CE 407 Notes
Multicomponent distillation short-cut examples

1. An eight-component mixture is to be distilled with 98 percent recovery of the light key in the distillate and 99 percent recovery of the heavy key in the bottoms. Feed composition and relative volatilities (assumed constant) are specified in the following table:

component i	$(x_F)_i$	$\alpha_i = \alpha_{i,HK}$
1	0.06	3.766
2	0.03	3.110
3	0.17	2.875
4 (LK)	0.25	1.910
5	0.06	1.409
6 (HK)	0.30	1.000
7	0.10	0.733
8	0.03	0.405

What will be the split of component 5 at infinite reflux ratio? State any assumptions or approximations you make, and justify *a posteriori*.

2. A four-component mixture (see table below) is to be distilled with 97.5 percent recovery of the light and heavy keys in the distillate and bottoms. Estimate the number of ideal stages required at a reflux ratio equal to 1.3 times the minimum.

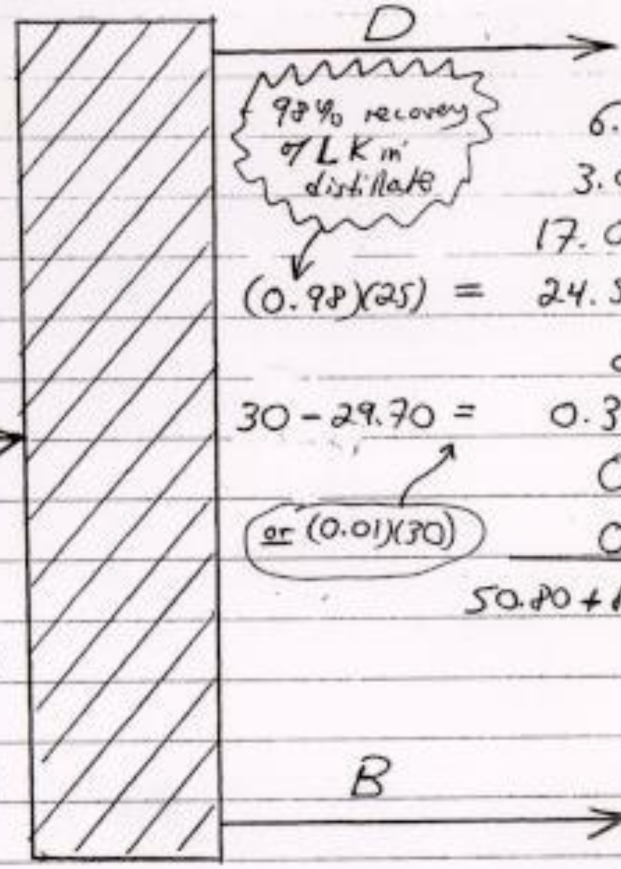
component i	mole fraction in feed $(x_F)_i$	relative volatility (with respect to HK) $\alpha_i = \alpha_{i,HK}$
1	0.08	3.09
2 (LK)	0.50	1.95
3 (HK)	0.39	1.00
4	0.03	0.52

The mixture enters the distillation column as saturated liquid.

PROBLEM 1

100 mol basis; $\text{cpnt}_i \equiv$ "component i"

(i) Set up mole numbers



F = 100 mol

6 mol cpnt_1
3 mol cpnt_2
17 mol cpnt_3
25 mol cpnt_4 (LK)
6 mol cpnt_5
30 mol cpnt_6 (HK)
10 mol cpnt_7
3 mol cpnt_8

D

98% recovery of LK in distillate

6.00 mol cpnt_1
3.00 mol cpnt_2
17.00 mol cpnt_3
$(0.98)(25) =$ 24.50 mol cpnt_4
δ mol cpnt_5
$30 - 29.70 =$ 0.30 mol cpnt_6 or $(0.01)(30)$
0 mol cpnt_7
0 mol cpnt_8
$50.80 + \delta$ mol total D

B

0 mol cpnt_1
0 mol cpnt_2
0 mol cpnt_3
$25 - 24.50 =$ 0.50 mol cpnt_4 or $(0.02)(25)$
$6 - \delta$ mol cpnt_5
$(0.99)(30) =$ 29.70 mol cpnt_6
10.00 mol cpnt_7
3.00 mol cpnt_8
$49.20 - \delta$ mol total B

99% recovery of HK in bottoms

Have <u>assumed</u> components 1–3 (lighter than LK) are entirely absent from bottoms and components 7, 8 (heavier than HK) are entirely absent from distillate

(ii) Fenske

Fenske eq. with $i = 4$ (LK) and $j = 6$ (HK):

$$N_{min} + 1 = \frac{\log\left[\dfrac{(x_D)_4 / (x_B)_4}{(x_D)_6 / (x_B)_6}\right]}{\log(\alpha_{4,6})}$$

$= \alpha_{4,HK} = \alpha_4$

Here have mult. num. and denom. in log by D/B

$$= \frac{\log\left[\dfrac{D(x_D)_4 / B(x_B)_4}{D(x_D)_6 / B(x_B)_6}\right]}{\log \alpha_4}$$

$$= \frac{\log\left[\dfrac{24.50 / 0.50}{0.30 / 29.70}\right]}{\log(1.910)} = 13.11$$

distributed component

Now apply Fenske eq. with $i = 5$ and $j = 6$:

$$N_{min} + 1 = \frac{\log\left[\dfrac{(x_D)_5 / (x_B)_5}{(x_D)_6 / (x_B)_6}\right]}{\log(\alpha_{5,6})}$$

$$= \frac{\log\left[\dfrac{D(x_D)_5 / B(x_B)_5}{D(x_D)_6 / B(x_B)_6}\right]}{\log \alpha_5}$$

or

$$\frac{D(x_D)_5}{B(x_B)_5} = \frac{\text{moles of } 5 \text{ in } D}{\text{moles of } 5 \text{ in } B} = \text{Split of component } 5$$

$$= \left(\frac{D(x_D)_6}{B(x_B)_6}\right) \alpha_5^{\,N_{min}+1}$$

$$= \left(\frac{0.30}{29.70}\right)(1.409)^{13.11} = \boxed{0.905}$$

(Can now determine δ: $0.905 = \frac{\delta}{6-\delta} \Rightarrow \delta = 2.85$ mol)

(iii) Go back and check approximation that there is negligible amount of ⑦, ⑧ in distillate and negligible amount of ①, ②, ③ in bottom product. For instance, Fenske eq. for $i=1$, $j=6$ can be written as:

$$N_{min} + 1 = \frac{\log\left[\frac{(x_D)_1 / (x_B)_1}{(x_D)_6 / (x_B)_6}\right]}{\log(\alpha_{1,6})}$$

$$= \frac{\log\left[\frac{D(x_D)_1 / B(x_B)_1}{D(x_D)_6 / B(x_B)_6}\right]}{\log \alpha_1}$$

or

$$B(x_B)_1 = D(x_D)_1 \left\{\left(\frac{D(x_D)_6}{B(x_B)_6}\right) \alpha_1^{N_{min}+1}\right\}^{-1}$$

$$= (6.00 \text{ mol}) \left\{\left(\frac{0.30}{89.70}\right) (3.766)^{13.11}\right\}^{-1}$$

$$\boxed{B(x_B)_1 = 1.67 \times 10^{-5} \text{ mol} = \text{negligibly small}^{*}}$$

Find similarly

$$\boxed{\begin{aligned} B(x_B)_2 &= 1.03 \times 10^{-4} \text{ mol} = \text{negligibly small} \\ B(x_B)_3 &= 1.63 \times 10^{-3} \text{ mol} = \text{negligibly small} \end{aligned}}$$

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* This number is SO SMALL it wouldn't be noticed even by Elroy's Pet Mushroom!

Also, Fenske eq. for $i = 7, j = 6$ can be written as:

$$N_{min} + 1 = \frac{\log\left(\frac{(x_D)_7 / (x_B)_7}{(x_D)_6 / (x_B)_6}\right)}{\log(\alpha_{7,6})}$$

$$= \frac{\log\left(\frac{D(x_D)_7 / B(x_B)_7}{D(x_D)_6 / B(x_B)_6}\right)}{\log \alpha_7}$$

or

$$D(x_D)_7 = B(x_B)_7 \left(\frac{D(x_D)_6}{B(x_B)_6}\right) \alpha_7^{N_{min}+1}$$

$$= (10.00 \text{ mol}) \left(\frac{0.30}{29.70}\right) (0.733)^{13.11}$$

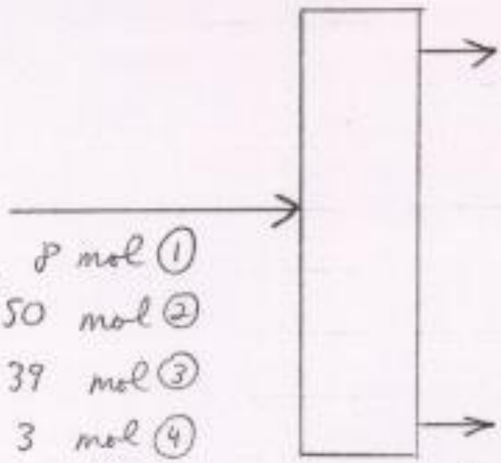
$$\boxed{D(x_D)_7 = 1.72 \times 10^{-7} \text{ mol} = \text{negligibly small}}$$

Find similarly

$$\boxed{D(x_D)_8 = 2.16 \times 10^{-7} \text{ mol} = \text{negligibly small}}$$

PROBLEM 2 Basis: 100 mol feed

8.000 mol ①
$(0.975)(50) = 48.750$ mol ②
$39 - 38.025 = 0.975$ mol ③
$D = 57.725$ mol



8 mol ①
50 mol ②
39 mol ③
3 mol ④

$50 - 48.750 = 1.250$ mol ②
$(0.975)(39) = 38.025$ mol ③
3.000 mol ④
$B = 42.275$ mol

(Assume no ① is present in B and no ④ is present in D.)

(i) N_{min} at infinite reflux ratio ("total reflux")

$$N_{min} + 1 = \frac{\log\left(\dfrac{D(x_D)_2 / B(x_B)_2}{D(x_D)_3 / B(x_B)_3}\right)}{\log \alpha_{2,3}}$$

$$= \frac{\log\left(\dfrac{48.750 / 1.250}{0.975 / 38.025}\right)}{\log (1.95)} = 11.0$$

$N_{min} = 10.0$

(ii) Minimum reflux ratio R_{min}

Eq. (18.29) on p. 602:

$$1 - \cancel{q} = \sum_{i=1}^{4} \frac{\alpha_i (x_F)_i}{\alpha_i - \varphi}$$

Sum over all components present in the feed

1 because feed is sat liq.

or

$$0 = \frac{(3.09)(0.08)}{3.09-\varphi} + \frac{(1.95)(0.50)}{1.95-\varphi} + \frac{(1.00)(0.39)}{1.00-\varphi} + \frac{(0.52)(0.03)}{0.52-\varphi}$$

Look for solution of this equation in the interval $\alpha_3 < \varphi < \alpha_2$, i.e., $1.00 < \varphi < 1.95$. WHY NOT USE MAPLE ?

```
elroy@elroy> maple

**NOTE** Maple/XMaple 5.4 is now the default. To use the
command-line interface, type maple. To use the graphical
interface type xmaple. To access the previous version of
Maple, type maple5.3/xmaple5.3

    |\^/|     Maple V Release 4 (WMI Campus Wide License)
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 \  MAPLE  /  reserved. Maple and Maple V are registered trademarks of
 <____ ____>  Waterloo Maple Inc.
      |       Type ? for help.
> f := phi -> 3.09 * 0.08 / (3.09 - phi) + 1.95 * 0.50 / (1.95 - phi)
>     + 1.00 * 0.39 / (1.00 - phi) + 0.52 * 0.03 / (0.52 - phi);
                     .2472         .9750         .3900        .0156
      f := phi -> ----------- + ----------- + ----------- + ----------
                  3.09 - phi    1.95 - phi    1.00 - phi    .52 - phi

> phi = fsolve(f(phi) = 0, phi=1..1.95);
                             phi = 1.256613339

> quit
bytes used=366324, alloc=327620, time=0.38
elroy@elroy>
```

Find $\varphi = 1.257$. Then R_{min} is given by Eq. (19.30):

Book calls this R_{D_m}

$$R_{min} + 1 = \sum_{i=1}^{3} \frac{\alpha_i (x_D)_i}{\alpha_i - \varphi}$$

Sum over only those components found in distillate

$$\overset{*}{=} \frac{(3.09)(0.1386)}{3.09-1.257} + \frac{(1.95)(0.8445)}{1.95-1.257} + \frac{(1.00)(0.0169)}{1.00-1.257}$$

$$= 2.54$$

$$R_{min} = 1.54$$

$*\quad (x_D)_1 = \frac{8.000}{57.725} = 0.1386, \quad (x_D)_2 = \frac{48.750}{57.725} = 0.8445, \quad (x_D)_3 = \frac{0.975}{57.725} = 0.0169$

(iii) Number of ideal stages @ operating reflux ratio.

$R = (1.3)\, R_{min} = (1.3)(1.544) = 2.00$

Use Gilliland correlation.

$$\frac{R - R_{min}}{R+1} = \frac{2.00 - 1.54}{2.00 + 1} = 0.15$$

From Fig. 19.5 on p. 609, $\frac{N - N_{min}}{N+1} \cong 0.48$

so

$$N = \frac{N_{min} + 0.48}{1 - 0.48}$$

From part (ii)

These number of stages include reboiler, so notation changes in this part of problem. $N_{min} = \underbrace{10.0}_{column} + \underbrace{1}_{reboiler}$
$= 11.0$

$$N = \frac{11.0 + 0.48}{0.52} = 22.1 \quad \text{(incl. reboiler)}$$

$\therefore$ need 21.1 ideal stages in column itself, or can say need 21.1 ideal stages + reboiler.

(For safety, might round up to 22 ideal stages in column itself.)