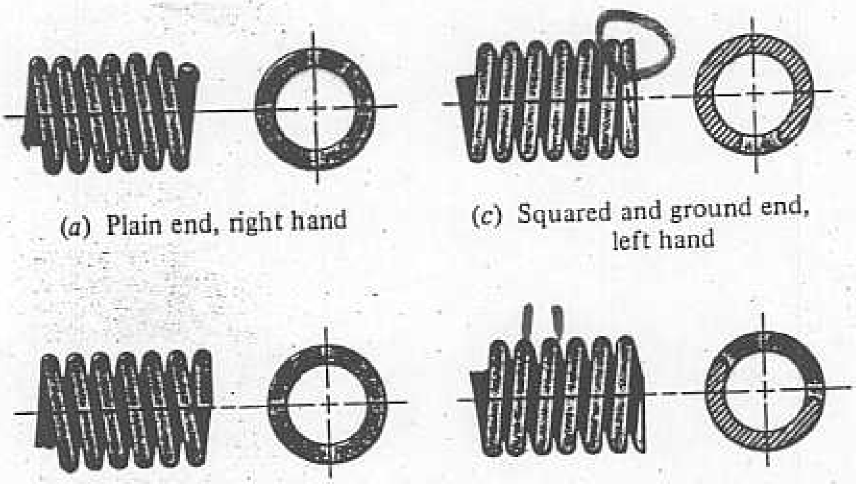
l ~ length of wire

$A = \pi d^2/4$ $J = \pi d^4/32$

(1) Stored energy due to twist

(2) Stored energy due to shear

eqs. 3-30 & 3-31



(*a*) Plain end, right hand

(*c*) Squared and ground end, left hand

(*b*) Squared or closed end, right hand

(*d*) Plain end, ground, left hand

FIGURE 10-5

Types of ends for compression springs: (*a*) both ends plain; (*b*) both ends squared; (*c*) both ends squared and ground; (*d*) both ends plain and ground.

	TYPE OF SPRING ENDS			
TERM	PLAIN	PLAIN AND GROUND	SQUARED OR CLOSED	SQUARED AND GROUND
End coils, N_e	0	1	2	2
Total coils, N_t	N_a	$N_a + 1$	$N_a + 2$	$N_a + 2$
Free length, L_0	$pN_a + d$	$p(N_a + 1)$	$pN_a + 3d$	$pN_a + 2d$
Solid length, L_s	$d(N_t + 1)$	dN_t	$d(N_t + 1)$	dN_t
Pitch, p	$(L_0 - d)/N_a$	$L_0/(N_a + 1)$	$(L_0 - 3d)/N_a$	$(L_0 - 2d)/N_a$

Source: Associated Spring–Barnes Group, *Design Handbook*, Bristol, Conn., 1981, p. 32.

TABLE 10-2

Formulas for Compression-Spring Dimensions. (N_a = Number of Active Coils)

$N = N_a$ (active coils)

Table 10-2

$$\therefore\ U = \frac{4F^2D^3N}{d^4G} + \frac{F^2DN}{d^2G}$$

deflection y, "vertically" in F direction is:

$$y = \frac{\partial U}{\partial F}$$

$$= \frac{8FD^3N}{d^4G} + \frac{\overset{2}{\cancel{4}}FDN}{d^2G}$$

$$\boxed{y \simeq \frac{8FD^3N}{d^4G}}$$

Scale

Spring rate, spring stiffness,

$$k = \frac{F}{y} = \frac{d^4G}{8D^3N}$$

NOTE HIGH POWERS OF d & D

Extensional Springs

Tension + bending (Fr_m)

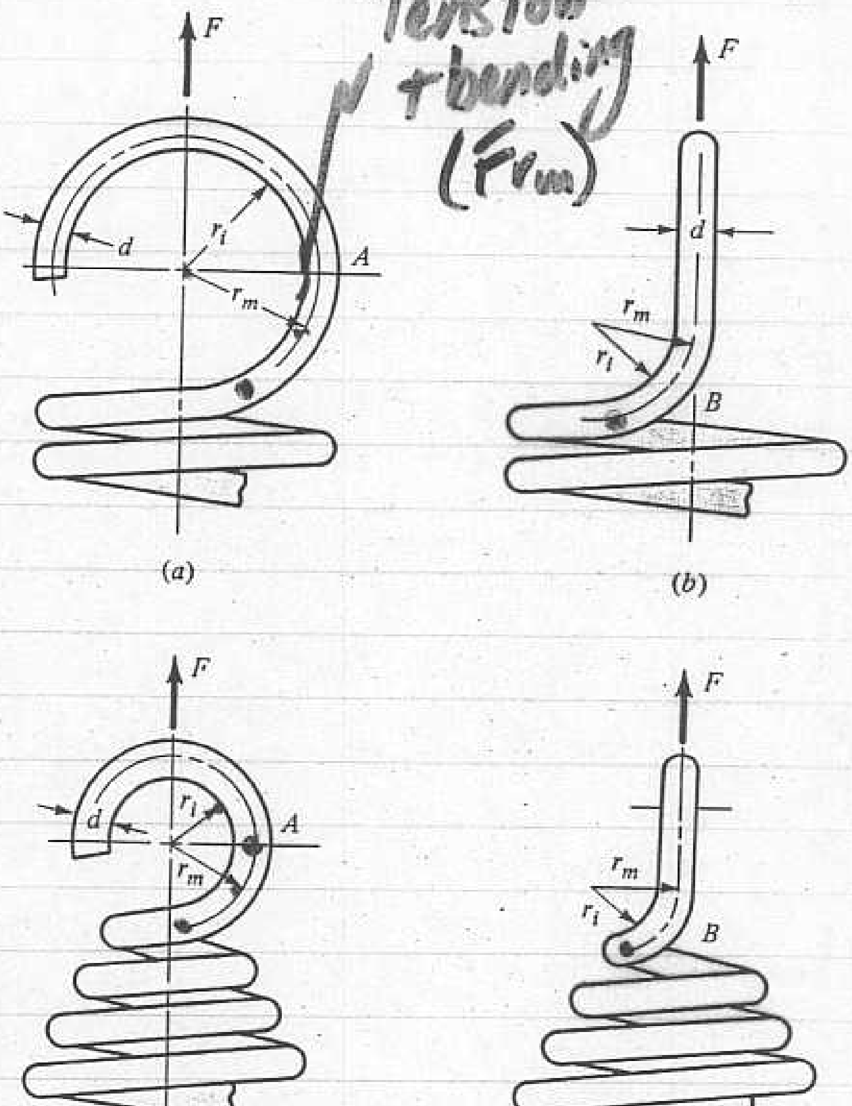


FIGURE 10-3

Ends for extension springs. (a) Usual design; stress at A is due to combined axial and bending forces. (b) Side view of part a; stress is mostly torsion at B. (c) Improved design; stress at A is due to combined axial and bending forces. (d) Side view of part c; stress at B is mostly torsion.

Stress Concentration:

$$\left(K = \frac{r_m}{r_i}\right)$$

Reduced Diameter gives smaller moment

d) Deflection due to F_s

$$y_s = \frac{F_s}{k} = \frac{6.80}{4.0} = \underline{\underline{1.70\text{ in}}}$$

e) Solid Length of spring (Table 10-2)

$$L_s = d(N_t + 1)$$

$$= 0.037(12.5 + 1) = \underline{\underline{0.50\text{ in}}}$$

f) Free length such that when unloaded (from F_s) there is no permanent set. (yield)

$$L_0 = \underbrace{L_s}_{\text{Solid height}} + \underbrace{y_s}_{\text{deflection due to } F_s} = 0.50 + 1.70 = \underline{\underline{2.20\text{in}}}$$

(Buckling will occur)

Buckling? $\dfrac{D}{\alpha} = \dfrac{2.63(0.40)}{0.5} = \underline{\underline{2.10\text{ in}}}$

Table 10-3

[redacted]

Stability

- Spring may buckle
- see 10-6 $L_0 < 2.63\frac{D}{\alpha}$

α From Table 10-3

Spring Materials

- Very strong
  - music wire
  - oil-tempered wire
  - chrome vanadium
  - ⋮

$$\boxed{S_{ut} = \frac{A}{d^m} \quad A \,\&\, m \text{ from Table 10-5}}$$

α

END CONDITION	CONSTANT α
Spring supported between flat parallel surfaces (fixed ends)	0.5
One end supported by flat surface perpendicular to spring axis (fixed); other end pivoted (hinged)	0.707
Both ends pivoted (hinged)	1
One end clamped; other end free	2

*Ends supported by flat surfaces must be squared and ground.

TABLE 10-3

End-Condition Constants α for Helical Compression Springs*

SIMILAR SPECIFICATIONS	DESCRIPTION
UNS G10850 AISI 1085 ASTM A228-51	This is the best, toughest, and most widely used of all spring materials for small springs. It has the highest tensile strength and can withstand higher stresses under repeated loading than any other spring material. Available in diameters 0.12 to 3 mm (0.005 to 0.125 in). Do not use above 120°C (250°F) or at subzero temperatures
UNS G10650 AISI 1065 ASTM 229-41	This general-purpose spring steel is used for many types of coil springs where the cost of music wire is prohibitive and in sizes larger than available in music wire. Not for shock or impact loading. Available in diameters 3 to 12 mm (0.125 to 0.5000 in), but larger and smaller sizes may be obtained. Not for use above 180°C (350°F) or at subzero temperatures
UNS G10660 AISI 1066 ASTM A227-47	This is the cheapest general-purpose spring steel and should be used only where life, accuracy, and deflection are not too important. Available in diameters 0.8 to 12 mm (0.031 to 0.500 in). Not for use above 120°C (250°F) or at subzero temperatures
UNS G61500 AISI 6150 ASTM 231-41	This is the most popular alloy spring steel for conditions involving higher stresses than can be used with the high-carbon steels and for use where fatigue resistance and long endurance are needed. Also good for shock and impact loads. Widely used for aircraft-engine valve springs and for temperatures to 220°C (425°F). Available in annealed or pretempered sizes 0.8 to 12 mm (0.031 to 0.500 in) in diameter
UNS G92540	This alloy is an excellent material for highly

MATERIAL	ASTM NO.	EXPONENT m	INTERCEPT	
			A, kpsi	A, MPa
Music wire[a]	A228	0.163	186	2060
Oil-tempered wire[b]	A229	0.193	146	1610
Hard-drawn wire[c]	A227	0.201	137	1510
Chrome vanadium[d]	A232	0.155	173	1790
Chrome silicon[e]	A401	0.091	218	1960

[a]Surface is smooth, free from defects, and has a bright lustrous finish.
[b]Has a slight heat-treating scale which must be removed before plating.
[c]Surface is smooth and bright with no visible marks.
[d]Aircraft-quality tempered wire; can also be obtained annealed.
[e]Tempered to Rockwell C49, but may be obtained untempered.
Source: Associated Spring–Barnes Group, *Design Handbook*, Bristol, Conn., 1981, p. 19.

Table 10-5

Strengths

Strength based on shear

$$S_{sy} = \underbrace{\tau_{all}}_{\text{allowable}} = \begin{cases} 0.45\, S_{ut} & \text{cold-drawn steel} \\ 0.50\, S_{ut} & \text{hardened, tempered} \\ 0.35\, S_{ut} & \text{austenitic stainless + non-ferrous} \end{cases}$$

$$S_{sy} = \tau_{all} = 0.56\, S_{ut} \quad \text{high tensile}$$

Normally use one of these values as appropriate

2x4 [redacted]

Example 10-1

- Helical compression spring
- No 16 (0.037 in ϕ) music wire
- Outer diam = $7/16$"
- squared ends, $N_t = 12\frac{1}{2}$

Find $<$ Torsional yield strength / Load at yield / stiffness, etc

a) Torsional yield strength

(Table 10-5)

$$S_{ut} = \frac{A}{d^m} \qquad A = 186 \text{ kpsi} \quad m = 0.163$$

$$= \frac{186}{(0.037)^{0.163}} = \underline{318 \text{ kpsi}}$$

$\llcorner$ Note high value

$S_{sy} = 0.45\, S_{ut}$ .... eq. 10-19

$= 0.45(318) = \underline{143 \text{ kpsi}}$

$\left(\tau = \underline{K_s} \dfrac{8FD}{\pi d^3}\right)$

b) <u>Load, F_s corresponding to yield strength</u> $(D_o = 7/16 = 0.437 \text{ in}$

$D = D_o - d = 0.437 - 0.037 = \underline{\underline{0.400}}$

$C = \dfrac{D}{d} = \dfrac{0.400}{0.037} = \underline{10.8}$

$K_s = \dfrac{2C+1}{2C} = \dfrac{2(10.8)+1}{2(10.8)} = \underline{1.046}$

$F_s = S_{sy}\left(\dfrac{\pi d^3}{8K_s D}\right) = \dfrac{\pi(143)(10^3)(0.037)^3}{8(1.046)(0.400)}$

replace $\underline{\tau}$ by yield strength in torsion, S_{sy}

$F_s = \underline{\underline{6.80 \text{ lb}}}$

c) <u>Spring stiffness (scale, rate)</u>

11.5 Mpsi

$k = \dfrac{d^4 G}{8D^3 N_a} = \dfrac{(0.037)^4(11.5\times10^6)}{8(0.400)^3(10.5)} = \underline{4.0} \dfrac{\text{lb}}{\text{in}}$

Table 10-2

$N_a = 12.5 - 2 = 10.5$

Read 10-8 + Example 10-2
- "Trial and error spring selection"

Omit 10-9

10-10 Critical Frequencies & Surging

- Can lead to rapid spring failure by fatigue
- Surging is due to excitation of spring resonance $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

- normally avoided by keeping force fluctuations well below "f" e.g. slow speeds/freqs re "f"

Fatigue Failure

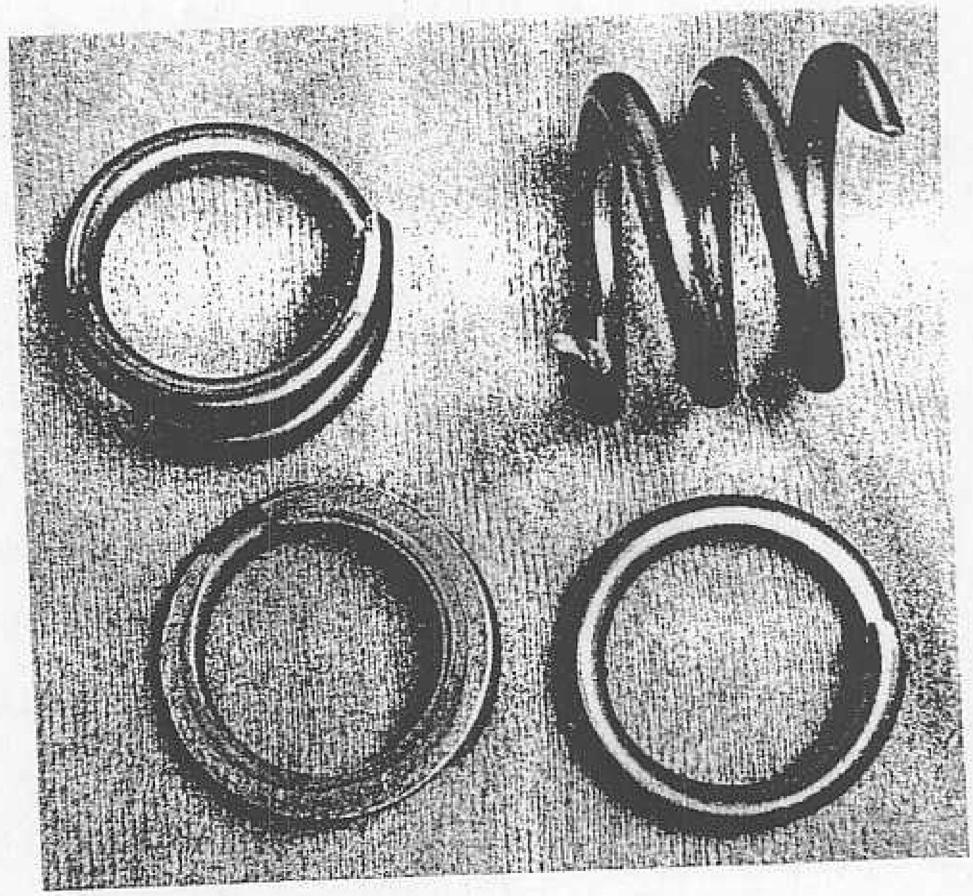


FIGURE 10-8

Valve-spring failure in an over-revved engine. Fracture is along the 45° line of maximum principal stress associated with pure shear.

Fatigue failure due to surging (valve spring in over-revved engine)

Fatigue loading of Springs

- often loaded between F_{max} & F_{min}

  eg Vehicle, machine mount

$$F_a = \frac{F_{max} - F_{min}}{2} \quad \text{altern.}$$

$$F_m = \frac{F_{max} + F_{min}}{2} \quad \text{mean}$$

$$\tau_a = K_B \left(\frac{8 F_a D}{\pi d^3} \right)$$

$$\tau_m = K_S \left(\frac{8 F_m D}{\pi d^3} \right)$$

- According to Zimmerli no modifying factors to endurance limit

not effect of S_{ut}

USE:

$S_{se} = \underline{45 \text{ kpsi}}$ (310 MPa)

(unpeened)

shear endurance

$S_{se} = \underline{67.5 \text{ kpsi}}$ (465 MPa)

(peened)

peening is surface hardening procedure

Example 10-4

- Helical comp spring
- Music wire 0.092 in dia
- O.D. 9/16 in Free length 4 1/8"
- 21 active coils Squared & ground ends
- Preload 5 lb
- Max load 35 lb

FIND FACTOR OF SAFETY AGAINST FATIGUE

$$K_S = \frac{2C+1}{2C} = 1.098 \qquad F_a = \frac{35-5}{2} = 15^{lb}$$

$$K_B = \frac{4C+2}{4C-3} = 1.287 \qquad F_m = \frac{35+5}{2} = 20^{lb}$$

$$D = 0.5625 - 0.092 = 0.4705 \qquad C = 5.11$$

$$\tau_a = K_B \frac{8 F_a D}{\pi d^3} = 1.287 \frac{8(15)(0.47)}{\pi(.092^3)} \times 10^{-3}$$

$$= \underline{29.7 \text{ kpsi}}$$

$$\tau_m = K_s \frac{8 F_m D}{\pi d^3} = 1.098 \left(\quad \right)$$

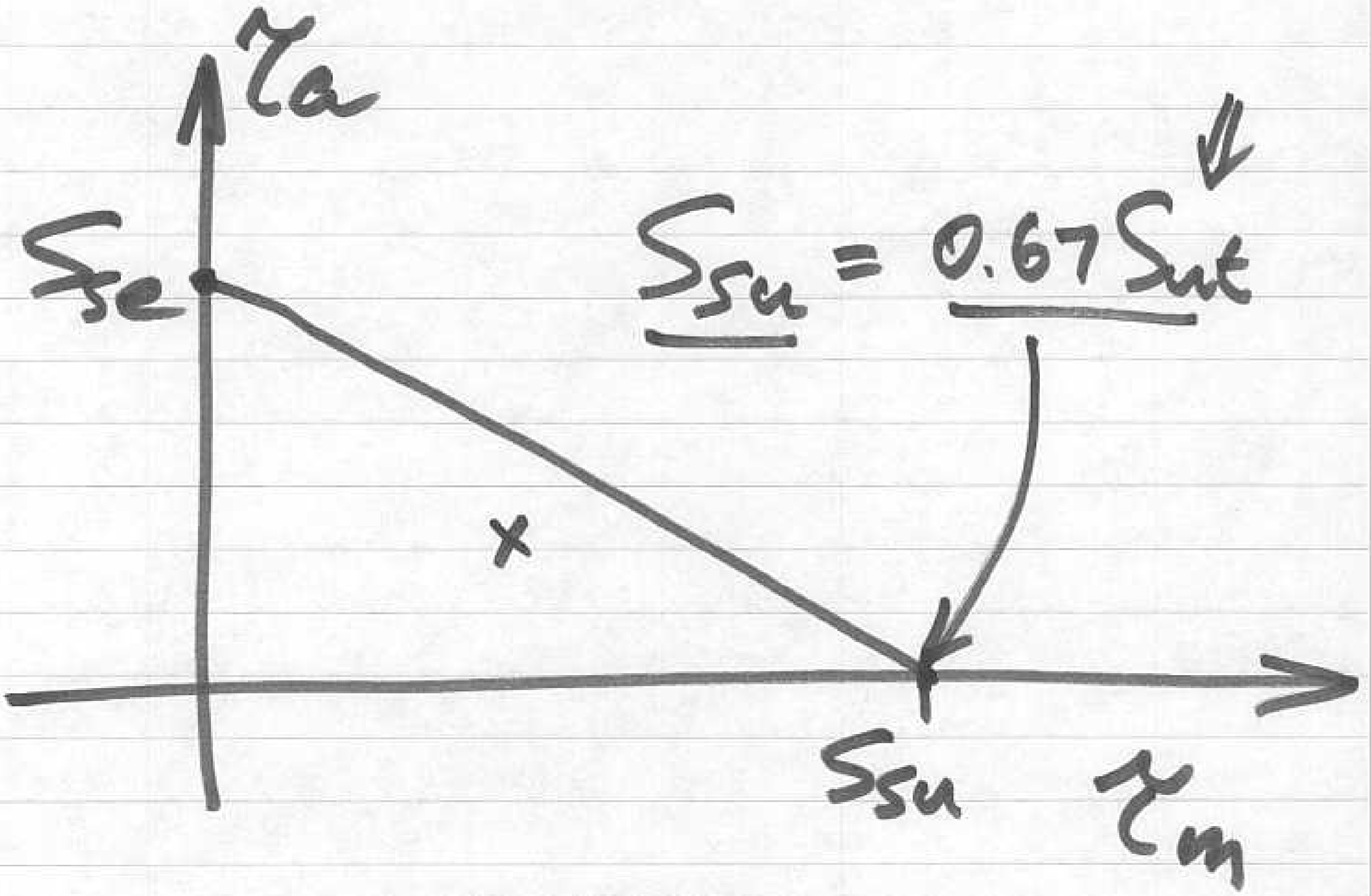
$$= \underline{33.8 \text{ kpsi}}$$

$$S_{ut} = \frac{186}{(0.092)^{0.163}}$$ Table 8-5

$$= \underline{274 \text{ kpsi}}$$

and $S_{su} = 0.67 S_{ut} = \underline{184} \text{ kpsi}$

Use "quasi-Goodman" for fatigue



$$\boxed{\frac{\tau_a}{S_{se}} + \frac{\tau_m}{S_{su}} = \frac{1}{n}}$$

Since peening not mentioned, assume unpeened:

$\therefore$ $S_{se} = 45$ kpsi

$$\frac{\tau_a}{S_{se}} + \frac{\tau_m}{S_{su}} = \frac{1}{n}$$

$$\frac{29.7}{45} + \frac{33.8}{184} = \frac{1}{n}$$

$$\boxed{n = 1.19}$$