HW#4 Solutions

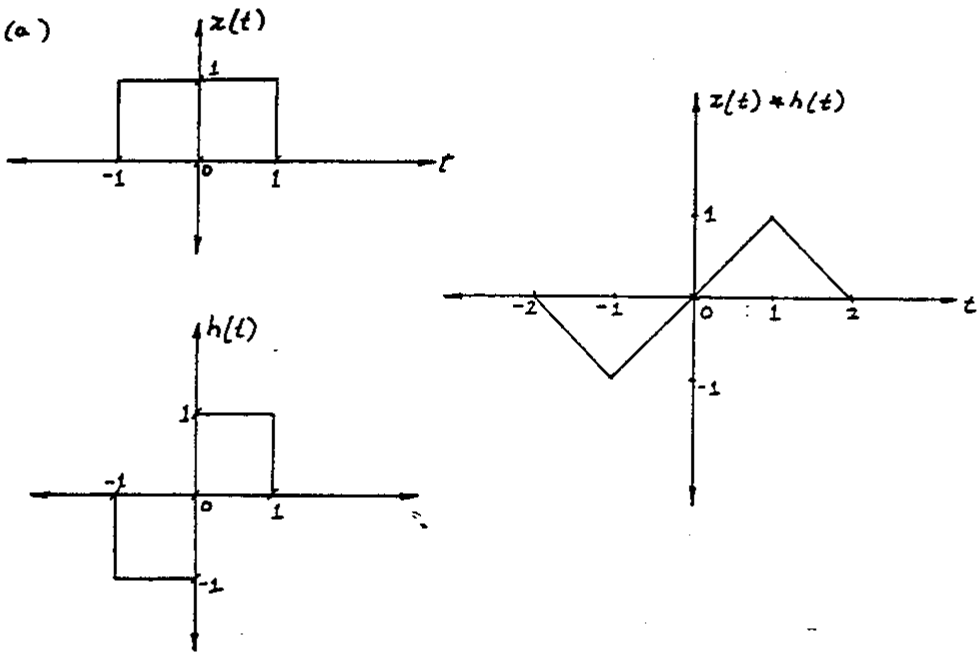
2.3

(i) $y(t) = [u(t+1) - u(t-1)]\,\mathrm{sgn}\,t * u(t)$

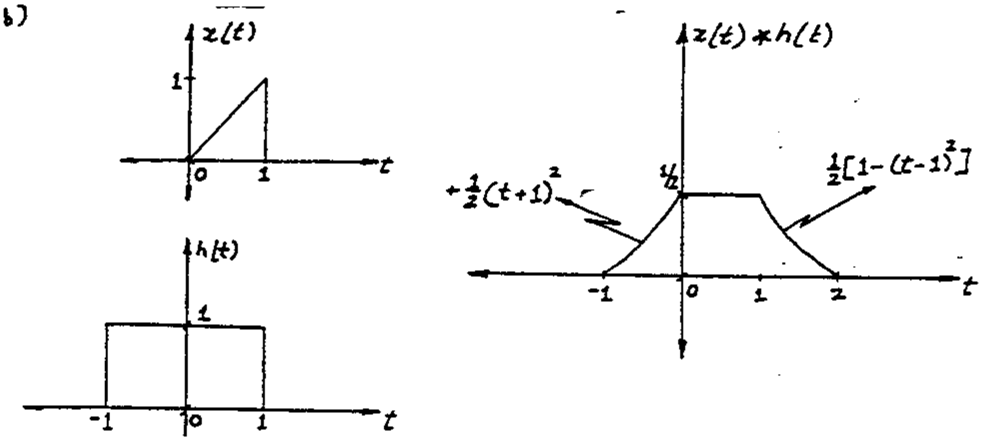
$$= [-u(t+1) + 2u(t) - u(t-1)] * u(t)$$

$$= \begin{cases} 0 & t \le -1 \\ -t-1 & -1 \le t \le 0 \\ t-1 & 0 \le t \le 1 \\ 0 & t \ge 1 \end{cases}$$

2-4 (a)



(b)



3.10 $x(t) = \sin t \qquad 0 \le t \le \pi \qquad x(t+\pi) = x(t)$

$$C_n = \frac{1}{T}\int_T x(t)\exp[-j2nt]\,dt$$

$$= \frac{1}{\pi}\int_0^{\pi} \sin t \, \exp[-j2nt]\,dt = \frac{1}{\pi}\int_0^{\pi}\left(\frac{\exp[jt]-\exp[-jt]}{2j}\right)\exp[$$

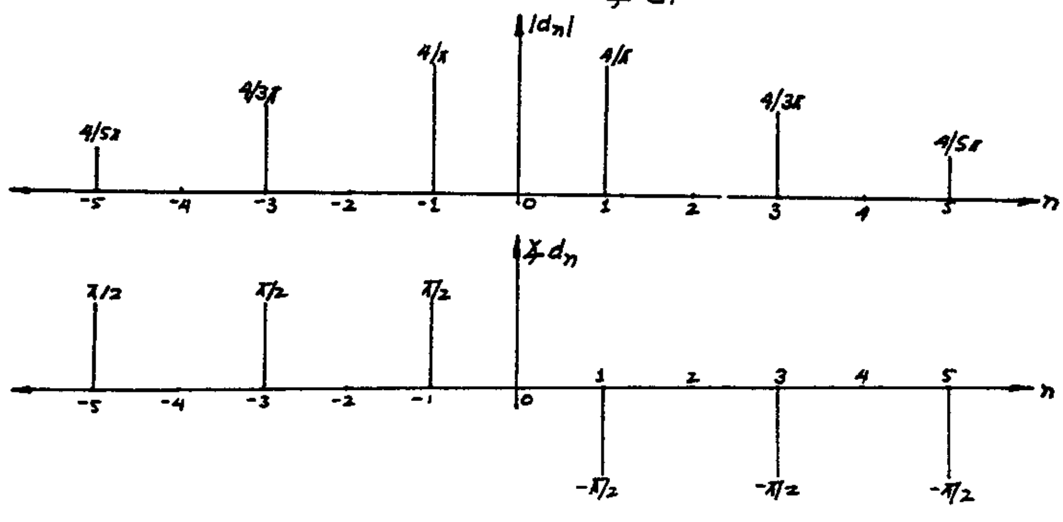
$$= \frac{2}{\pi}\left(\frac{1}{1-4n^2}\right)$$

3.12

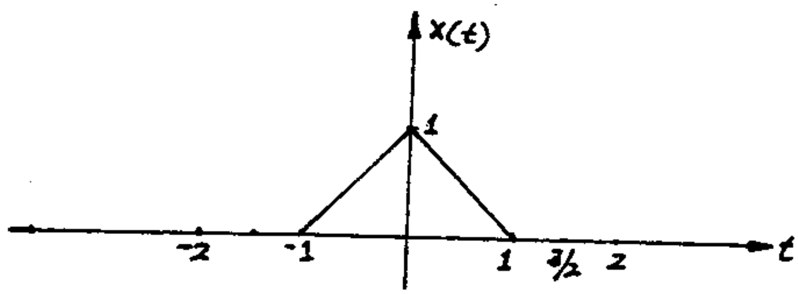
(e) $x(t)$ in (e) is equal to 2 times of $x(t)$ in (a)

$$x(t) = 2\sum_{n=-\infty}^{\infty} C_n \exp[jn\pi t]$$

$\Longrightarrow$ $d_n = 2C_n$ and $\measuredangle d_n = \measuredangle C_n$



(h)



$x(t)$ is an even function

$$C_n = \frac{1}{T}\int_T x(t)\exp[-jn\omega_0 t]\,dt$$

$$= \frac{2}{T}\int_0^{\frac{T}{2}} x(t)\cos n\omega_0 t\,dt$$

with $\quad T = 3 \qquad \omega_0 = \frac{2\pi}{T} = \frac{2\pi}{3}\,, \quad C_0 = 0$

$$C_n = \frac{2}{3}\int_0^1 (1-t)\cos\frac{2n\pi}{3}t\,dt$$

$$= \frac{2}{3}\left[\frac{3}{2n\pi}\sin\left(\frac{2n\pi}{3}\right) - \frac{3}{2n\pi}\left[\sin\frac{2n\pi}{3} + \frac{3}{2n\pi}\left(\left(\cos\frac{2n\pi}{3} - 1\right)\right)\right]\right]$$

$$= -\frac{2}{3}\left[\left(\frac{3}{2n\pi}\right)^2\left(\cos\frac{2n\pi}{3} - 1\right)\right] = -\frac{3\left[\cos\left(\frac{2n\pi}{3}\right) - 1\right]}{(2\pi^2)\,n^2}$$

C_n is real