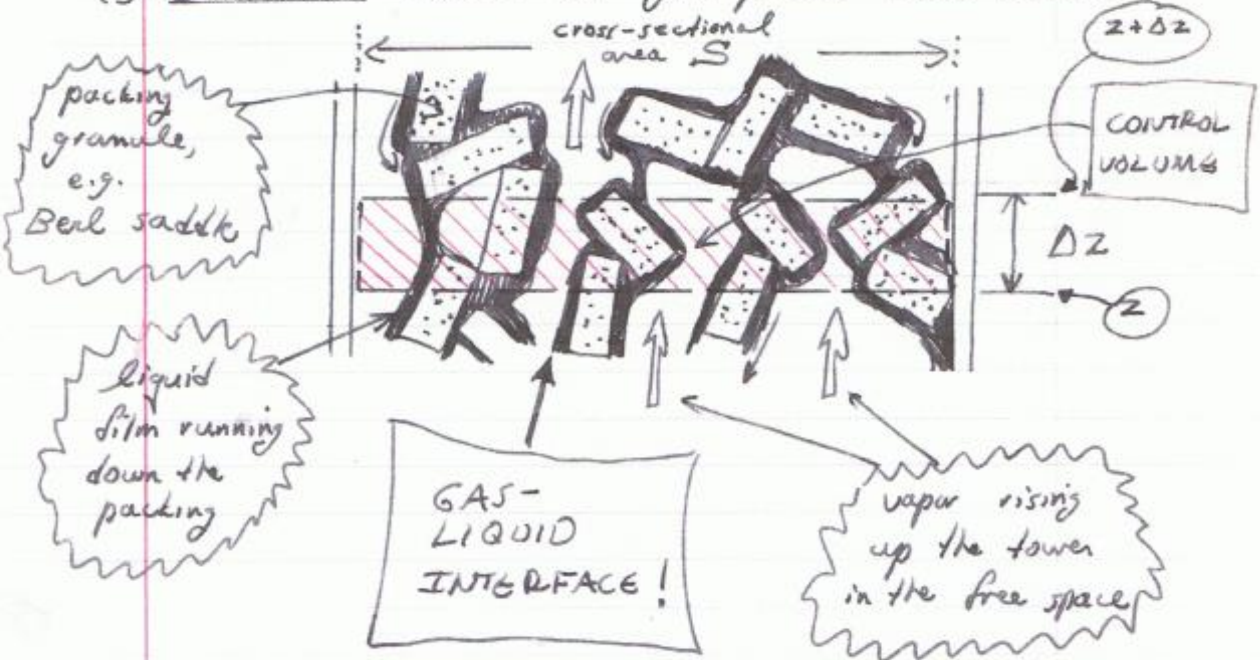
Problem can be worked out four different ways:

## Agony joy trauma #1

(i) Derivation based on gas-phase mass transfer.



Considering gas region (which has an Elroy-joyous complicated shape) of control volume:

$$\begin{bmatrix}\text{rate solute in} \\ \text{(at bottom)}\end{bmatrix} = Vy\Big|_z .$$

$$\begin{bmatrix}\text{rate solute out} \\ \text{(at top)}\end{bmatrix} = Vy\Big|_{z+\Delta z}$$

$$\begin{bmatrix}\text{rate solute out} \\ \text{(from gas} \to \text{liquid} \\ \text{through gas-liquid} \\ \text{interface)}\end{bmatrix} = \underbrace{\underbrace{k_y(y-y_i)}_{\left(\frac{\text{mol solute}}{\text{area}\cdot\text{time}}\right)} \cdot \underbrace{\overset{\text{interfacial area per packed volume}}{a}\ \overset{\text{volume of control vol}}{\widetilde{S\Delta z}}}_{(\text{area for mass transfer})}}_{\frac{\text{mol solute}}{\text{time}}}$$

YES!

at steady state, rate in = rate out

$\Rightarrow$

$$Vy\Big|_z = Vy\Big|_{z+\Delta z} + k_y(y-y_i)\, a\, S\, \Delta z$$

or

$$\frac{Vy\Big|_{z+\Delta z} - Vy\Big|_z}{\Delta z} = -k_y\, a\, S\,(y-y_i)$$

As $\Delta z \to 0$ (thin slice),

$$\frac{d}{dz}(Vy) = -k_y\, a\, S\,(y-y_i)$$

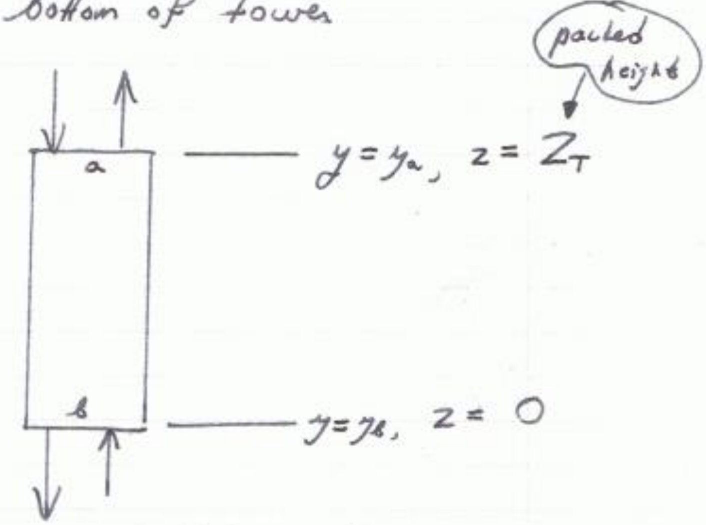
Dilute mixture $\Rightarrow$ $V \approx$ constant

$$\Rightarrow \quad V \frac{dy}{dz} = -k_y a \, S (y - y_i)$$

Now separate variables (because Elroy says so!)

$$\Rightarrow \quad dz = -\frac{(V/S)}{k_y a} \frac{dy}{y - y_i}$$

Integrate from bottom of tower




$$\int_0^{Z_T} dz = \underbrace{\frac{-(V/S)}{k_y a}}_{\text{const.}} \int_{y_b}^{y_a} \frac{dy}{y - y_i}$$

or

$$\boxed{Z_T = \frac{V/S}{k_y a} \int_{y_a}^{y_b} \frac{dy}{y - y_i}}$$

<u>Note</u>: If $y - y_i$ is constant, then can pull $(y-y_i)^{-1}$ out of integral, so that

$$N_y = \frac{y_b - y_a}{y - y_i} = \frac{\text{(total conc. change over tower)}}{\text{(driving force for mass transfer)}}$$

If $y - y_i$ not constant, then you would expect

$$N_y = \frac{y_b - y_a}{\overline{(y - y_i)}},$$

average driving force

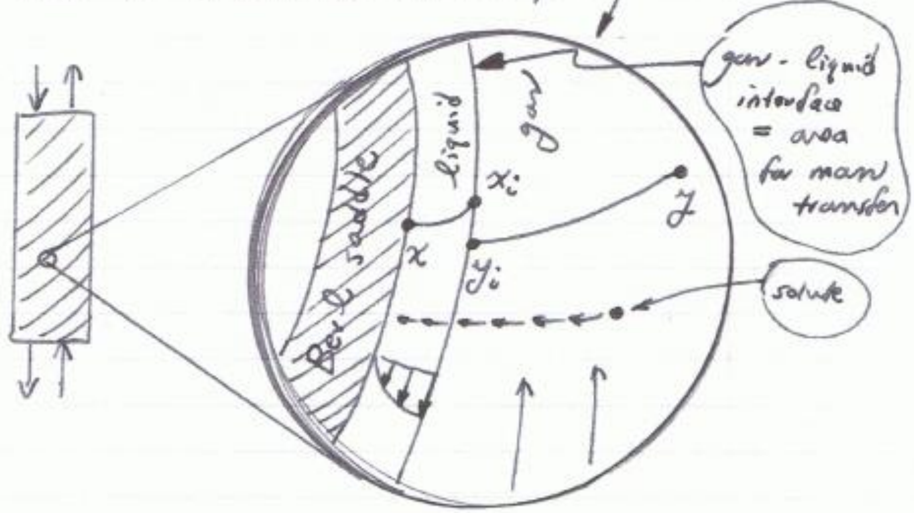
but what average to take? If op. line and equil. curve both perfectly straight, then the correct mean is the

LOGARITHMIC MEAN

... which brings us to an important question:

(ii) <u>How do you calculate the interfacial gas-phase solute mole fraction $y_i$?</u>

Time to fetch the microscope




$$\begin{bmatrix}\text{rate of solute}\\ \text{transfer from bulk}\\ \text{gas to interface}\end{bmatrix} = \begin{bmatrix}\text{rate of solute}\\ \text{transfer from interface}\\ \text{to bulk liquid}\end{bmatrix}$$

$$\underbrace{k_y(y-y_i)}_{\text{flux}}\,\underbrace{(a\,\cancel{\Delta V})}_{\text{area for mass transfer in a little control vol. } \Delta V} = \underbrace{k_x(x_i-x)}_{\text{flux}}\,(a\,\cancel{\Delta V})$$

or $$y - y_i = -\left(\frac{k_x}{k_y}\right)(x - x_i)$$

Equil. at interface $\Rightarrow$ $y_i = y^*(x_i)$, i.e. $y_i$ and $x_i$ satisfy the equil. relation.

For general use $H_y = \frac{V/S}{k_y a}$ (see page 3 or book eq. (22-19)).
Also (analogously) $H_x = \frac{L/S}{k_x a}$ (book eq. (22-20)).
$\therefore$

$$\underbrace{\frac{k_x}{k_y}} = \frac{(L/S)}{H_x a} \cdot \frac{H_y a}{(V/S)} = \underbrace{\left(\frac{L}{V}\right) \frac{H_y}{H_x}}$$

Also, for dilute systems $L/V \approx$ const.
= slope of nearly-straight operating line †.
$\therefore$ here (using bottom of tower $(x, y) = (x_b, y_b)$),

$$\frac{L}{V} = \frac{y_b - y_a}{x_b - x_a} = \frac{0.009 - 0.001}{0.08 - 0} = 0.1$$

Then

$$\frac{k_x}{k_y} = \frac{L}{V} \frac{H_y}{H_x} = (0.1)\left(\frac{0.36\ m}{0.24\ m}\right) = 0.15$$

YET ANOTHER "also"!

Also in this problem equil. relation is $y = 0.06x$ so that

$$y_i = 0.06 x_i \quad \text{or} \quad x_i = \frac{y_i}{0.06}.$$

Use these facts in the eqr. at bottom of page 5 $\Rightarrow$

$$y - y_i = (-0.15)\left(x - \frac{y_i}{0.06}\right)$$

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† We ALL remember the exact eq. for the op. line. We also ALL remember that at low concentration the op. line is nearly a straight line, given to good approx. by

$$y - y_a = \frac{L}{V}(x - x_a) \quad \text{from solute balance.}$$

Solve for $y_i$ $\Rightarrow$

$$y_i = \frac{y + 0.15\,x}{3.5}$$

(a) at top of tower

$$y_i = \frac{0.001 + 0.15(0)}{3.5} = 2.857 \times 10^{-4}$$

(b) at bottom of tower

$$y_i = \frac{0.009 + 0.15(0.08)}{3.5} = 0.006$$

Then

$(y - y_i)_a$ = driving force at top of tower
$= 0.001 - 0.0002857 = 0.0007143$

$(y - y_i)_b$ = driving force at bottom of tower
$= 0.009 - 0.006 = 0.003$

and $\overline{(y - y_i)_L}$ = logarithmic mean† of $(y-y_i)_a$ and $(y-y_i)_b$

$$= \frac{0.003 - 0.0007143}{\ln\left(\frac{0.003}{0.0007143}\right)} = 1.593 \times 10^{-3}$$

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† The logarithmic mean of two numbers A and B is $(A - B)/\ln(A/B)$.

(iii) Let's finish this problem up!

$$N_y = \frac{y_b - y_a}{(y - y_i)_L} = \frac{0.009 - 0.001}{1.593 \times 10^{-3}}$$

$$= 5.022$$

Finally $Z_T = H_y N_y = (0.36 \text{ m})(5.022)$

$$\boxed{Z_T = 1.81 \text{ m}}$$

Agony joy trauma # 2

(i) By same kind of derivation (already done in class!) based on solute flux given by overall mass transfer coefficient, i.e.

$$N_A = \text{flux} = K_y (y - y^*),$$

solute

$= y^*(x)$ = vapor conc. that would be in equil. with bulk liquid

bulk liquid conc.

get equation

$$\boxed{Z_T = \underbrace{\frac{V/S}{K_y a}}_{\text{call this } H_{Oy}} \underbrace{\int_{y_a}^{y_b} \frac{dy}{y - y^*}}_{\text{call this } N_{Oy}}}$$

From mass transfer theory we ALL know that †

$$\frac{1}{K_y} = \frac{1}{k_y} + \frac{m}{k_x}$$

slope of equil. curve

$$\therefore \quad \frac{V/S}{a K_y} = \frac{V/S}{a k_y} + \frac{V/S}{a} \frac{m}{k_x}$$

or

$$\underbrace{\frac{V/S}{a K_y}}_{H_{oy}} = \underbrace{\frac{V/S}{a k_y}}_{H_y} + \underbrace{\frac{L/S}{a k_x}}_{H_x} m \cdot \frac{V}{L}$$

This is where the equation

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† OK, one MORE TIME!

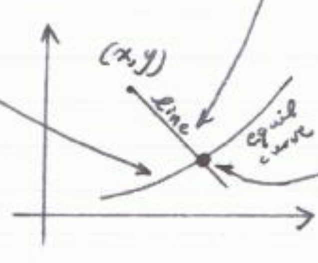
$$k_y (y - y_i) = k_x (x_i - x)$$

rate solute from bulk gas to interface = rate solute from interface to bulk liquid

$$\Rightarrow (y_i - y) = -\frac{k_x}{k_y} (x_i - x)$$

at a given cross-section of tower, $x$ and $y$ have definite values. ∴ this eq. represents linear relation that must be satisfied by $x_i$ and $y_i$. And we also know that $x_i$ and $y_i$ must lie on equil. curve.

∴ $(x_i, y_i)$ is point of intersection



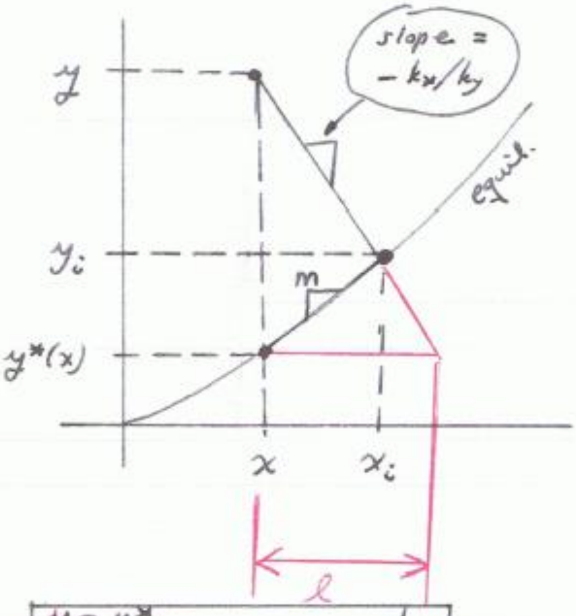

(continued next page.)

$$\boxed{H_{O_y} = H_y + m \cdot \frac{V}{L} \cdot H_x}$$

(Eq. 22-26 in book) comes from. <u>Note</u>: $V/L = \underbrace{(V/S)}_{G_M} / \underbrace{(L/S)}_{L_M}$ and book uses notation

$G_M = V/S$ = molar velocity (superficial, i.e., based on total tower cross-section) and similarly $L_M = L/S$.

† (continued) Now let's take a closer look at that graph.




$$\boxed{\frac{y - y^*}{\ell} = |\text{slope}| = \frac{k_x}{k_y}} \quad (1)$$

By similarity of triangles

$$\boxed{\frac{y - y_i}{x_i - x} = \frac{y - y^*}{\ell}} \quad (2)$$

Also, if m = slope of chord drawn to equil. curve, then

$$\boxed{x_i - x = \frac{y_i - y^*}{m} = \frac{y - y^* - (y - y_i)}{m}} \quad (3)$$

(continued next page)

(a) at top of tower $y^* = (0.06)(0) = 0$

$\therefore (y-y^*)_a = 0.001 - 0 = 0.001$

(b) at bottom of tower $y^* = (0.06)(0.08) = 0.0048$

$\therefore (y-y^*)_b = 0.009 - 0.0048 = 0.0042$

Logarithmic mean

$$\overline{(y-y^*)}_L = \frac{0.0042 - 0.001}{\ln\left(\frac{0.0042}{0.001}\right)} = 2.230 \times 10^{-3}$$

Then

$$N_{Oy} = \frac{y_b - y_a}{\overline{(y-y^*)}_L} = \frac{0.009 - 0.001}{2.230 \times 10^{-3}} = 3.587$$

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† (continued)

Eqs. (2) + (3) $\Rightarrow$

$$\frac{(y-y_i)\quad m}{(y-y^*) - (y-y_i)} = \frac{y-y^*}{\ell} = \frac{k_x}{k_y}$$

by eq. (1)

Solve for $(y-y_i) \Rightarrow$

$$(y-y_i) = \frac{k_x/k_y}{m + k_x/k_y}(y-y^*)$$

Then

$$N_A = k_y(y-y_i) = \frac{k_x}{m + k_x/k_y}(y-y^*)$$

$$= \left(\left(\frac{1}{\frac{m}{k_x} + \frac{1}{k_y}}\right)\right)(y-y^*)$$

call this $K_y$

$$K_y = \frac{1}{\frac{1}{k_y} + \frac{m}{k_x}} \quad \text{or} \quad \boxed{\frac{1}{K_y} = \frac{1}{k_y} + \frac{m}{k_x}}$$

also,

$$H_{oy} = H_y + \frac{m}{L/V} H_x$$

slope of equil. curve

$$= (0.36\ m) + \frac{(0.06)}{0.1}(0.24\ m)$$

slope of op. line already calculated on p. 6

$$H_{oy} = 0.504\ m$$

Finally, $Z_T = H_{oy} N_{oy} = (0.504 m)(3.587)$

$$\boxed{Z_T = 1.81\ m}$$

Agony joy trauma # 3

Same kind of derivation based on flux expression

$$N_A = k_x (x_i - x)$$

solute

and liquid portion of control volume

gives

$$\boxed{Z_T = \underbrace{\frac{(L/S)}{k_x a}}_{H_x} \cdot \underbrace{\int_{x_a}^{x_b} \frac{dx}{x_i - x}}_{N_x}}$$

We already computed $y_i$ at top and bottom of tower on p. 7. By equil. relation

(a) at top of tower

$$x_i = \frac{y_i}{0.06} = \frac{2.857 \times 10^{-4}}{0.06} = 0.004762$$

Then

$$(x_i - x)_a = 0.004762 - 0 = 0.004762$$

(b) at bottom of tower

$$x_i = \frac{y_i}{0.06} = \frac{0.006}{0.06} = 0.1$$

Then

$$(x_i - x)_b = 0.1 - 0.08 = 0.02$$

Logarithmic mean is

$$\overline{(x_i - x)}_L = \frac{0.02 - 0.004762}{\ln\left(\frac{0.02}{0.004762}\right)} = 1.062 \times 10^{-2}$$

so

$$N_x = \frac{x_b - x_a}{\overline{(x_i - x)}_L} = \frac{0.08 - 0}{1.062 \times 10^{-2}} = 7.533$$

Finally,

$$Z_T = H_x N_x = (0.24\ m)(7.533)$$

$$\boxed{Z_T = 1.81\ m}$$

$$\boxed{Z_T = \frac{L/S}{K_x a} \int_{x_a}^{x_b} \frac{dx}{x^* - x}}$$

$$\underbrace{\frac{L/S}{K_x a}}_{H_{Ox}} \quad \underbrace{\int_{x_a}^{x_b} \frac{dx}{x^* - x}}_{N_{Ox}}$$

(a) at top of tower

$$x^* = x \text{ in equil with } y = \frac{y}{0.06}$$

$$= 0.001/0.06 = 0.01667$$

so

$$x^* - x = 0.01667 - 0 = 0.01667$$

(b) at bottom of tower

$$x^* = y/0.06 = 0.009/0.06 = 0.15$$

so

$$x^* - x = 0.15 - 0.08 = 0.07$$

Logarithmic mean

$$\overline{(x^* - x)}_L = \frac{0.07 - 0.01667}{\ln\left(\frac{0.07}{0.01667}\right)} = 0.03717$$

Then

$$N_{Ox} = \frac{x_B - x_a}{(x^* - x)_L} = \frac{0.08 - 0}{0.03717} = 2.152$$

Next, by eq. (22-29) in book,

L/V was calculated on p. $\boxed{6}$

$$H_{Ox} = H_x + \frac{1}{m} \cdot \frac{L}{V} H_y$$

$$= 0.24\,m + \left(\frac{1}{0.06}\right)(0.1)(0.36\,m)$$

$$= 0.84\,m$$

Finally,

$$Z_T = H_{Ox} N_{Ox} = (0.84\,m)(2.152)$$

$$\boxed{Z_T = 1.81\,m}$$