CE407 SEPARATIONS

Lecture 20
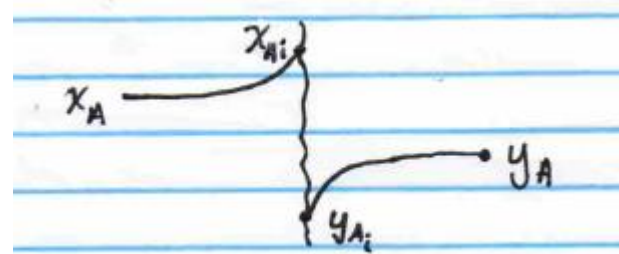
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Overall Mass Transfer Coefficients   McSH pp 546-548

- The purpose of using Overall Mass Transfer Coefficients is to avoid the need to determine the mole fractions at the interface


- We saw last lecture how to determine the flux if we know the bulk and interfacial mole fractions in both phases

- We can hope to sample the bulk phases in order to obtain information about the bulk mole fractions

- Sampling the interface itself would be extremely difficult
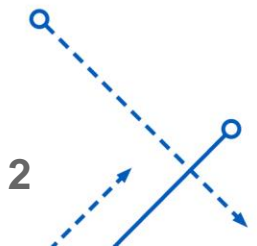
Overall Mass Transfer Coefficients

- Start with bulk mole fractions x_A and y_A

- Draw line with slope $= -\dfrac{k_x}{k_y}$

  - This line represents all of the combinations of x_{Ai} and y_{Ai} that satisfy the requirement that the flux is equal in both phases

- Intersection of this line with the equilibrium curve determines the pair x_{Ai} and y_{Ai} that also satisfy the requirement that the interfacial mole fractions are in equilibrium with one another: $y_{Ai} = y^*(x_{Ai})$ 🔴

- y_A^* is the value of y_A that would be in equilibrium with x_A 🔵

  - i.e. the value of y_A on the **EQ** curve for $x = x_A$

  - This is **NOT** the value at the interface

  - This is the equilibrium value of vapor mole fraction that is in contact with a liquid of mole fraction $x = x_A$

- Remember, if x_A and y_A were in equilibrium there would be no mass transfer!

3
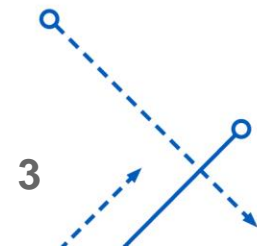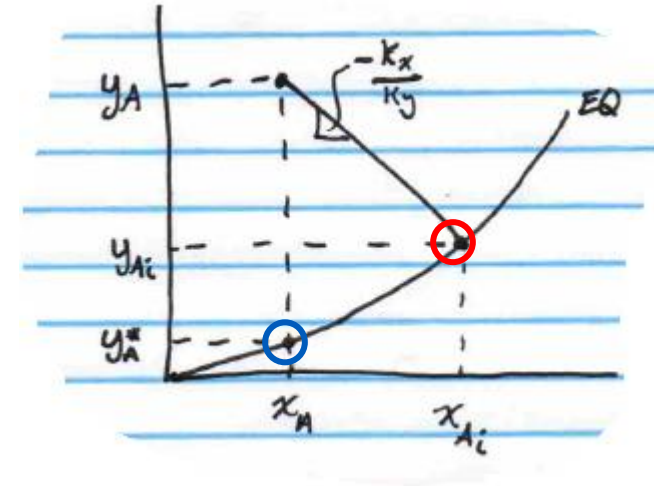
Overall Mass Transfer Coefficients

- Draw a line from (x_A, y_A^*) to (x_{Ai}, y_{Ai})

- Slope $= \dfrac{Rise}{Run} = \dfrac{y_{Ai} - y_A^*}{x_{Ai} - x_A} = m$

- Note that: $\quad y_A - y_A^* = (y_A - y_{Ai}) + (y_{Ai} - y_A^*)$ $\boxed{1}$

- From slope we know that: $\quad (y_{Ai} - y_A^*) = m * (x_{Ai} - x_A)$ $\boxed{2}$

- Remember: $\quad N_A = k_y'(y_A - y_{Ai}) = k_x'(x_{Ai} - x_A)$

  - Using dilute approximation that $k_x' \approx k_x$ and $k_y' \approx k_y$

  - $\therefore (y_A - y_{Ai}) = \dfrac{N_A}{k_y}$ and $(x_{Ai} - x_A) = \dfrac{N_A}{k_x}$ $\boxed{3}$

- Substitute $\boxed{2}$ and $\boxed{3}$ into $\boxed{1}$

$$y_A - y_A^* = \frac{N_A}{k_y} + m\frac{N_A}{k_x}$$
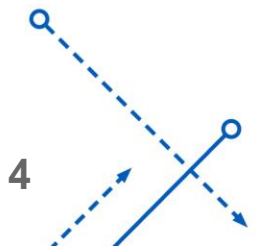
$$= N_A\left(\frac{1}{k_y} + \frac{m}{k_x}\right)$$

- **m** can be approximated as the slope of the EQ curve in the region of interest
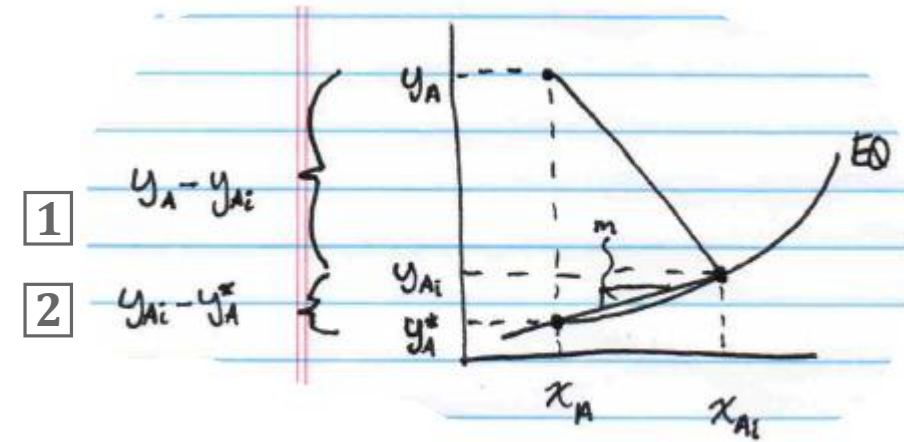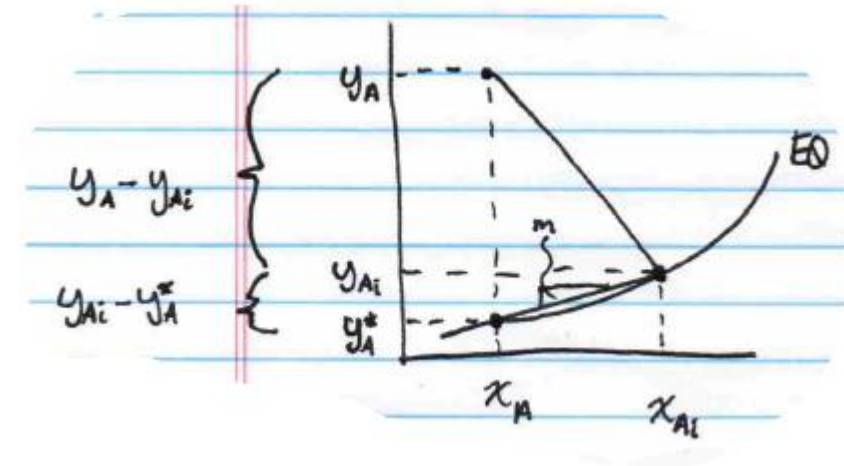
Overall Mass Transfer Coefficients

$$y_A - y_A^* = N_A \left(\frac{1}{k_y} + \frac{m}{k_x} \right)$$

$$N_A = \left(\frac{1}{k_y} + \frac{m}{k_x} \right)^{-1} (y_A - y_A^*)$$

$$N_A = K_y(y_A - y_A^*)$$

$$\boxed{K_y = \left(\frac{1}{k_y} + \frac{m}{k_x} \right)^{-1}}$$

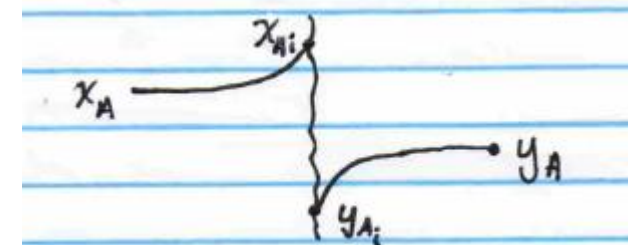- K_y is the **Gas Phase Overall Mass Transfer Coefficient**

Overall Mass Transfer Coefficients

$$N_A = K_y(y_A - y_A^*)$$

new coefficient   new driving force

- K_y is the **Gas Phase Overall Mass Transfer Coefficient**

- With this expression we can calculate flux with y_A^* and y_A and not have to calculate interfacial mole fractions

- There is **NO** coefficient or expression to express the flux in terms of x_A and y_A
  - $(x_A - y_A)$ does **NOT** tell you **ANYTHING** about flux
  - You need to account for the equilibrium relationship of solute in the liquid versus the vapor at the interface

- Because we have a flow of liquid and vapor, the bulk phase mole fractions x_A and y_A do not reach equilibrium

- Because $x_A \neq x_{Ai}$ and $y_A \neq y_{Ai}$ there exists a driving force on both sides of the interface – that is why we have a flux of solute from one phase to the other
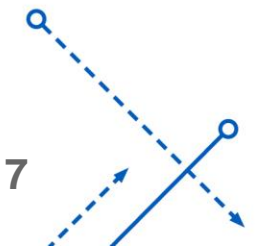
Overall Mass Transfer Coefficients

- Overall Liquid Phase Mass Transfer Coefficient

$$N_A = K_x(x_A^* - x_A)$$

$$K_x = \left(\frac{1}{k_x} + \frac{1}{m\,k_y}\right)^{-1}$$

- x_A^* is the liquid mole fraction that would be in equilibrium with y_A bulk phase gas mole fraction

- Note that m appears in a different location in the equation for liquid phase versus gas phase

7

Overall Mass Transfer Coefficients

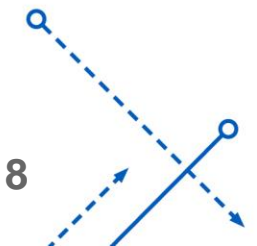- Four different expressions for Molar Flux

$$N_A = k_y(y_A - y_{Ai})$$

$$N_A = k_x(x_{Ai} - x_A)$$

Need to calculate interfacial mole fractions

$$N_A = K_y(y_A - y_A^*)$$

$$N_A = K_x(x_A^* - x_A)$$

Need to calculate Overall Mass Transfer Coefficients

Molar Flux
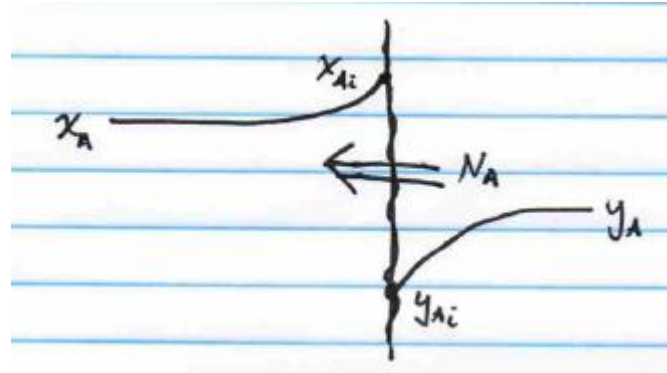
- All of our expressions have been for this case:
  - Liquid shown on left of interface
  - Vapor shown on right of interface
  - Positive Flux indicates solute flow from right to left
  - If flux is a negative number that indicates solute flow is from left to right
- It is a good idea to draw a picture to make sure you understand the sign convention for your case

Mass Transfer Coefficients

- Alternate Units

  - Concentration: $c = \frac{moles}{volume}$

$$N_A = k_y(y_A - y_{Ai}) = k_y(y_A - y_{Ai}) * \frac{c}{c} = \frac{k_y}{c}(c_A - c_{Ai})$$

$$= k_c(c_A - c_{Ai})$$

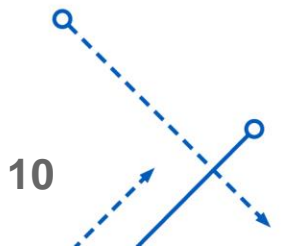$$\boxed{k_c = \frac{k_y}{c}}$$

  - Partial Pressure: $\frac{P}{RT} = c$ (from Ideal Gas Law) and partial pressure $P_i = P * y_i$

$$N_A = k_y(y_A - y_{Ai}) = k_y(y_A - y_{Ai}) * \frac{P/RT}{c}$$

$$= \frac{k_y}{cRT}(P\, y_A - P\, y_{Ai}) = k_G(p_A - p_{Ai})$$

$$\boxed{k_G = \frac{k_y}{cRT} = \frac{k_c}{RT}}$$

  - Correlations generally give k_c, which has dimensions of velocity, $length/time$

Correlations for Mass Transfer Coefficients

- Correlations are interpreted using Dimensionless Groups

- The dimensionless groups will be composed of:
  - L = a characteristic length
    - Diameter for round or spherical objects – we will use Diam to avoid confusion with diffusivity, D
  - U = characteristic velocity
  - ρ = fluid density
  - μ = fluid viscosity
  - ν = fluid kinematic viscosity = μ/ρ

- We are looking at mixtures of Solute A and Solvent B

- For dilute mixtures, the fluid properties may be approximated by using pure solvent B properties
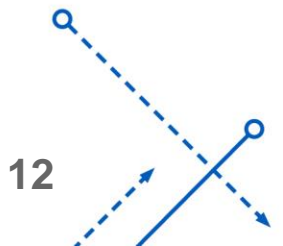
Reynolds Number

$$Re = \frac{\rho\,U\,L}{\mu} = \frac{(\rho\,U)\,L}{\mu} = \frac{G\,L}{\mu}$$

where $G = \rho\,U$, Mass Velocity

- $$Re = \frac{U\,L}{\mu/\rho} = \frac{U\,L}{\nu}$$

  where $\nu = {}^{\mu}/_{\rho}$, Kinematic Viscosity

- Reynolds number is the ratio of inertial to viscous forces

Schmidt Number

$$S_c = \frac{\nu}{D} = \frac{\mu}{\rho\, D}$$

- Where ν is representative of the diffusivity of momentum
- Where D is the diffusivity of solute A

- Schmidt number is the ratio of momentum diffusivity (ν) to mass diffusivity

13

Sherwood Number

$$S_h = \frac{k_c \, L}{D}$$

- $\therefore \quad k_c = \frac{D}{L} S_h$

- Sherwood Number is the ratio of convective mass transfer to the rate of diffusive mass transfer

14

Stanton Number

$$St_m = \frac{k_c}{U}$$

$$= \frac{k_c L}{D} * \frac{\nu}{U\,L} * \frac{D}{\nu} = \frac{k_c L}{D} * \left(\frac{U\,L}{\nu} * \frac{\nu}{D}\right)^{-1}$$

$$St_m = \frac{S_h}{R_e S_c}$$

- Stanton Number is the ratio of heat transferred into a fluid relative to the thermal capacity of the fluid

Colburn j Factor for Mass Transfer

$$j_M = \frac{k_c}{U}\left(\frac{\mu}{\rho\,D}\right)^{2/3} = St_M Sc^{2/3} = \frac{Sh}{R_e Sc^{1/3}}$$

- It has been observed that $j_M \approx \frac{1}{2}\,Fanning\ Friction\ Factor$

$$j_M \approx \frac{f_{Fanning}}{2}$$

- The Fanning Friction factor is a well known function of the Reynolds Number in pipe flow

Correlations for Mass Transfer Coefficients

- The correlations typically give you one of the three following:

$$S_h \quad or \quad St_M \quad or \quad j_M$$

- Any one of which can be converted to the other and can be used to get k_c

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Mass Transfer to Walls of a Pipe with Turbulent Flow
McSH pp 549

$$S_h = 0.023 \ R_e{}^{0.8} \ S_c^{1/3}$$

- Which is equivalent to

$$j_M = 0.023 \ R_e{}^{-0.2}$$

Mass Transfer in Gas Phase within Wetted Wall Tower

- "Wall" the gas sees is a wavy surface of the liquid running down the tower

$$S_h = 0.023 \ R_e^{0.81} \ S_c^{0.44}$$

- Result is very similar to walls of pipe correlation

- The difference is due to the waviness of the fluid surface versus the smooth pipe wall

Flow Perpendicular Past a Single Cylinder

$$S_h = 0.61 \, R_e^{1/2} \, S_c^{1/3}$$

Flow Perpendicular Past a Single Sphere

$$S_h = 2.0 + 0.6 \, R_e^{1/2} \, S_c^{1/3}$$

Flow Through a Packed Bed of Spherical Particles

$$S_h = 1.17 \ R_e^{0.585} \ S_c^{1/3}$$

$$j_M = 1.17 \ R_e^{-0.415}$$

- The U used to calculate R_e is based on SUPERFICIAL VELOCITY

$$\frac{Volumetric\ Flow}{Total\ Cross-sectional\ Area\ of\ Bed}$$

- Even though part of the area is blocked by spheres

* Consult Text for Ranges of Applicability *