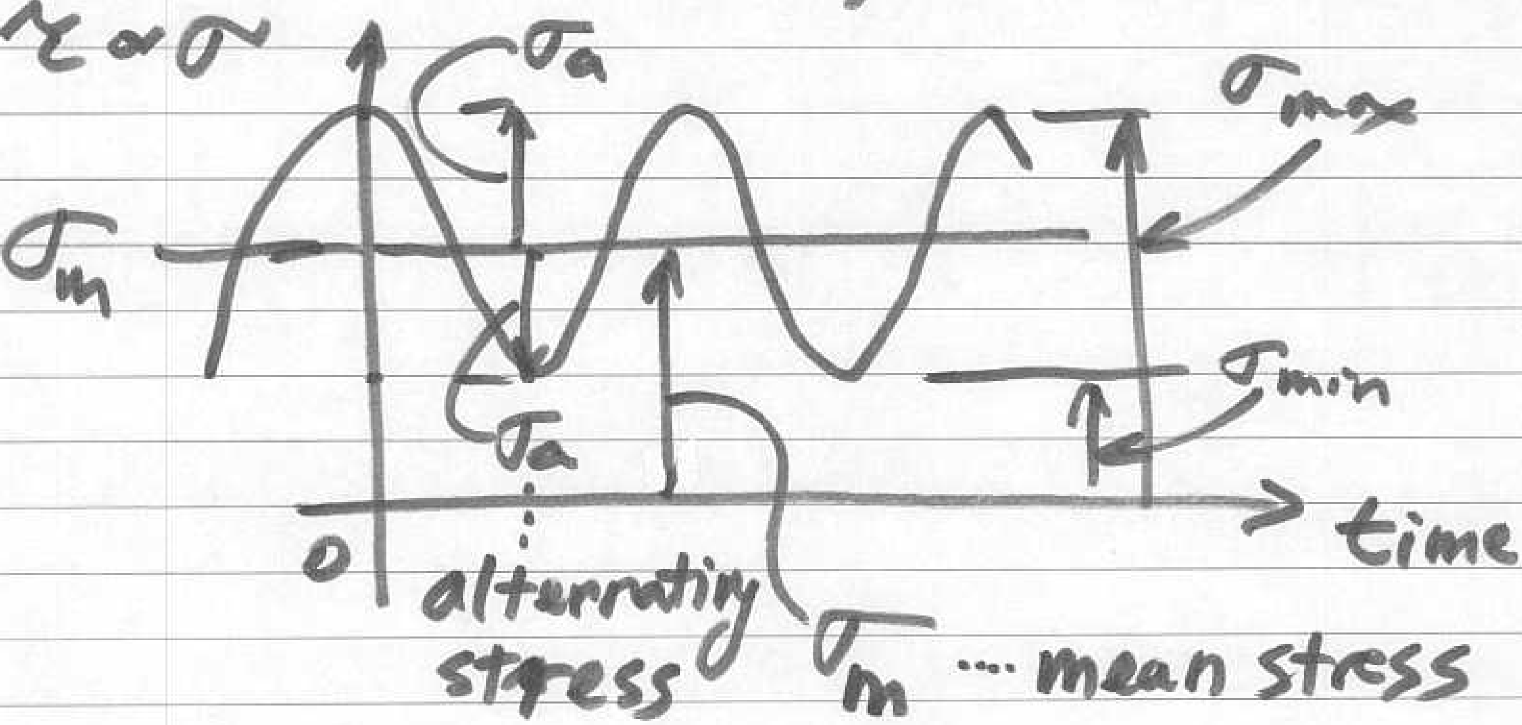
FATIGUE STRENGTH UNDER GENERAL FLUCTUATING LOADS

- WE HAVE, UP TO NOW, LOOKED AT "ALTERNATING" OR "FULLY-REVERSED" LOADING WHERE STRESS FLUCTUATES BETWEEN $\pm\sigma_a$ or $\pm\tau_a$

- MORE GENERALLY, WE CAN HAVE



- σ_m can be +ve or -ve

ALSO

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$$ alternating stress

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$ mean stress

σ_{max} & σ_{min} can be computed from max & min loading:

F_{max} & F_{min} Force

M_{max} & M_{min} Moment

T_{max} & T_{min} Torque

∴ Max & min refer to loads at a particular location over time, eg. rotating shaft, or other fluctuating forces, loads

$\therefore$ S_e & S_f apply <u>directly</u> to cases where $\sigma_m = 0$

$\therefore$ When $\sigma_{min} = 0$, i.e. fluctuations between 0 and σ_{max} we call it "repeated loading"

and $\sigma_a = \dfrac{\sigma_{max}}{2}$ and $\sigma_m = \dfrac{\sigma_{max}}{2}$

... e.g. loading of rotating gear teeth.

QUESTION: HOW DOES THE PRESENCE OF A MEAN + ALTERNATING STRESS AFFECT FATIGUE STRENGTH / ENDURANCE?

ANS: if $\sigma_m > 0$ (tensile)
Strength is reduced
if $\sigma_m \leq 0$ (compressive)
Strength is unchanged.

GRAPHICALLY:

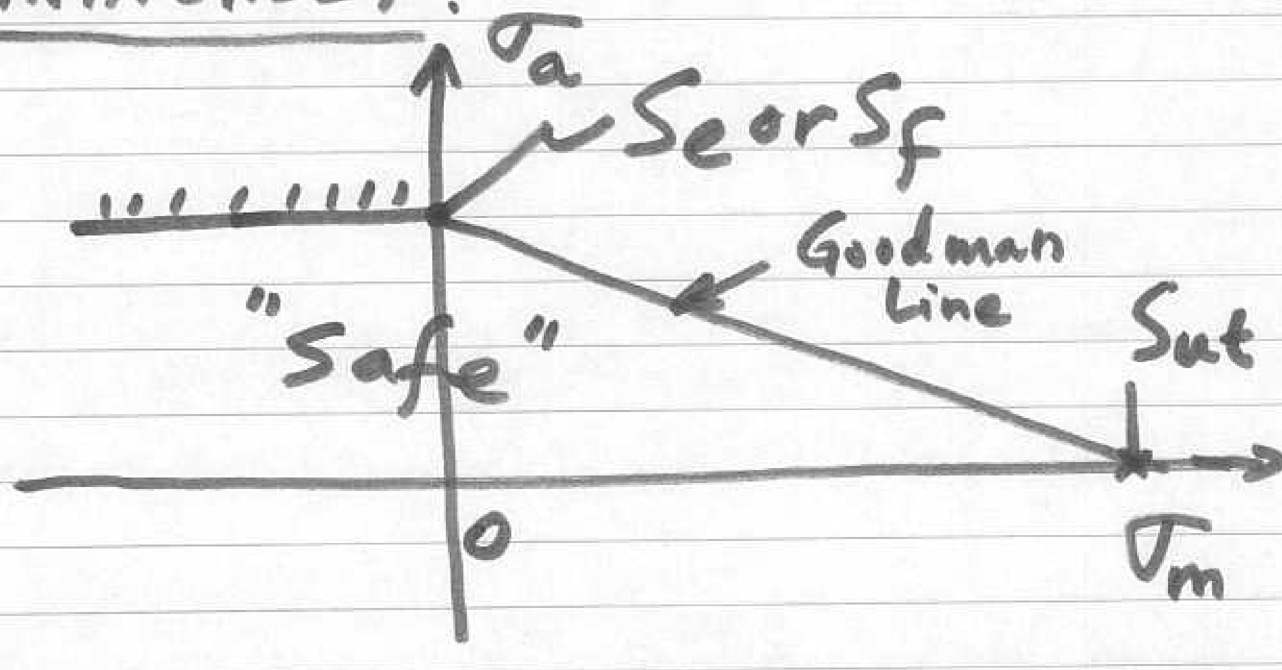
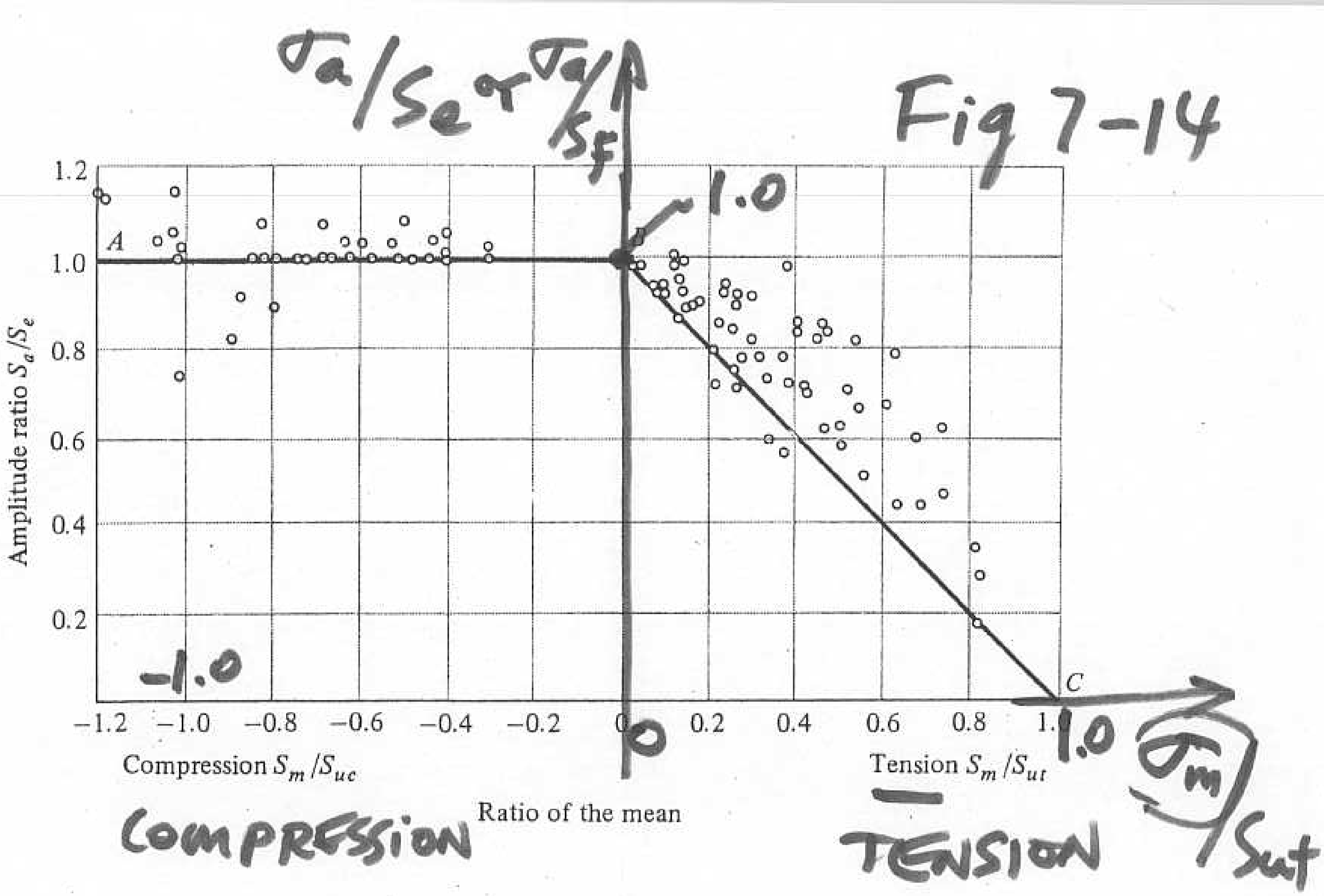


Fig 7-14



SOME DATA

NOTE: WHEN $\sigma_m = 0$, only have σ_a
THEN ONLY NEED S_e or S_f

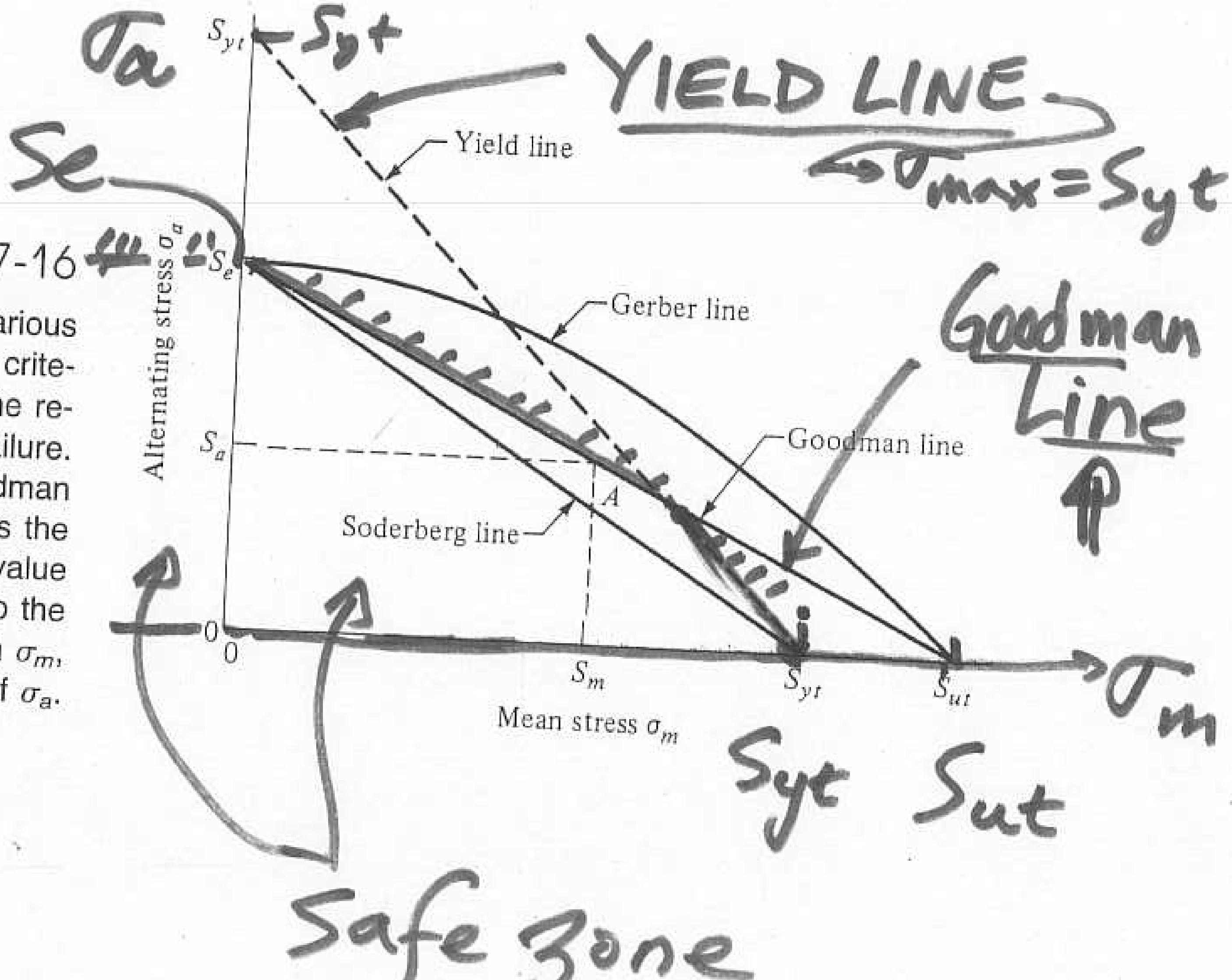
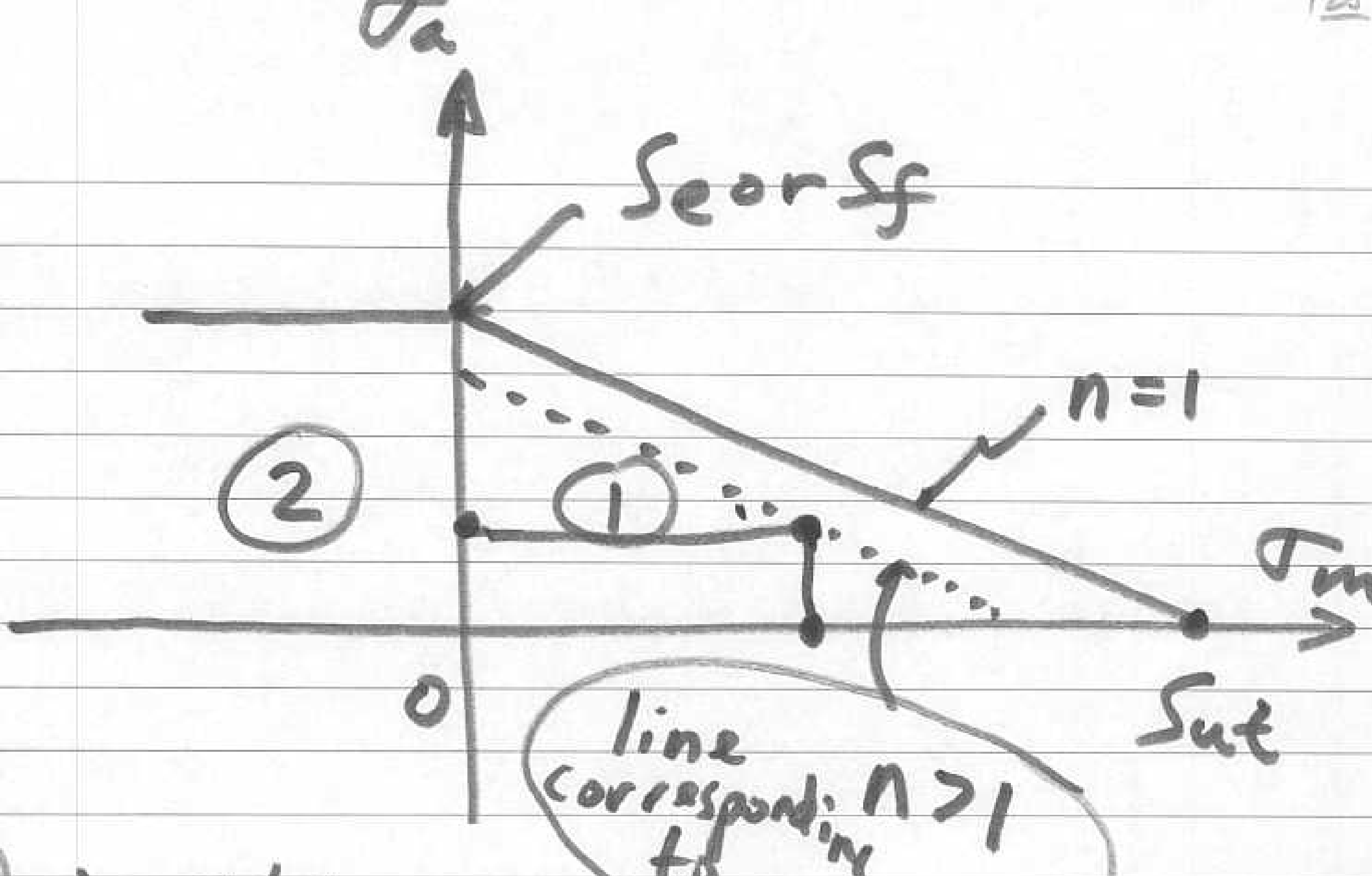


FIGURE 7-16 Fatigue diagram showing various criteria of failure. For each criterion, points on or outside the respective line indicate failure. Some point A on the Goodman line, for example, gives the strength S_m as the limiting value of σ_m corresponding to the strength S_a, which, paired with σ_m, is the limiting value of σ_a.

For $\sigma_m > 0$ use Goodman Line

For $\sigma_m \leq 0$ use relations for $\sigma_m = 0$



① in this zone

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n}$$

{also check that: $S_{yt} > \sigma_{max}$ for static failure or $n = \dfrac{S_{yt}}{\sigma_{max}}$}

② in this zone

$$n = \frac{S_e}{\sigma_a} \text{ or } \frac{S_f}{\sigma_a}$$

Example: Determine Factor of Safety

i). $\begin{cases} \sigma_a = 5 \text{ kpsi.} \\ \sigma_m = 3 \text{ kpsi} \end{cases} \rightarrow$ 1st quad

$S_e = 20$ kpsi.

$S_{ut} = 100$ kpsi

Goodman Line

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n}$$

$$\frac{1}{n} = \frac{5}{20} + \frac{3}{100} = 0.28$$

$$\boxed{n = 3.57}$$

STATIC

$$n_s = \frac{S_{yt}}{\sigma_a + \sigma_m}$$

$$n_s = S_{yt}/\sigma_{max}$$

No mean stress

ii) $\sigma_a = 5 \text{ kpsi}$

$\sigma_m = 0$

$S_e = 20 \text{ kpsi}$

$S_{ut} = 100 \text{ kpsi}$

STATIC

$n_s = \dfrac{S_{yt}}{\sigma_a + \sigma_m}$

$$\boxed{n = \frac{S_e}{\sigma_a} = \frac{20}{5} = \underline{\underline{4.0}}}$$

iii) $\sigma_a = 5 \text{ kpsi}$

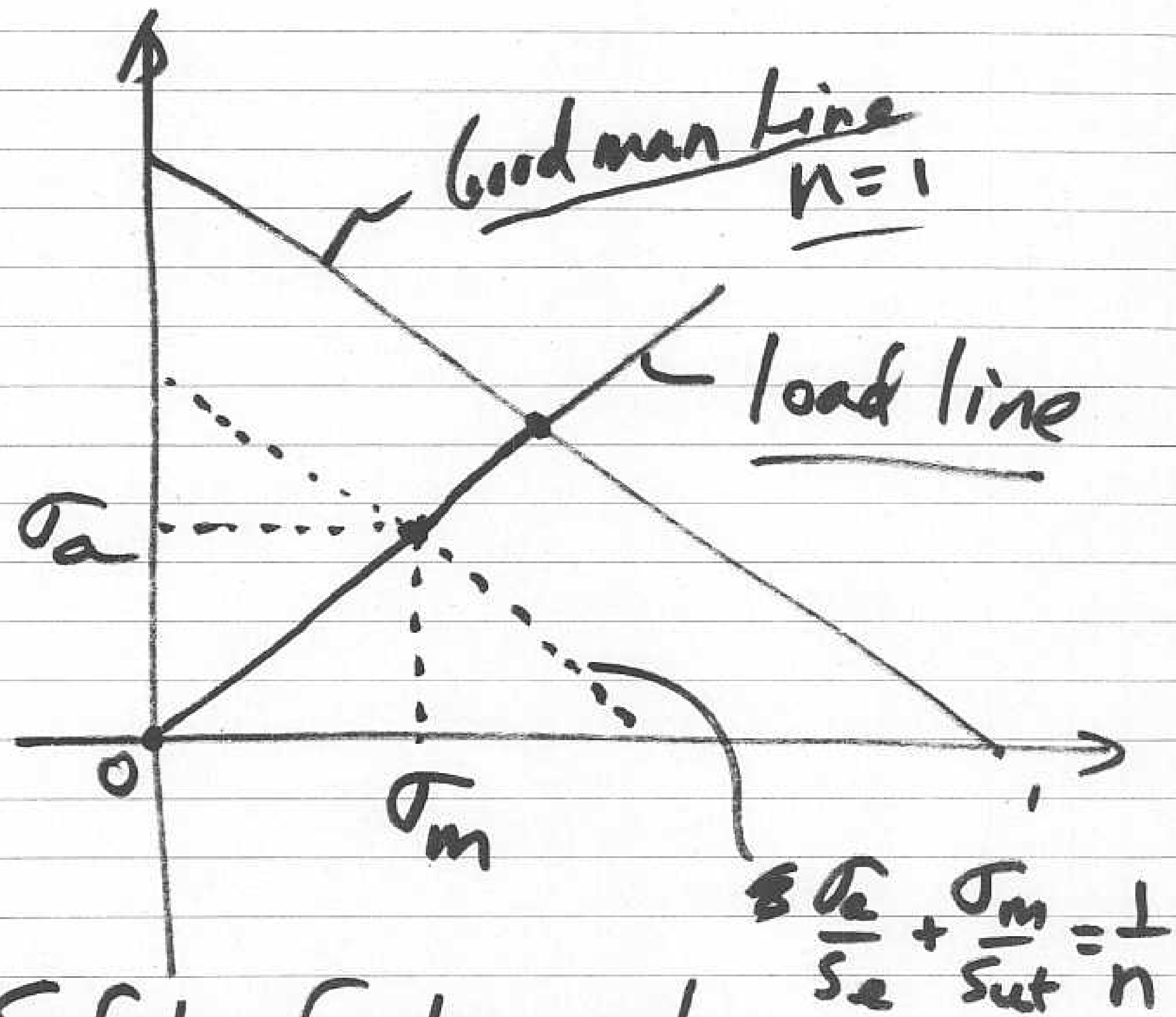
$\boxed{\sigma_m = -28 \text{ kpsi}}$

$S_e = 20 \text{ kpsi}$

$S_{ut} = 100 \text{ kpsi}$

$$n = \frac{S_e}{\sigma_a} = \frac{20}{5} = \underline{\underline{4.0}}$$

Concept of Load-line



- Safety factor calc assumes load-line through origin
- Use this assumption unless told otherwise

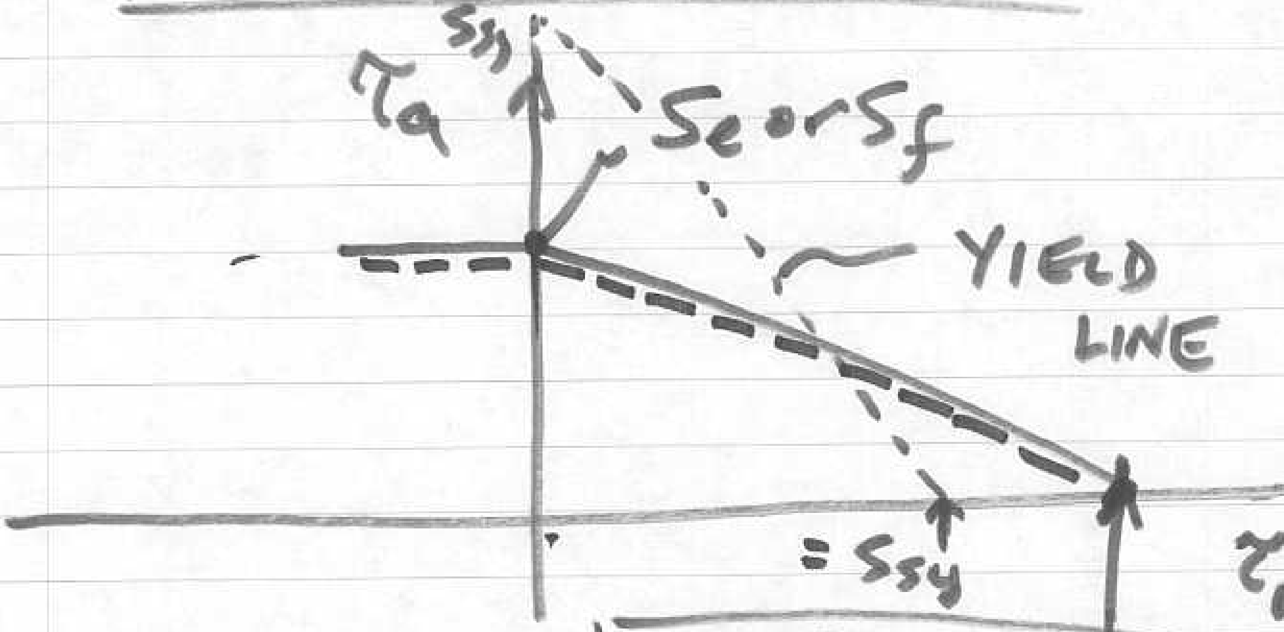
Assignment #4
Due Mon Jan 17th

#1. 7-1 from text
#2. 7-5 (reversed bending, see sect 5-4 for tensile strength)
#3. 7-11 (infinite life and static failure (yielding))
#4. A 1.25 inch diameter hot-rolled steel rod has a 0.125 inch diameter hole drilled through it. Its (ultimate) tensile strength is 60 kpsi. The rod is subjected to a reversed (alternating) torque of 2000 in-lbs. Estimate the factor of safety for infinite life and against yielding. What would be the safety factor if a life of 20,000 cycles were needed? ... $S_y = 45$ kpsi

TORSIONAL FATIGUE PURE SHEAR

... ALTERNATING + MEAN STRESS

QUASI-GOODMAN DIAGRAM



$$\boxed{0.67 S_{ut} = S_{su}}$$

$S_{sy} = \underline{0.577 S_{yt}}$ for Yielding

For Quasi Goodman Line 1st quadrant

$$\frac{\tau_a}{S_e} + \frac{\tau_m}{S_{su}} = \frac{1}{n}$$

WHAT IF LOADING IS COMBINED?

I ALTERNATING ONLY.

e.g. σ_{xa}, σ_{ya}, τ_{xya}

- Calculate VON-MISES (σ_a') (... using principal stresses)
- Obtain fully correct Se or Sf for BENDING
- CONSERVATIVE APPROACH.
- Define safety factor, n

$$n = \frac{S_e}{\sigma_a'} \quad \text{or} \quad \frac{S_f}{\sigma_a'}$$

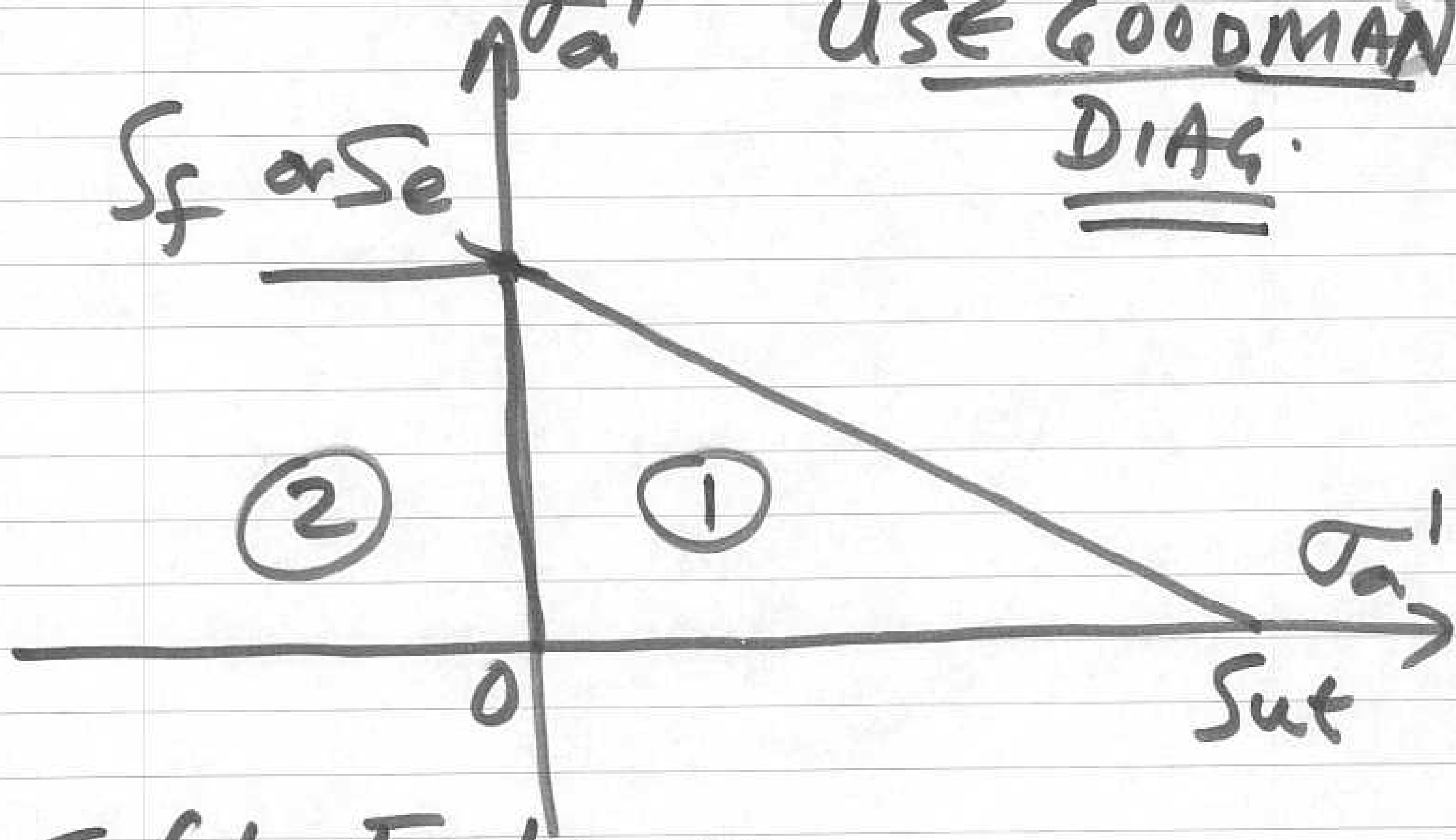
infinite Life ... finite Life

II ALTERNATING + MEAN

e.g. $\sigma_{xa}, \sigma_{ya}, \tau_{xya} \Rightarrow \underline{\underline{\sigma_a'}}$

and $\sigma_{xm}, \sigma_{ym}, \tau_{xym} \Rightarrow \underline{\underline{\sigma_m'}}$

USE GOODMAN DIAG.



Safety Factor, n

① $\dfrac{\sigma_a'}{S_e} + \dfrac{\sigma_m'}{S_{ut}} = \dfrac{1}{n}$ ② $\dfrac{\sigma_a'}{S_e} = \dfrac{1}{n}$

ADDITIONAL NOTE FOR COMBINED LOADING PROBS

i.e. $\sigma_{xa}, \sigma_{ya}, \tau_{xya} \Rightarrow \sigma_{1a}, \sigma_{2a} \Rightarrow \sigma_a'$

$\sigma_{xm}, \sigma_{ym}, \tau_{xym} \Rightarrow \sigma_{1m}, \sigma_{2m} \Rightarrow \sigma_m'$

∴ APPLY STRESS CONCENTRATION FACTORS K_f & K_{fs} (SHEAR) TO INCREASE ALTERNATING (NOT MEAN) STRESSES. DO NOT USE $k_e = 1/K_f$ TO REDUCE STRENGTH IN THESE CASES.

∴ DO NOT USE K_f FOR MEAN STRESSES (EXCEPT FOR BRITTLE MTLS)

SECTION 7-16 DEALS WITH CUMULATIVE FATIQUE WHEN DIFFERENT LEVELS OF FLUCTUATING LOADS ARE PRESENT AT DIFFERENT TIMES

... "Palmgren - Miner Rule"

.... WE WILL <u>NOT</u> consider

SECTION 7-17 FRACTURE MECHANICS APPROACH

... WE WILL <u>NOT</u> consider

7-18 SURFACE FATIGUE STRENGTH

- Failure by pitting, surface fatigue in gears & rolling bearings & other contacts
- Repeated loading assumed
- SURFACE FATIGUE STRENGTH

Brinell Hdness

$$\underline{S_c} = (0.4 H_B - 10) \text{ kpsi}$$

OR $$S_c = (2.76 H_B - 70) \text{ MPa}$$

STEEL

Compare with 5-20 for S_u

$$S_u = 0.45 H_B \text{ kpsi}$$

$$S_u = 3.10 H_B \text{ MPa}$$

<u>NOTE THAT, GENERALLY</u>

$S_c > 0.5\,S_u$

.... this is due to the hydrostatic compression during contact loading

.... <u>also compare p_{max} NOT σ_a</u>

.... <u>S_c applies for</u>

<u>EXACTLY $10^8 = N$</u>

<u>cycles of loading</u>

... <u>NO ENDURANCE LIMIT</u>

.... if S_c @ some other value of N is needed, call it S_{cN}

For contact fatigue it's been found that:

$$S_{CN}^{a} N = \text{Constant} \quad \ldots \text{Palmgren 1940's}$$

$$3 < a < 3.3$$

How metals behave!

$$\therefore \quad S_{c_1}^{a} N_1 = S_{c_2}^{a} N_2$$

$$S_{c_2} = \left(\frac{N_1}{N_2}\right)^{1/a} S_{c_1}$$

Let $N_1 = 10^8$, $S_{c_1} = S_c$

$N_2 = N$, $S_{c_2} = S_{CN}$

$$\boxed{S_{CN} = \left(\frac{N_1}{N_2}\right)^{1/a} S_c}$$

e.g. if $a = 3$
and $N = 10^5$

$$S_{cN} = \left(\frac{10^8}{10^5}\right)^{1/3} S_c$$

$$\boxed{S_{cN} = 10\, S_c}$$

At any # of cycles
Factor of safety is:

$$n = \frac{S_{cN}}{p_{max}}$$

MAE 311 Miniproject 1

Due March 19, 2003

Worth five per cent of course grade

To be submitted:

A written report with a title page plus 2-3 pages of written description, plus any illustrations or figures.This is to be an individual report, written in your own words. Cite all sources of information. Relate to MAE 311 course material as appropriate.

Choose one of three topics:

1. *A recent newsworthy mechanical failure*. e.g., the Alaska Airlines crash, recently traced to a maintenance problem. Discuss nature of failure, technical aspects, failure mechanisms or design issues, as appropriate. DOW-JONES INTERACTIVE

2. *A commercial mechanical product or component that that has become available within the past ten years*. Identify and describe the product or component. What's technically interesting about

it? What seems to have prompted the introduction of the product, new materials, demand for the product, cost/technical advantage, innovation, some combination?

MACHINE DESIGN, DESIGN NEWS:

3. Document how and why a longstanding product or component has evolved over the past 20 to 60 years. How is the product better? What advances in technology, regulation or competitive pressures have contributed to the evolution?

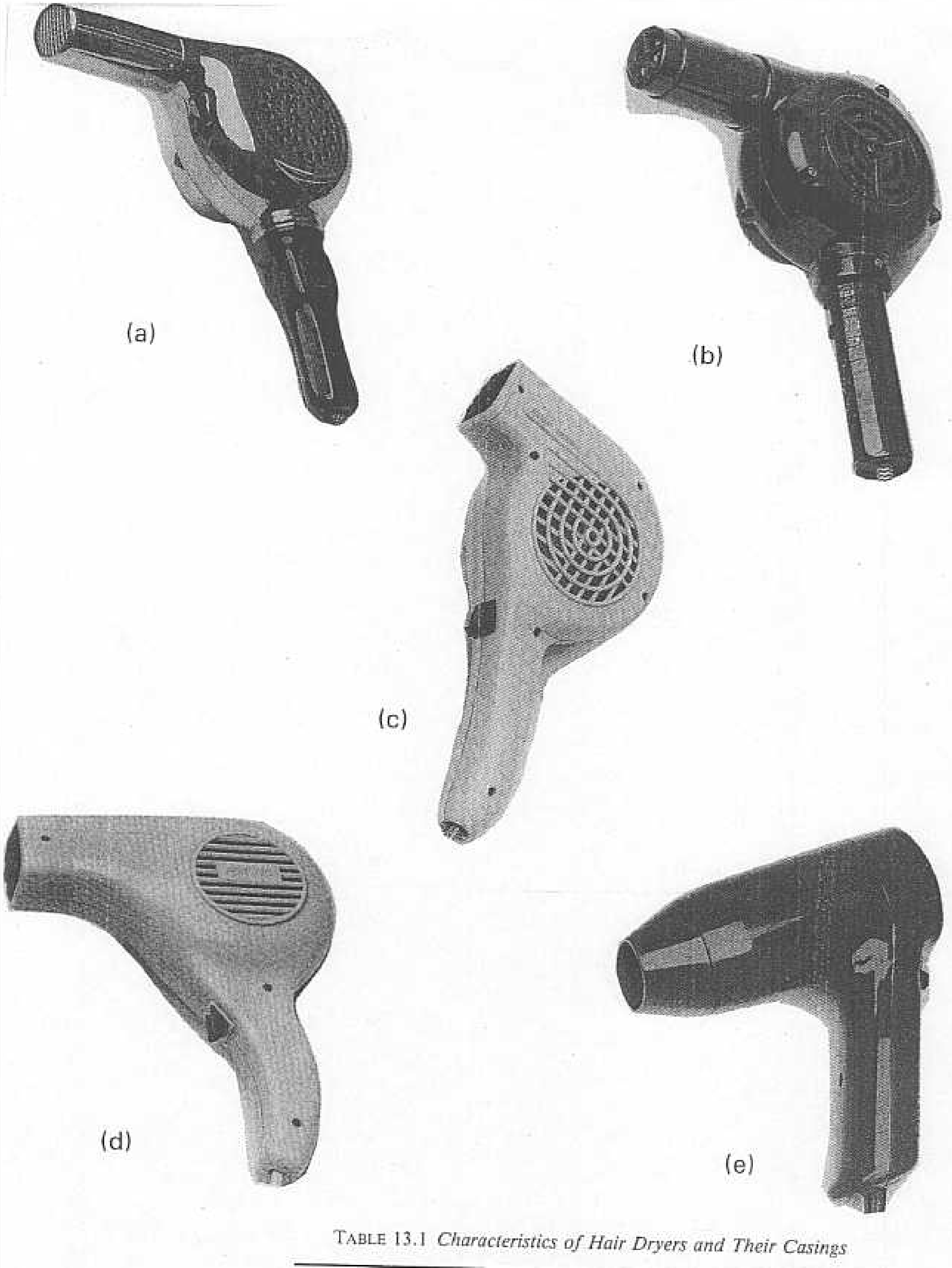


TABLE 13.1 *Characteristics of Hair Dryers and Their Casings*

Model and Date	Power (W)	Weight (kg)	Parts	Fasteners
Schott, 1940	300	1.0	5	7
Ormond, 1950	500	0.85	5	8
Morphy-Richards, 1960	400	0.82	3	6
Pifco, 1965	300	0.80	3	4
Braun, 1986	1200	0.27	3	1

From: Ashby Mtls Selection in Mech. Design (1993)

FIG 13.2 Hair dryers: (a) a metal hair dryer of about 1950; (b) a bakelite dryer, almost identical in form to (a); (c) a plastic dryer of 1960, still influenced by "metal" thinking, but with attractive moulding; (d) a dryer of 1965 — it has fewer fasteners than (c), but is undistinguished in design; (e) a hair dryer of 1986, exploiting fully and effectively the properties of polymers, and with a racy, youthful look. Their characteristics are given in Table 13.1.

February 2003

automotive engineering international

SAE 2003 World Congress

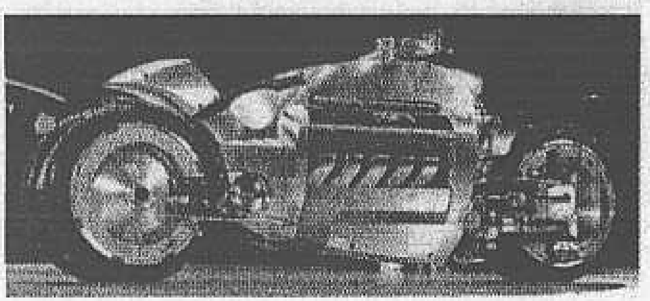
Concepts from Detroit

OUTRAGEOUS!

SAE International

Dodge Tomahawk

With the **Dodge** Tomahawk, **DaimlerChrysler**'s designers have wrought an industrially styled platform for the Dodge Viper's aluminum 8.3-L OHV V10. The Tomahawk, which was named after both the hatchet and cruise missile, rolls on four wheels, but is effectively a motorcycle. The front and rear wheels roll in closely spaced pairs, but are spread far enough apart that the Tomahawk can stand on its own, without a kickstand.



Such a radical vehicle, with 500 hp (373 kW) and 525 lb•ft (712 N•m) in an extra-large 1500-lb (680-kg) motorcycle package, looks like an extreme styling exercise aimed at garnering attention at the auto show. But closer inspection reveals a machine that is not only rideable, but which has a strong chance of being developed into a limited-production vehicle.

Each of the front wheels is carried by a separate automotive-style unequal-length control-arm suspension system, and the wheels steer individually rather than as a pair about an axis between them. The control arms are fabricated from polished billet aluminum. Steering effort is acceptably light when stationary, and would lighten further in motion. The design team carefully studied previous single-sided control-arm-design bikes, such as the **Elf** (petroleum)-backed Grand Prix racing bike of the late 1980s to aid the development, according to designer Mark Walters.

Each of the rear wheels is carried on its own motorcycle-style single-sided swing arm, and each is driven by its own chain drive from the Tomahawk's two-speed transmission. Each of the four 20-in wheels carries a rim-mounted brake rotor made of stainless steel in the front and cast iron in the rear. Front wheels carry a pair of four-piston aluminum calipers each, for 16 pistons total, and the rears carry a single four-piston caliper each, for eight pistons total. The wheels wear custom-made **Dunlop** symmetrical motorcycle tires. The Tomahawk has no frame, so all suspension parts attach to mounting plates on the engine.

Cooling the huge engine could be a challenge, and the Tomahawk uses a pair of hidden aluminum radiators mounted above the engine's intake manifold and fed by a turbine-style cooling fan. The riding characteristics of a four-wheeled motorcycle are also of interest, but DaimlerChrysler claims the Tomahawk can lean to a 45° angle.

The engine's unmuffled exhaust passes between the rear wheels, and the unequal-length headers give the engine a crackle that is not present in the Viper. The lightened flywheel also lends the Tomahawk the urgency of a race machine, even when stationary.

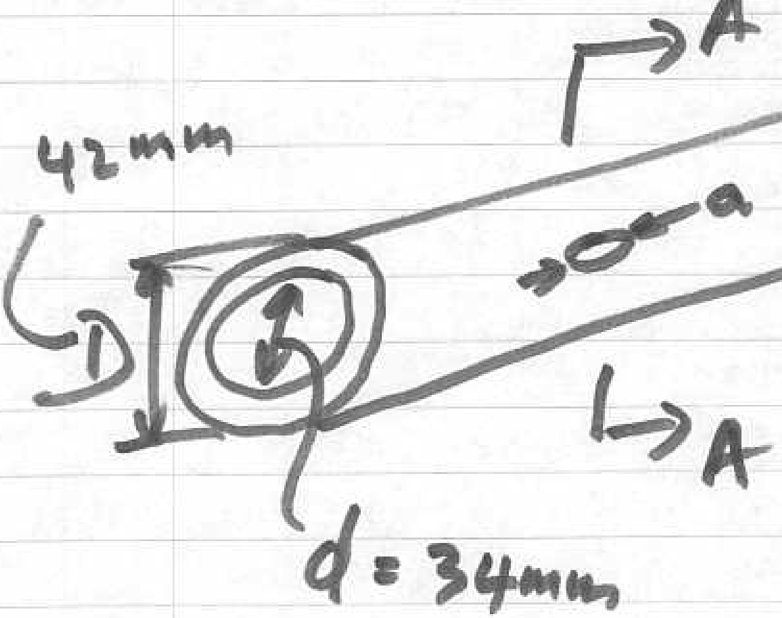
The Tomahawk was built by the vintage racing specialist shop **RM Motorsports**, whose technicians have tested the bike by riding it about 45 mi (72 km). They report that it handles much like a drag racing bike, so it isn't meant for carving corners, but it can be ridden safely on public roads.

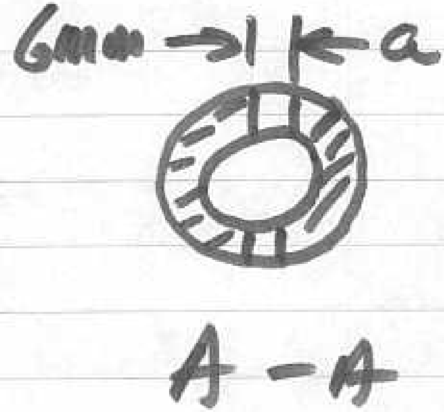
DaimlerChrysler has made no official decision on the Tomahawk, but it is openly contemplating production of 100 bikes, to be built by RM, which would sell for $150,000-200,000. Computer projections put top speed at a theoretical 300 mph (483 km/h).

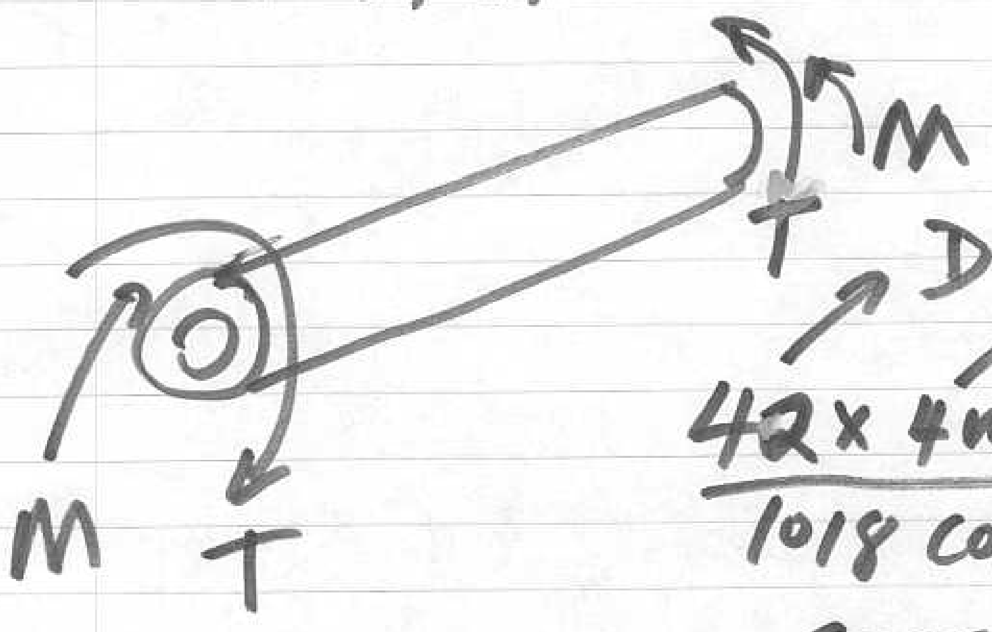
Dan Carney

1431

VARIATION ON EXAMPLE 7-8







D $\quad \dfrac{D-d}{2}$

42 x 4mm AISI 1018 COLD DRAWN STEELTUBE

6 mm ⌀ hole = a

T = const = 120 N-m

M = reversed bending = 150 N-m

Find Factor of Safety for Infinite Life:

From A-20 $\quad S_{ut} = 440\ MPa$

$S_{yt} = 370\ MPa$

σ_{xa}, τ_{xym} $\quad S_e' = 220\ MPa$

FOR ALT. STRESS

Determine $k_a, k_b \quad k_c, k_d, k_e = 1$

increase stress by K_f

$$S_e = k_a k_b S_e'$$

$$k_a = a S_{ut}^{\,b} = 1.58(440)^{-0.085} = .942$$

$$k_b = \left(\frac{\quad}{7.62}\right)^{-0.1133} = \left(\frac{42}{7.62}\right)^{-0.1133}$$

$$k_b = 0.824$$

$$S_e = (0.942)(.824)(220) = 170\ MPa$$

Still need $K_f = 1 + q(K_t - 1)$

From Fig A-16 $K_t = 2.37$

ALSO

A-16

NOTCH DIMS.

$$\left.\begin{aligned} Z_{net} &= 3.31 \times 10^3\ mm^3 \\ J_{net} &= 15.5 \times 10^4\ mm^4 \end{aligned}\right\}$$

$\sigma = \frac{M}{Z}$

$\tau = \frac{TD}{J2}$

$\Rightarrow q_{bending} = 0.78$ from 5-16 Fig

$$K_f = 1 + 0.78(2.37 - 1) = \underline{\underline{2.07}}$$

$$\sigma_a = \sigma_{nom} K_f = K_f \frac{M}{Z_{net}}$$

$$= \frac{2.07(150)}{3.31\,cm^3} = \underline{\underline{93.8\ MPa}}$$

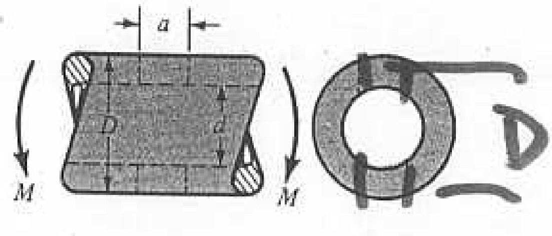
$$\tau_{xym} = \frac{T}{J_{net}}\left(\frac{D}{2}\right) = \underline{\underline{16.3\ MPa}} = \underline{\underline{\tau_m}}$$

TABLE A-16

Approximate Stress-Concentration Factors K_t for Bending of a Round Bar or Tube with a Transverse Round Hole
[The Nominal Bending Stress Is $\sigma_0 = M/Z_{net}$, where Z_{net} is a Reduced Value of the Section Modulus and Is Defined by

$$Z_{net} = \frac{\pi A}{32D}(D^4 - d^4)$$

Values of A Are Listed in the Table. Use $d = 0$ for a Solid Bar]



	d/D					
	0.9		0.6		0	
a/D	A	K_t	A	K_t	A	K_t
0.050	0.92	2.63	0.91	2.55	0.88	2.42
0.075	0.89	2.55	0.88	2.43	0.86	2.35
0.10	0.86	2.49	0.85	2.36	0.83	2.27
0.125	0.82	2.41	0.82	2.32	0.80	2.20
0.15	0.79	2.39	0.79	2.29	0.76	2.15
0.175	0.76	2.38	0.75	2.26	0.72	2.10
0.20	0.73	2.39	0.72	2.23	0.68	2.07
0.225	0.69	2.40	0.68	2.21	0.65	2.04
0.25	0.67	2.42	0.64	2.18	0.61	2.00
0.275	0.66	2.48	0.61	2.16	0.58	1.97
0.30	0.64	2.52	0.58	2.14	0.54	1.94

Source: R. E. Peterson, *Stress Concentration Factors*, Wiley, New York, 1974, pp. 146, 235.

$K_t = 2.37$

from interpolation

$a/D = 6/42 = 0.143$ $d/D = 34/42 = .81$

To find principal stresses $\Rightarrow \sigma'$

$\sigma_a' = \sigma_{1a} = \sigma_a = \underline{\underline{93.8\,MPa}}$ Pure bending

$\tau_m = \sigma_{1m} = \underline{16.3\,MPa}$
$= -\sigma_{2m} = \underline{-16.3\,MPa}$ } Pure shear (see next page)

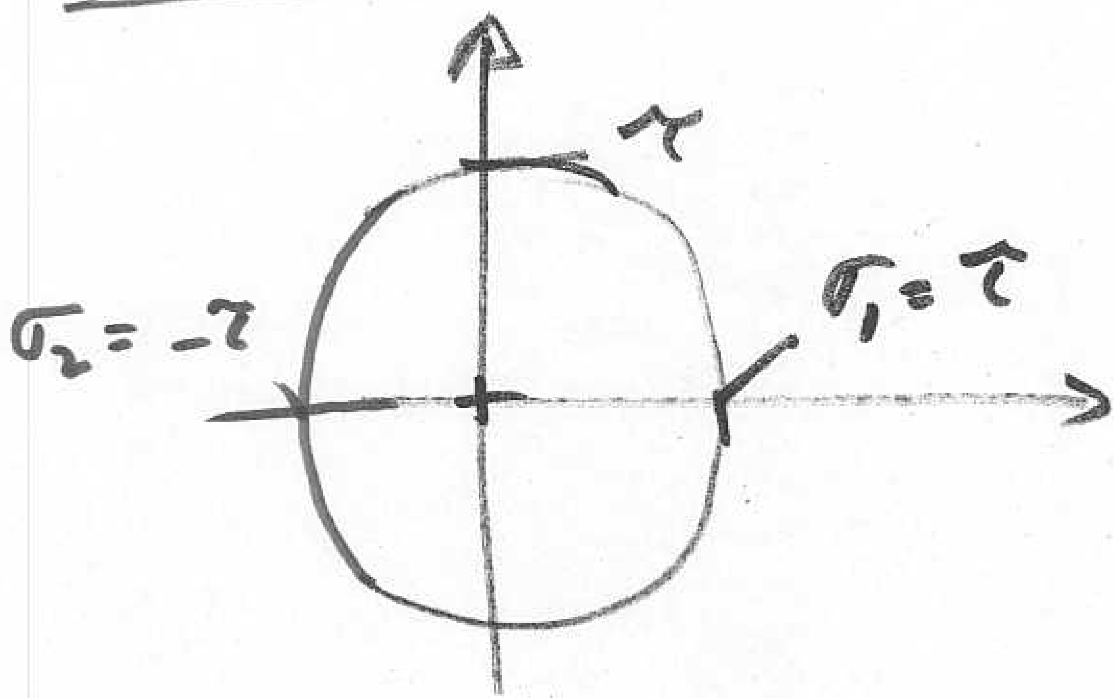
Factor of safety ... FATIGUE

$$\frac{1}{n} = \frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{S_{ut}}$$ (see next page for σ_m')

$$= \frac{93.8}{170} + \frac{28.3}{440} = 0.62$$

$$\boxed{n = 1.62}$$

RECALL: PURE SHEAR



$$\sigma_m' = \sqrt{\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2}$$

$$= \sqrt{16.3^2 + 16.3^2 + 16.3^2}$$

$$= \sqrt{797}$$

$$\sigma_m' = 28.3\ \mathrm{MPa}$$

CHECK FOR YIELDING

$$\sigma' = \sqrt{\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2}$$

for only σ_x, τ_{xy}

$$\sigma_m + \sigma_a = \sigma_{max} \qquad \tau_{xy_{max}} = \tau_a + \tau_m$$

THEN:

$$\sigma'_{max} = \sqrt{\underbrace{\sigma_{x_{max}}}_{45.3 = M/Z}^2 + 3\,\underbrace{\tau_{xy_{max}}}_{48.9}^2}$$

$$= \underline{53.4 \text{ MPa}}$$

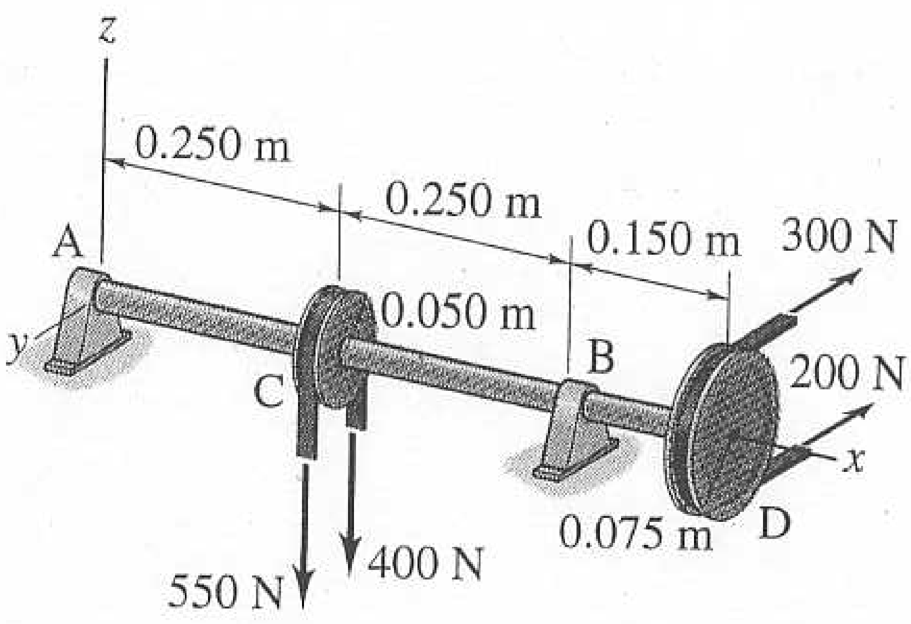
NO STRESS CONCENTRATION

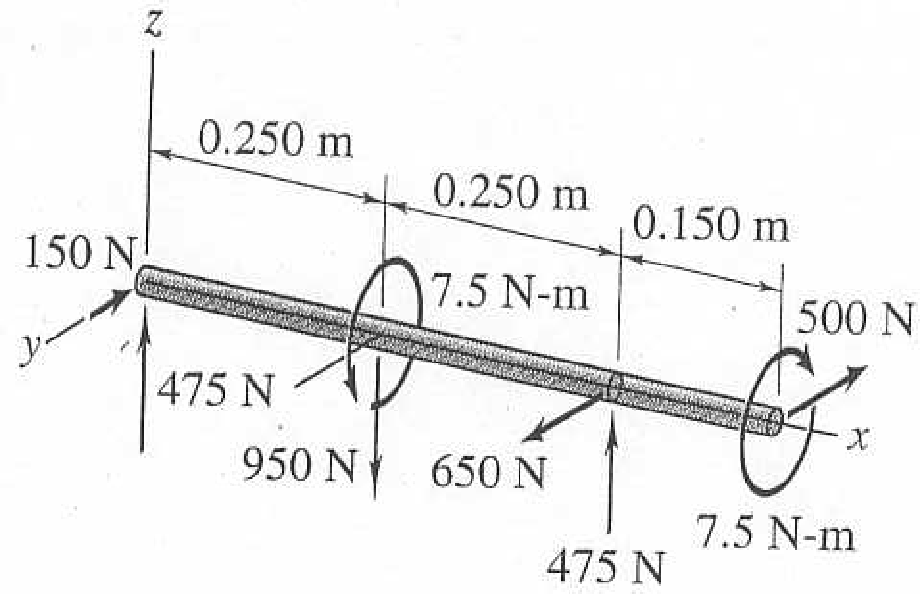
$$n = \frac{S_{yt}}{\sigma'_{max}} = \frac{370}{53.4}$$

$$\boxed{n = 6.93}$$

Much larger n than for fatigue

COMBINED ALTERNATING + MEAN LOADS ... EXAMPLE

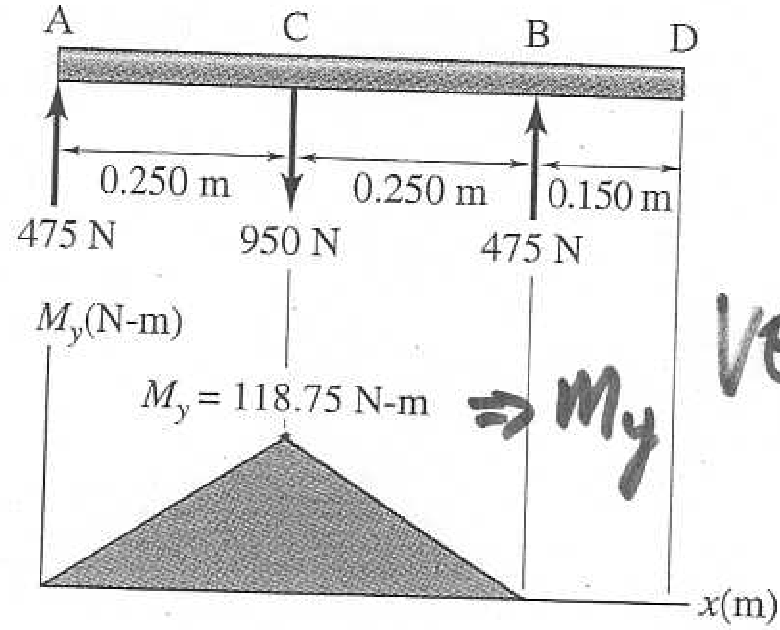




∴ AS SHAFT ROTATES MOMENT IN VERTICAL & HORIZONTAL PLANES CAUSE alternating stress, σ_a

∴ STEADY TORQUE BETWEEN C & D CAUSES τ_m

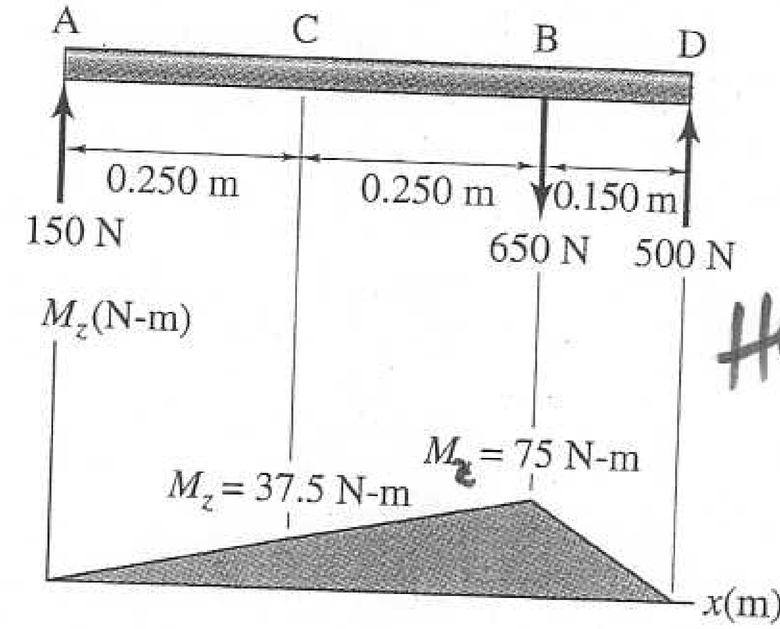
MOMENT
X-Z PLANE
M_y



⇒ M_y

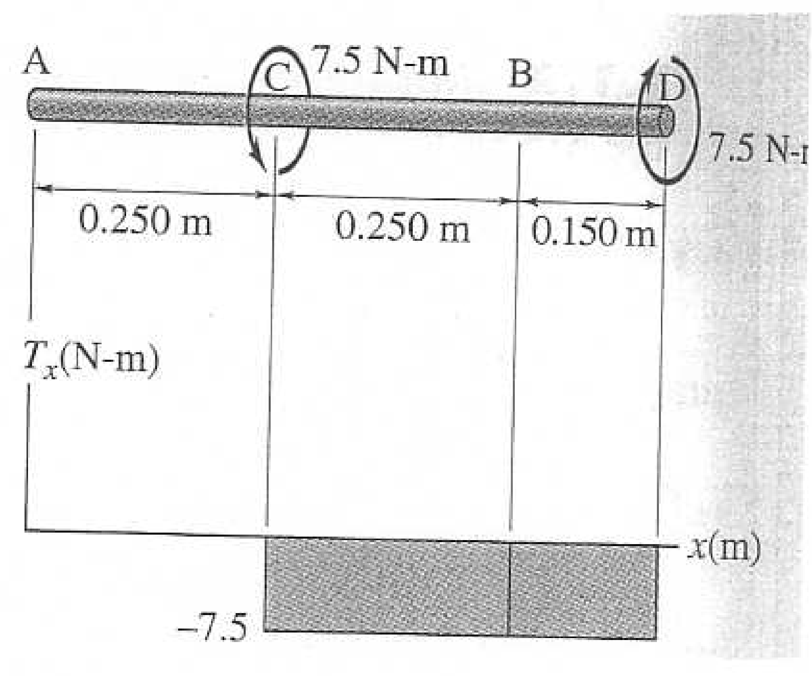
VERT

MOMENT
X-Y PLANE
M_z



HORIZ

TORQUE
T_x



LOCATION OF MINIMUM FACTOR OF SAFETY

... since diameter is constant look for location of max loading

... max bending moment @ C

$$\underline{M_{max}} = \sqrt{M_y^2 + M_z^2}$$

$$= \sqrt{118.75^2 + 37.5^2}$$

$$\boxed{M_{max} = 124.5 \text{ N-m}}$$

... also have torque T acting @ C

... ∴ Min "n" is @ C.

.... MAY BE ELSEWHERE IF diam ≠ const

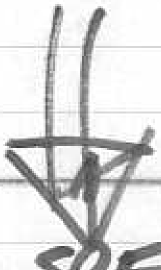
POSSIBLE QUESTIONS ... MTL KNOWN

÷ SHAFT DIAMETER FOR GIVEN LIFE AND SAFETY FACTOR*

÷ GIVEN DIAMETER, WHAT IS SAFETY FACTOR FOR INFINITE OR FINITE LIFE

÷ WHAT SORT OF SURFACE FINISH SHOULD BE SPECIFIED?

÷ etc



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* IF NOT SPECIFIED, ASSUME SAFETY FACTOR $n = 1$
