Optimal Sidelobe Suppression for Biphase Codes

J. M. Baden and M. N. Cohen
Georgia Tech Research Institute
Atlanta GA 30332

1 Abstract

In this paper a technique is developed for the generation of filters for pulse compression sidelobe reduction. This technique has two advantages over the standard Wiener filtering technique. Firstly, it minimizes sidelobe energy over multiple input sequences, and secondly it can shape the sidelobe energy through the application of a weighting function that indicates which sidelobes are most important to reduce. Two example applications are provided, showing the effectiveness of this technique in generating filters for orthogonal code pairs and in generating filters for compressions with low sidelobes near the main peak.

2 Introduction

Biphase pulse compression is widely used in radar systems. In biphase compression a transmit signal is phase-modulated with a binary code, i.e. the phase of the transmit signal is 0 degrees relative to a local reference for a "+1" in the binary code, and 180 degrees for a "−1". In general the compressed waveform is obtained by correlating the received signal with the binary code that was used to modulate the phase of the transmit signal. This is referred to as *matched filtering.*

The correlation function for a biphase code is invariably contaminated with range sidelobes. Several techniques exist that generate *mismatched filters* to minimize these range sidelobes. The sidelobe reduction available through the use of mismatched filtering varies depending on the code and type of mismatched filter being used, and also varies depending on the length of the filter (the code and filter may be of different lengths.) Several techniques [1, 2, 3, 4, 5] minimize the total energy in the range sidelobes (the integrated sidelobe level, or ISL, of the compressed pulse). Linear programming techniques have been applied to the problem of minimizing the peak sidelobe [6].

The technique described in this paper uses an approach similar to that of the integrated sidelobe level reduction techniques, but generates a filter that minimizes sidelobe energy for multiple input sequences, and allows for specification of the importance of reducing specific range sidelobes.

3 Definitions

Biphase pulse compression relies on the correlation function:

$$s_i = \sum_{j=0}^{p} x_j a_{j-i}, \quad i = -p, ..., p, \tag{1}$$

where s_i is the correlator output at bin number i, x_j is code bit j, and a_j is filter bit j. Here it is also assumed that p is an integer representing the greater of the code and filter lengths, and that the code is zero-padded to the same length as the filter. The term *biphase* is used to indicate that the transmit code bits (x_i) are either +1 or -1; indicating either 0 degrees or 180 degrees phase shift relative to a local reference. A *polyphase* signal is one for which the phase may assume values other than 0 and 180 degrees.

A significant problem inherent in biphase pulse compression is that the correlation of the receive signal and its matched filter does not yield a perfect impulse; i.e., it does not yield $s_i = 0$ for all $i \neq 0$ as it ideally might. Any s_i term for $i \neq 0$ is referred to as a *range sidelobe.* The zero-offset correlation value (s_0) is referred to as the *main peak.* The difference between a pulse compression waveform and a simple pulse waveform may be found primarily in the existence and values of these sidelobes. The size of the main peak of an autocorrelation is a measure of the signal-to-noise (S/N) ratio improvement that is attained through the use of the given code. The longer the code, the better the S/N ratio improvement. The sidelobes can greatly limit the usefulness of a code regardless of the strength of the main peak. This is because the sidelobes are effectively "self-noise" which may be much larger than system noise, and nullify many of the benefits of using pulse compression in the first place. Codes are usually chosen for a given application based on their length (for S/N ratio and resolution improvements) and their sidelobe levels.

There are two frequently used sidelobe measures. The first is the *peak sidelobe level*, or *PSL.* The peak sidelobe is simply the largest sidelobe in the correlation of a code and its filter. The peak sidelobe level is usually expressed as a ratio of the peak sidelobe to the main peak and is expressed in decibels. The second measure is the *integrated sidelobe level*, or *ISL.* This refers to the total energy in all of the sidelobes. It is usually expressed as a ratio of the total sidelobe energy to main peak energy and is expressed in decibels. The main peak will be degraded when mismatched filtering is used.

As mentioned above, when mismatched filtering is used, the main peak will not be as large as it would have been with matched filtering. This loss is referred to as the *loss in processing gain*, or *LPG.* LPG is expressed in decibels as the ratio of the mismatched peak to the matched peak.

4 Weighted Sidelobe Reduction Filtering

In this section the mathematical basis for weighted sidelobe reduction filtering will be given. The derivation is a generalization of the derivations given in [4] for Wiener filters and in [7] for Optimal ISL filters. Figure 1 illustrates standard Wiener filtering. The input to the filter is $x(n)$; the filter weights are $a(n)$; the desired output is $d(n)$; and the error is $e(n) = d(n) - [x(n) * a(n)]$. Wiener filtering minimizes the expected error $\sum_n |e(n)|^2$.

The Wiener filtering technique will now be modified to generate a single filter that minimizes the energy in the difference between the desired output of the correlation of a set of codes with the filter and the actual correlator output. A weighting function is included so that errors in definable regions of the correlation sidelobes of certain codes with the filter may be counted more heavily than other sidelobes. Figure 2 shows a diagram for Weighted Mismatched Filtering. The changes as compared with Figure 1 are the additional dimensions for the sequences x, d, and e, and the energy weighting function $w(n,m)$. The terms $x(n,m)$, $d(n,m)$, $e(n,m)$, and $a(n)$ are all complex; $w(n,m)$ is real. The only unknown is the sequence $a(n)$.

The problem now is to minimize

$$\begin{aligned} E &= \sum_{n=-p}^{p} \sum_{m=1}^{l} |e(n,m)|^2 \\ &= \sum_{m=1}^{l} \sum_{n=-p}^{p} [e(n,m)e^*(n,m)] \end{aligned} \tag{2}$$

where l is the row dimension of x, i.e. the number of codes, p is the length of the code, and e^* is the complex conjugate of e. The summation over $n = -p, -p+1, \ldots, p-1, p$ encompasses all range sidelobes, and the summation over $m = 1, \ldots, m$ encompasses the multiple input sequences. Note that the error term may be expressed as

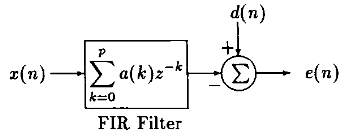

Figure 1: Standard Wiener Filtering

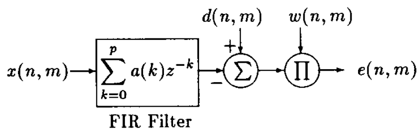

Figure 2: Weighted Mismatch Filtering

$$e(n,m) = w(n,m)\left[d(n,m) - \sum_{i=0}^{p} x(i,m)a(i-n)\right]. \quad (3)$$

The problem is to find $a(n)$, for all n, such that E will be a minimum. The method is to set $\frac{\partial E}{\partial a(k)} = 0$ for $k = 0, \ldots, p$. The real and imaginary parts may be worked separately; thus, set

$$\frac{\partial E}{\partial a_r(k)} = \frac{\partial E}{\partial a_i(k)} = 0 \quad k = 0, \ldots, p, \quad (4)$$

where a_r is the real part of a, and a_i is the imaginary part. Taking the partial derivative with respect to $a_r(k)$ of both sides of Eq. 2 gives:

$$\frac{\partial E}{\partial a_r(k)} = \sum_{m=1}^{l}\sum_{n=-p}^{p}\Big[\; e(n,m)\frac{\partial}{\partial a_r(k)}e^*(n,m) + e^*(n,m)\frac{\partial}{\partial a_r(k)}e(n,m)\Big]. \quad (5)$$

Eq. 3 may be used to calculate the derivatives of $e(n,m)$ and $e^*(n,m)$:

$$\begin{aligned} e(n,m) &= w(n,m)\left[d(n,m) - \sum_{i=0}^{p} x(i,m)a(i-n)\right] \quad (6)\\ &= w(n,m)\left[d(n,m) - \sum_{i=0}^{p} x(i,m)[a_r(i-n) + ja_i(i-n)]\right] \end{aligned}$$

so

$$\frac{\partial}{\partial a_r(k)}e(n,m) = -w(n,m)x(n+k,m) \quad (7)$$

and

$$\frac{\partial}{\partial a_r(k)}e^*(n,m) = -w(n,m)x^*(n+k,m). \quad (8)$$

So Eq. 5 becomes

$$\frac{\partial E}{\partial a_r(k)} = -\sum_{m=1}^{l}\sum_{n=-p}^{p}\Big[\; e(n,m)w(n,m)x^*(n+k,m) + e^*(n,m)w(n,m)x(n+k,m)\Big]. \quad (9)$$

Since one term in the summation is the complex conjugate of the other, and the terms are added together, Eq. 9 may be simplified as:

$$\frac{\partial E}{\partial a_r(k)} = -2\sum_{m=1}^{l}\sum_{n=-p}^{p}\Re[e(n,m)w(n,m)x^*(n+k,m)]. \quad (10)$$

Similarly, for the imaginary part:

$$\frac{\partial E}{\partial a_i(k)} = -\sum_{m=1}^{l}\sum_{n=-p}^{p}\Big[\; e(n,m)\frac{\partial}{\partial a_i(k)}e^*(n,m) + e^*(n,m)\frac{\partial}{\partial a_i(k)}e(n,m)\Big] \quad (11)$$

and

$$\frac{\partial}{\partial a_i(k)}e(n,m) = -jw(n,m)x(n+k)\Phi(n+k,m) \quad (12)$$

and

$$\frac{\partial}{\partial a_i(k)}e^*(n,m) = jw(n,m)x^*(n+k,m). \quad (13)$$

So Eq. 11 becomes

$$\frac{\partial E}{\partial a_i(k)} = \sum_{m=1}^{l}\sum_{n=-p}^{p}\Big[\; je(n,m)w(n,m)x^*(n+k,m) - je^*(n,m)w(n,m)x(n+k,m)\Big]. \quad (14)$$

Since one term in the summation is the complex conjugate of the other, and the second term is subtracted from the first, Eq. 14 may be simplified as:

$$\frac{\partial E}{\partial a_i(k)} = -2\sum_{m=1}^{l}\sum_{n=-p}^{p}\Im[e(n,m)w(n,m)x^*(n+k,m)]. \quad (15)$$

Remembering that the partials are set to zero, adding Eq. 10 to Eq. 15 gives

$$\sum_{m=1}^{l}\sum_{n=-p}^{p} e(n,m)w(n,m)x^*(n+k,m) = 0. \quad (16)$$

Substituting Eq. 3 into Eq. 16 gives

$$\sum_{m=1}^{l}\sum_{n=-p}^{p} w^2(n,m)x^*(n+k,m)\left[d(n,m) - \sum_{i=0}^{p} x(i,m)a(i-n)\right] = 0 \quad (17)$$

which can be written as

$$\sum_{m=1}^{l}\sum_{n=-p}^{p} w(n,m)x^*(n+k,m)w(n,m)\sum_{i=0}^{p} x(i,m)a(i-n) = \sum_{m=1}^{l}\sum_{n=-p}^{p} w^2(n,m)x^*(n+k,m)d(n,m). \quad (18)$$

Since there are $p+1$ unknowns (the filter weights) and $p+1$ linear equations in the unknowns $a(k)$ (one equation for each k in Eq. 4, and therefore in Eq. 18), this may be rewritten in matrix form as $RA = Y$, where $R_{k,l}$ is the coefficient of $a(l)$ in equation k. Then, $R_{k,l}$ and Y_k are given by:

$$\begin{aligned} R_{k,l} &= \sum_{m=1}^{l}\sum_{n=-p}^{p} w^2(n,m)x^*(n+k,m)x(l+n,m) \quad (19)\\ Y_k &= \sum_{m=1}^{l}\sum_{n=-p}^{p} w^2(n,m)x^*(n+k,m)d(n,m). \quad (20) \end{aligned}$$

The values for the filter weights, $a(i)$, may be obtained by multiplying $Y = (Y_k)$ by the inverse of R.

5 Tracking Waveform Filters

Weighted Mismatched Filtering may be used to generate filters that produce relatively sidelobe-free regions within the correlation. Such filters may be of use in tracking applications or in target recognition systems, where it may be particularly desirable to have low sidelobes close to the main peak. By appropriately choosing a weighting function to represent this requirement, very low close-in sidelobes may be achieved at the expense of higher sidelobes farther out.

Figures 3 and 4 show a twenty bit MPS (minimum peak sidelobe) code correlated with its matched filter and with a 32-bit Optimal ISL filter. Figure 5 shows the compression of the 20 bit MPS code through a 32 bit tracking filter designed for a 30 bin null region around the peak. The weighting function weighted the peak by a value of 0.5, sidelobes near the peak by 1.0, and outer sidelobes by 0.001. The sidelobes near the peak are effectively eliminated over a 30 bin region,

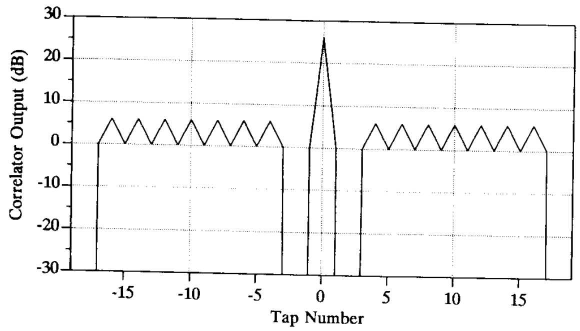

Figure 3: 20 Bit MPS Code, Matched Filter

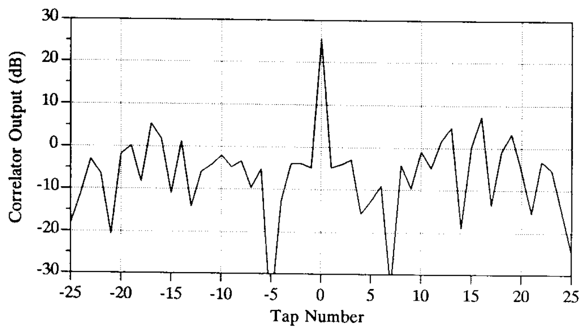

Figure 4: 20 Bit MPS Code, Optimal ISL Filter

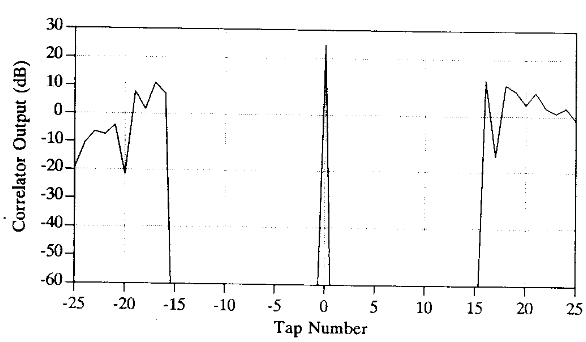

Figure 5: 20 Bit MPS Code, 30 Bin Tracking Filter

though the peak sidelobe has risen by about 6 dB. The LPG is 0.5 dB for the Optimal ISL filter and 1.2 dB for the tracking filter. The width of the achievable null region is limited; if the specified null region is too wide, then it is impossible to achieve zero sidelobes over the full region, and the resulting filter will be very similar to the Optimal ISL filter.

6 Orthogonal Filtering

In this report, the term Orthogonal Filter refers to a filter that correlates well with one code and is relatively "blind" to another code. That is, the peak value of either code's correlation with its respective filter should be substantially greater than the peak of the cross-correlation of either of the codes with the other code's filter. For two codes to be considered truly orthogonal, the cross-correlation of the two codes should be identically zero. Orthogonal coding may be of use in polarimetrics, where the polarization scattering matrix might be generated on a single pulse if the two polarization channels are coded orthogonally. Orthogonal coding may also be of use in communication systems for channel multiplexing, or in radar scenarios where more than one radar may be using the same frequency in a given arena.

Weighted Mismatched Filtering offers a chance of generating a filter that minimizes the energy in the sidelobes of the correlation of the code for which good compression is desired plus the energy in the correlation of filter with the code for which orthogonality is desired. To this end, let the number of codes $l = 2$ and $x(n,1) = c_1(n)$ be a code for which a large peak and low sidelobes are desired and let $x(n,2) = c_2(n)$ be a code for which the filter is to be as orthogonal as possible; i.e. c_2 correlated with the filter should be near zero at all taps. The weighting function may be set to unity for all n for both $m = 1$ and $m = 2$, and $d(n,m)$ may be set to zero everywhere but the peak of the first code; i.e., $d(n,m) = 0$ for $n \neq 1$ or $m \neq 1$, and $d(0,1) = 1$.

In previous work [8, 9], MPS codes with good orthogonality were discovered. Figure 6 shows the autocorrelation of one of a pair of 17 bit MPS codes that were noticed for their natural orthogonality. This code will be referred to as code A. Figure 7 shows the cross-correlation of code A with its counterpart, code B.

The weighting function described above was used to generate a filter that compressed well with code A and was as orthogonal as possible to code B. Figures 8 and 9 show the correlations of codes A and B, respectively, with the Optimal Orthogonal Filter. Though not obvious from the plots, the overall sidelobe energy (the sum of the sidelobe energy in Figure 8 and the total energy in Figure 9) drops by about one decibel for the Optimal Orthogonal Filter. Since the length of the filter is 51 bits, it is not expected that a much longer filter would improve the results significantly. Further analysis showed that the Optimal Orthogonal Filter was negligibly better than the Optimal ISL filter in terms of total sidelobe energy. Analysis for other codes showed some variation in performance, though no codes were found for which the Optimal Orthogonal Filter showed significant improvement over Optimal ISL filtering.

Given the disappointing results for orthogonality using the weighting function described above, a weighting function was defined for generating filters that minimized sidelobes in a region of the compressed waveforms rather than over the full sidelobe extent. The weighting function was set equal to 1.0 for the 8 sidelobes closest to the peak of the correlation of the filter and code A, and the 8 sidelobes at the center of the correlation of the filter and code B. This proved to be more effective. Figures 10 and 11 show the compressions of code A and code B with the filter generated with this new weighting function. The sidelobe energy in the eight bins around the peak are essentially zero for both code A and code B. Note that the peak for code A has dropped by about three dB from the matched filter. An orthogonal filter was also generated to provide good compression with code B and low correlation with code A. The results are not shown here but are similar to that shown in Figures 10 and 11.

Preliminary attempts to enlarge the null region for these particular codes were not successful, though it may be possible to obtain a larger null region for longer codes and filters.

7 Summary

The weighted mismatched filtering described in this paper makes it possible to generate filters that minimize the sidelobes in specific regions of the correlation of a number of input codes. This generalization of other optimal mismatched filtering techniques allows the sidelobe energy to be shaped rather than simply reduced.

Weighted Mismatched Filtering has been applied to two different applications. It proved to be very effective at producing filters for

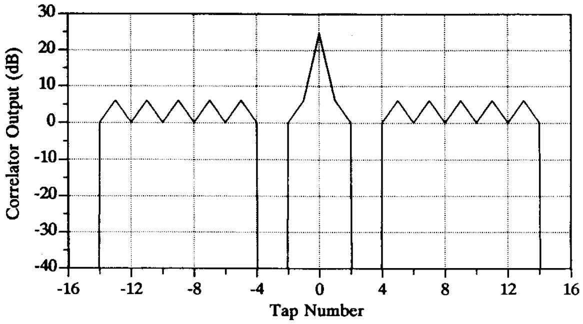

Figure 6: Code A Matched Filter Compression

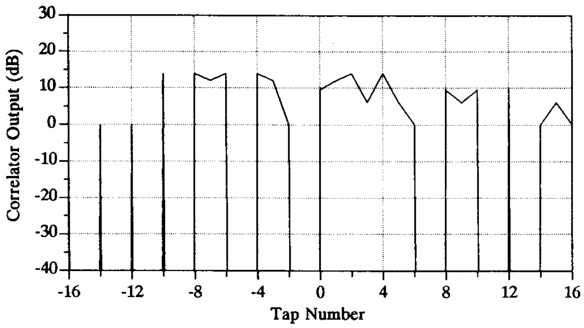

Figure 7: Code B compression Through Code A Matched Filter

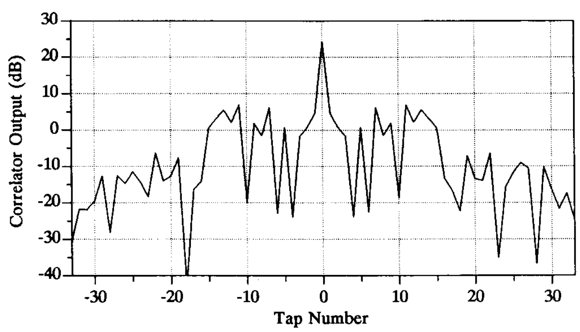

Figure 8: Code A Compression, Optimal Orthogonal Filter

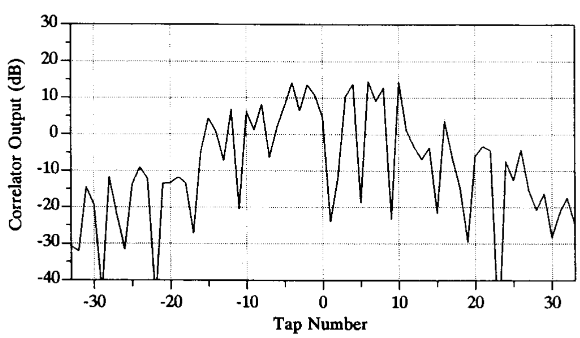

Figure 9: Code B Compression, Optimal Orthogonal Filter

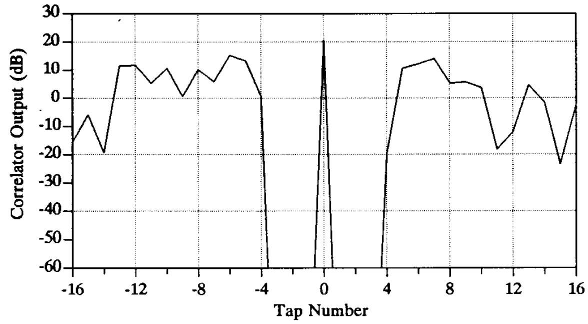

Figure 10: Code A Compression, 8 Bin Null Region Orthogonal Filter

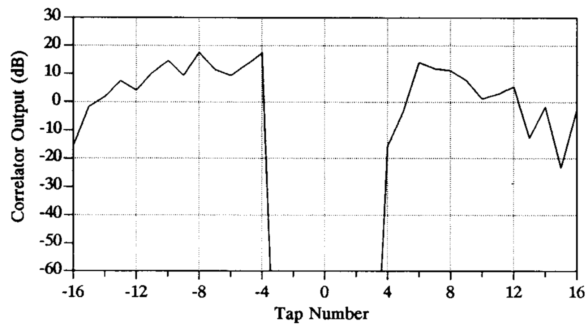

Figure 11: Code B Compression, 8 Bin Null Region Orthogonal Filter

tracking waveforms by producing wide null regions around the main peak. These filters may also be of use in high range resolution signature measurements if the codes and filters utilized are sufficiently long to permit null regions significantly wider than the target extent.

The results for orthogonal filtering were poor when orthogonality was desired over all range bins, but was very effective at producing null regions near the compression peak. These null regions, and larger null regions for longer codes, if attainable, may be utilized for single pulse measurement of the polarization scattering matrix or for channel multiplexing in communications systems.

References

[1] E. L. Key, E. N. Fowle, and R. D. Haggarty. A method of sidelobe suppression in phase-coded pulse compression systems. Technical report, MIT Lincoln Laboratory, August 1959. Technical Report No. 209.

[2] August W. Rihaczek and Roger M. Golden. Range sidelobe suppression for barker codes. *IEEE Transactions on Aerospace and Electronic Systems*, Volume AES-7, November 1971.

[3] Martin H. Ackroyd and F. Ghani. Optimum mismatched filters for sidelobe suppression. *IEEE Transactions on Aerospace and Electronic Systems*, Volume AES-9, March 1973.

[4] N. Wiener. *Extrapolation, Interpolation, and Smoothing of Stationary Time Series, With Engineering Applications.* Wiley, New York, 1949.

[5] Marvin N. Cohen and J. M. Baden. Pulse compression coding study. Technical report, Georgia Tech Research Institute, Atlanta GA, 1983. Project Number A-3366.

[6] Steven Zoraster. Minimum peak range sidelobe filters for binary phase-coded waveforms. *IEEE Transactions on Aerospace and Electronic Systems*, Volume AES-16, January 1980.

[7] Marvin N. Cohen. Binary phase-coded pulse compression. Technical report, Norden Systems, Norwalk, Conn, 1979. Internal Report No. 1293-R-0021.

[8] Marvin N. Cohen and Philip E. Cohen. Near-perfect biphase codes and optimal filtering of their range sidelobes. In *Proceedings of the 18th European Microwave Conference*. Stockholm, Sweden, September 1988.

[9] Marvin N. Cohen, J. M. Baden, and Philip E. Cohen. Biphase codes with minimum peak sidelobes. In *IEEE National Radar Conference*. Dallas Texas, 1989.