CE407 SEPARATIONS

Lecture 16

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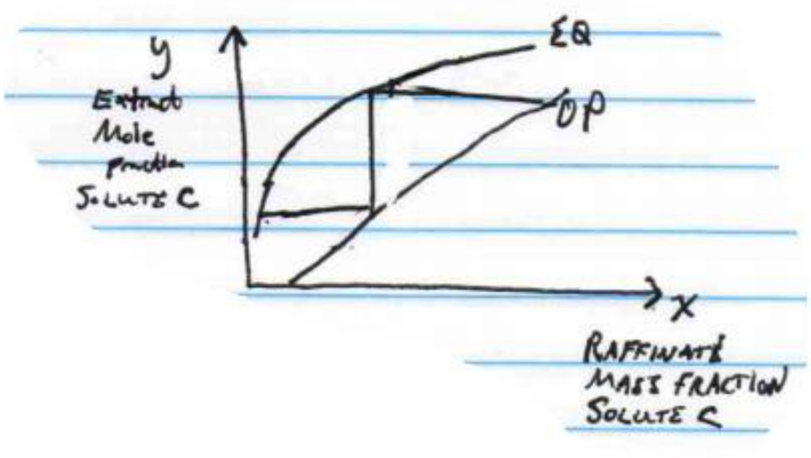
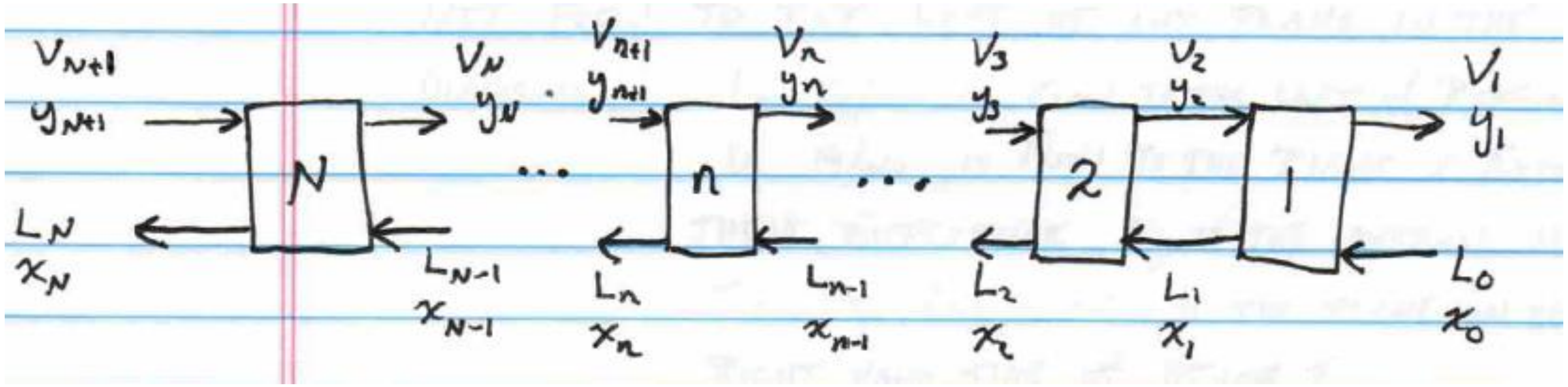
Multi-Stage Countercurrent LLE McSH pp783-786, Treybal pp 451-452

- In the last lecture we saw how to determine the composition and flowrates of the exiting flows from a multi-stage countercurrent process
- In order to determine the required number of stages for an extraction train we need to generate an Operating Line (which is, of course, a mass balance…)
  - It is very much a curve for an LLE extraction train
- With the operating line we can then do a McCabe-Thiele analysis



Multi-stage Countercurrent LLE Operating Line

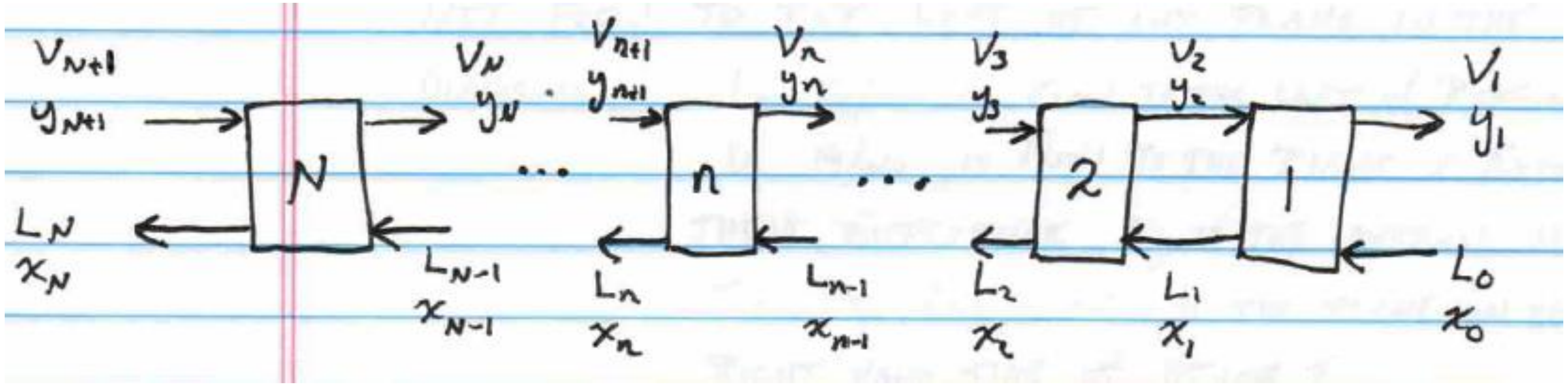
- The function of the operating line is to be able to pick a value of **x** at any point in the process and predict what the value of **y** is at that same location
- Use mass balance
  - Control volume will be from the right of stage 1 up to and including an arbitrary stage **n**
  - We know the compositions y_1 and x_0, so if we select a value of x we can determine corresponding value of y



LLE Mass Balance

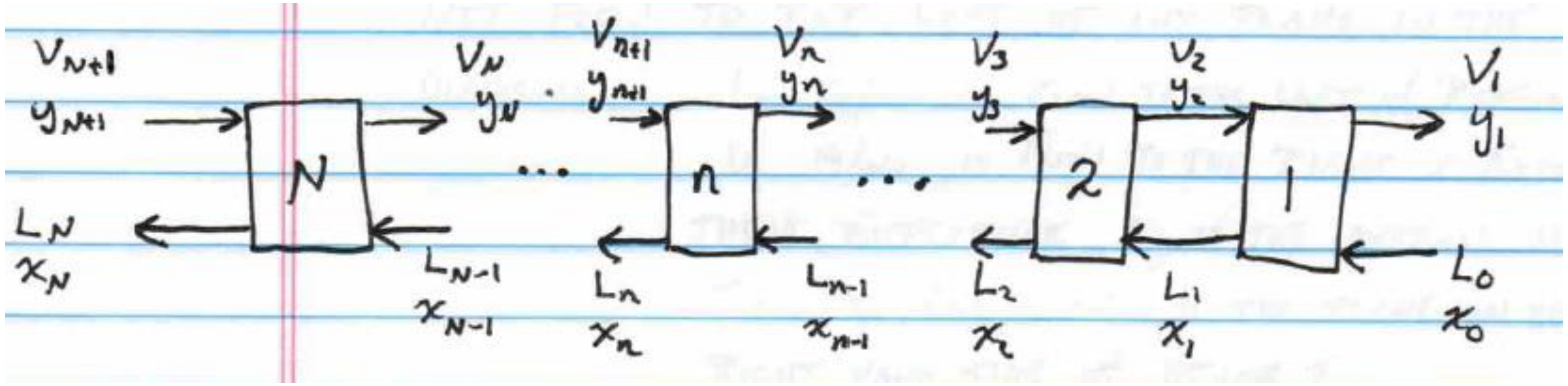
$$L_0 + V_{N+1} = L_N + V_1 = M \quad \Rightarrow \quad L_0 - V_1 = L_N - V_{N+1} = \Delta$$

$$L_0 x_0 + V_{N+1} y_{N+1} = L_N x_N + V_1 y_1 = M x_M \quad \Rightarrow \quad L_0 x_0 - V_1 y_1 = L_N x_N - V_{N+1} y_{N+1} = \Delta x_\Delta$$



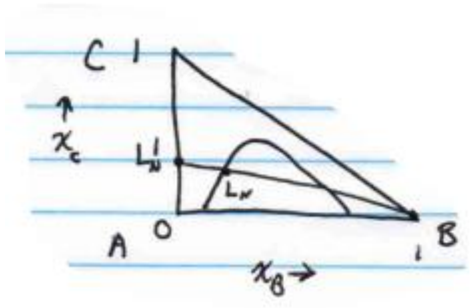
LLE Mass Balance

- Δ and Δx_Δ were also defined in terms of the plane to the left of stage **N**
  - The terms Δ and Δx_Δ must have equal values at both planes and, indeed, at ANY plane in the system. This needs to be true due to the conservation of total mass and conservation of mass of solute
- Think of it like a pipe:
  - If the flow is a given rate and direction at any point in the pipe, it MUST be that same rate and direction at any point throughout the pipe
  - Yes, the raffinate flow and the extract flow are varying from stage to stage, but the difference between them is constant

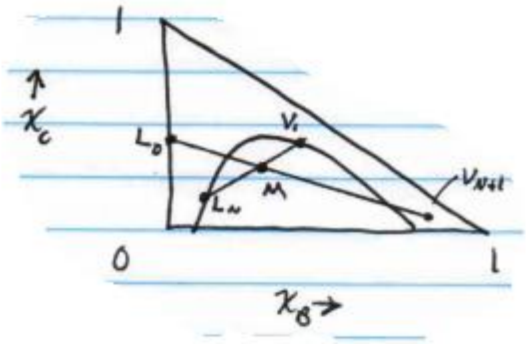


Determining Δ , the mixture point

- Review: First Diagram
  - Determine the actual raffinate composition $\mathbf{L_N}$ by drawing line from $\mathbf{L'_N}$ (solvent free raffinate composition) to pure solvent corner
  - Intersection with phase boundary on diluent rich side is $\mathbf{L_N}$



- 2nd Diagram
  - Plot $\mathbf{L_N}$, $\mathbf{L_0}$, $\mathbf{V_{N+1}}$ ($\mathbf{V_{N+1}}$ may not be pure solvent)
  - Draw line from $\mathbf{L_0}$ to $\mathbf{V_{N+1}}$, locate **M**
  - Extend a line from $\mathbf{L_N}$ through **M** up to solvent rich side of phase boundary
  - Intersection of that line with the phase boundary locates $\mathbf{V_1}$
  - Mass balance gives **R** and **E**



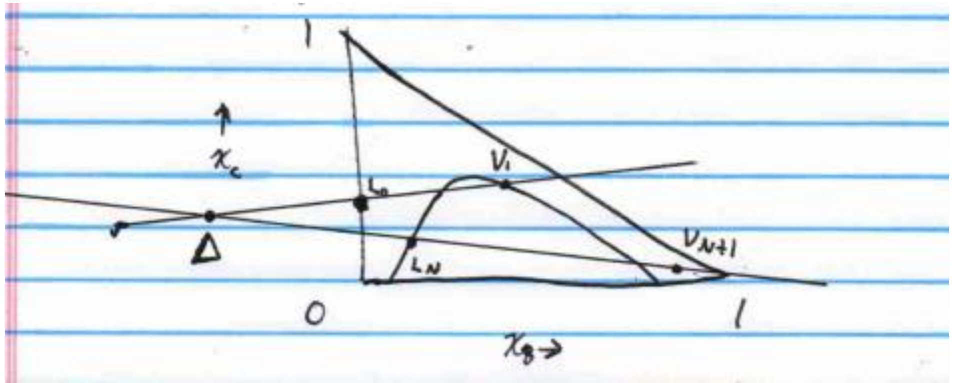
- Now forget about $\mathbf{L'_N}$, **M**, and all the intersecting lines, etc
- We just want to keep $\mathbf{L_N}$, $\mathbf{L_0}$, $\mathbf{V_{N+1}}$, and $\mathbf{V_1}$

Back to the Mass Balance Equations

$$L_0 - V_1 = L_N - V_{N+1} = \Delta \quad \Rightarrow \quad L_0 = V_1 + \Delta$$

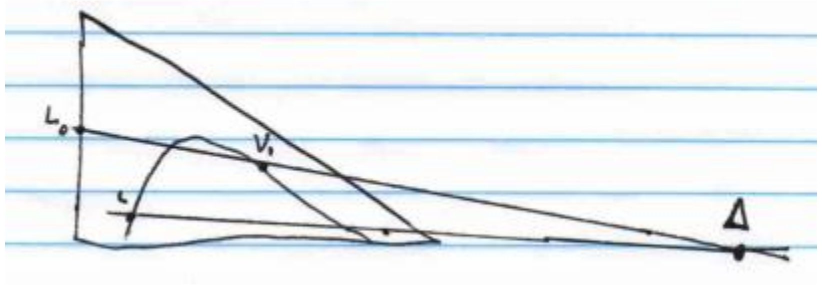
$$L_0 x_0 - V_1 y_1 = L_N x_N - V_{N+1} y_{N+1} = \Delta x_\Delta \quad \Rightarrow \quad L_0 x_0 = V_1 y_1 + \Delta x_\Delta$$

- Also $\quad L_N = V_{N+1} + \Delta$
- And $\quad L_N x_N = V_{N+1} y_{N+1} + \Delta x_\Delta$
- That has a form indicating that L_0 is a mixture of V_1 and Δ and also that L_N is a mixture of V_{N+1} and Δ
- We know that a mixture lies on a line between the two original compositions
- If we draw and extend the lines $\overline{L_0 V_1}$ and $\overline{L_N V_{N+1}}$ their intersection is the one point that satisfies both mass balances
- That intersection is Δ



Δ , the mixture point

- Notice that **Δ** is not even on the triangle. It has impossible mass fractions. It is only a mathematical construct with no physical significance at all!
- Depending upon the locations of $\mathbf{L_N}$, $\mathbf{L_0}$, $\mathbf{V_{N+1}}$, and $\mathbf{V_1}$, **Δ** can lie to the left or right of the phase diagram triangle



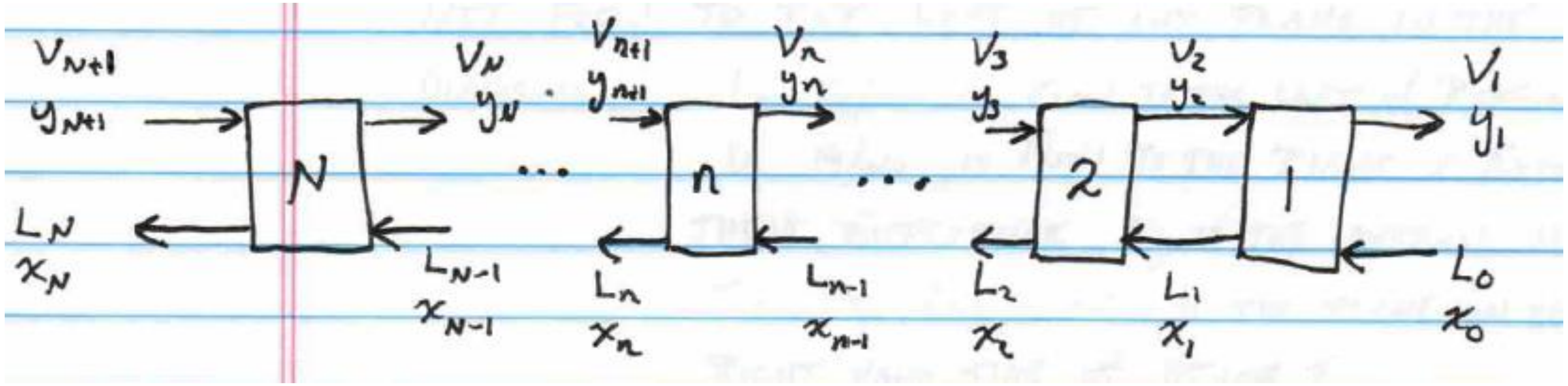
Using **Δ** , the mixture point

- Now we will set the control volume to include stage 1 up to and including stage **n**, an arbitrary stage

$$L_0 + V_{n+1} = L_n + V_1 = M \quad \Rightarrow \quad L_0 - V_1 = L_n - V_{n+1} = \Delta$$

$$L_0 x_0 + V_{n+1} y_{n+1} = L_n x_n + V_1 y_1 = M x_M \quad \Rightarrow \quad L_0 x_0 - V_1 y_1 = L_n x_n - V_{n+1} y_{n+1} = \Delta x_\Delta$$

- **Δ** is the same number as we calculated graphically using stages **1** to **N**
- Remember it is the net flow to the left and has to be constant

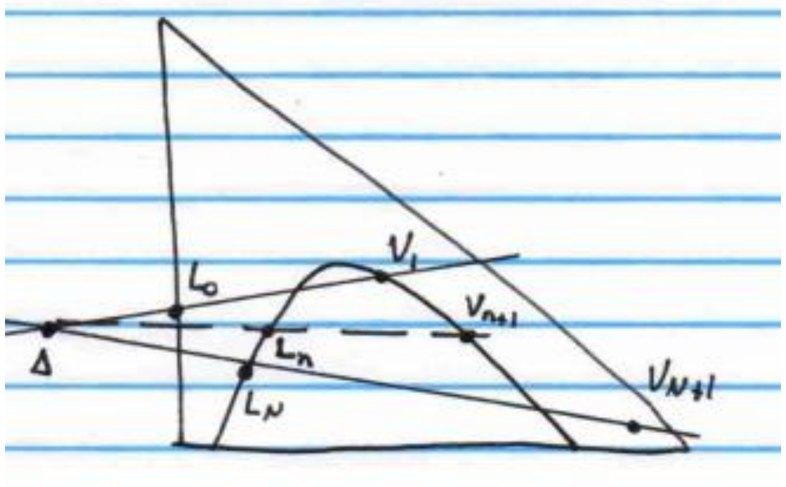


Using $\boldsymbol{\Delta}$, the mixture point

- The mass balance can be rearranged to be

$$L_n = V_{n+1} + \Delta \quad \text{and} \quad L_n x_n = V_{n+1} y_{n+1} + \Delta x_\Delta$$

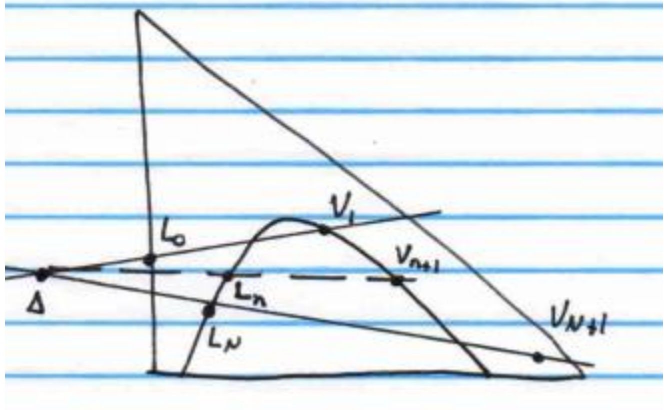
- Which, once again, looks like a mixture of Δ and V_{n+1}
- Note that with the exception of L_0 and V_{N+1}, the flows coming out of each stage are in equilibrium with each other. That means that they must reside on the phase boundary curve
- If we draw arbitrary lines radiating from Δ then they will intersect the left and right hand boundaries of the phase diagram in pairs corresponding to pairs of (x_n, y_{n+1})



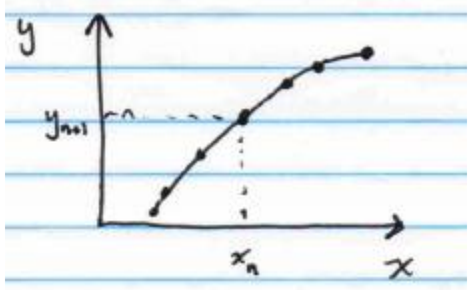
Using Δ , the mixture point

- We can draw as many of these lines as we want and generate a table of points on the Operating Line

x_n	y_{n+1}
~	~
~	~
~	~



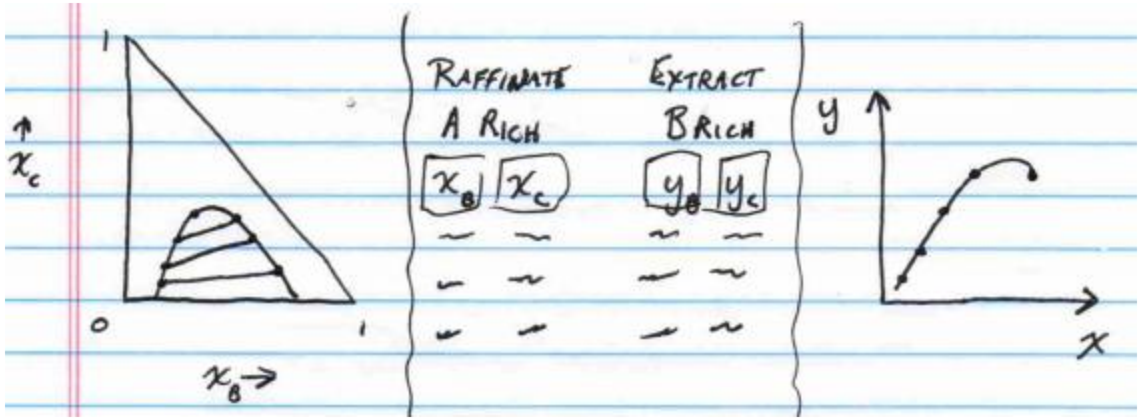
- Now we can plot the operating line



Equilibrium Curve

McSH pp 784 (figure 23.10), Treybal pp 453 (Figure 10.23)

- Equilibrium Data can be presented three ways



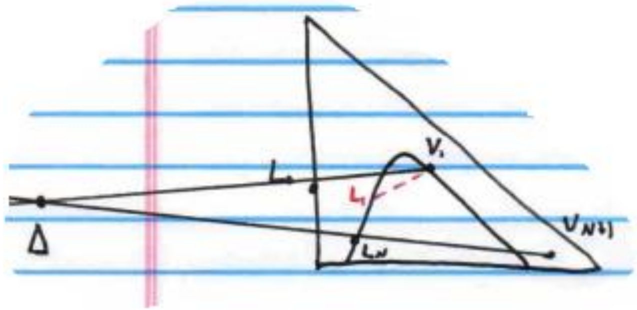
Ternary Tabular x-y Graphical

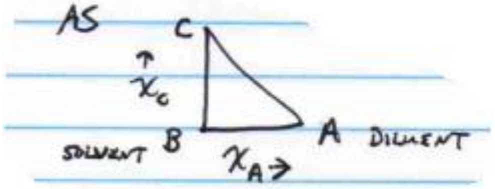
- Pairs of points in the table can be used to plot tie lines on ternary diagram
- Tie line end points can be used to generate table
- $\mathbf{x_c}$ and $\mathbf{y_c}$ from table or from tie lines can be used to generate x-y graphical

Counting Stages - Hunter-Nash Ternary Method

- From methods we looked at earlier, we know **Δ**, $\mathbf{L_0}$ and $\mathbf{V_1}$
- $\mathbf{L_1}$ and $\mathbf{V_1}$ are in equilibrium and therefore must be on a tie line drawn in red, so we can determine $\mathbf{L_1}$
- $\mathbf{L_1}$ and $\mathbf{V_2}$ are on the Operating Line together, so if we extend line from **Δ** through $\mathbf{L_1}$ we will locate $\mathbf{V_2}$
- $\mathbf{V_2}$ is on a tie line with $\mathbf{L_2}$, and $\mathbf{L_2}$ is on the Operating Line with $\mathbf{V_3}$, etc, etc
- Continue until you reach an L value that is at or below $\mathbf{L_N}$
- The subscript you end on (say $\mathbf{L_5}$) is the number of stages required.
- There will be interpolation of ties lines which introduces error and uncertainty
- Caution: If you look up Hunter-Nash in literature it often uses

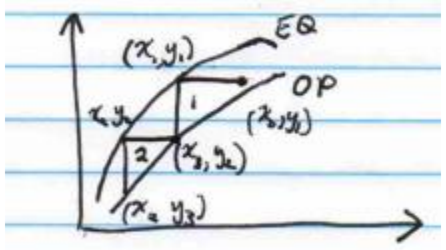
  a ternary diagram defined as:

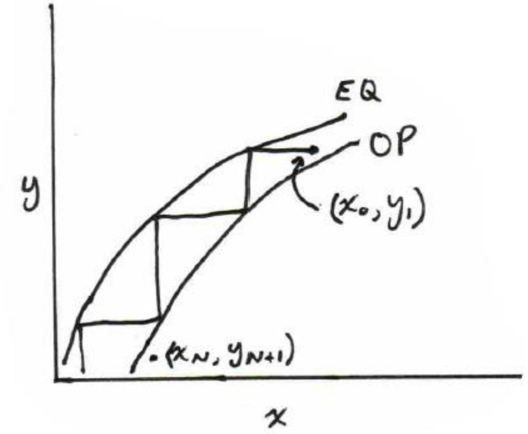


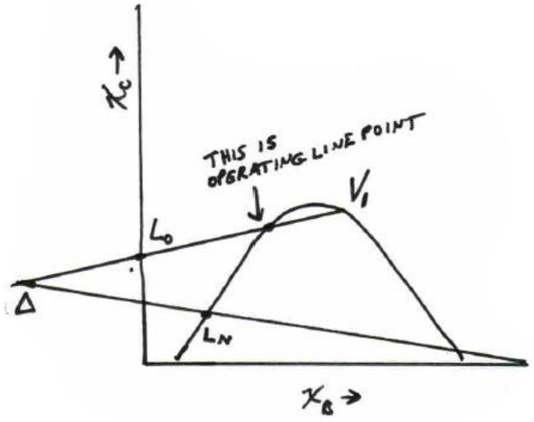


Counting Stages - McCabe-Thiele Method

- This is pretty much the same as we have done for other Unit Operations
- It is inherently more accurate than Hunter-Nash method
- Notice that points (**x_0**, **y_1**) and (**x_N**, **y_{N+1}**) are NOT on the Operating Line!
- Entering liquids **L_0** and **V_{N+1}** are not in equilibrium with anything and therefore are not on the phase boundary and therefore not part of the Operating Line
  - DO NOT EVER Draw them as being on the OP Line!!!
- See Video of LLE Countercurrent Example 3b on course website notes



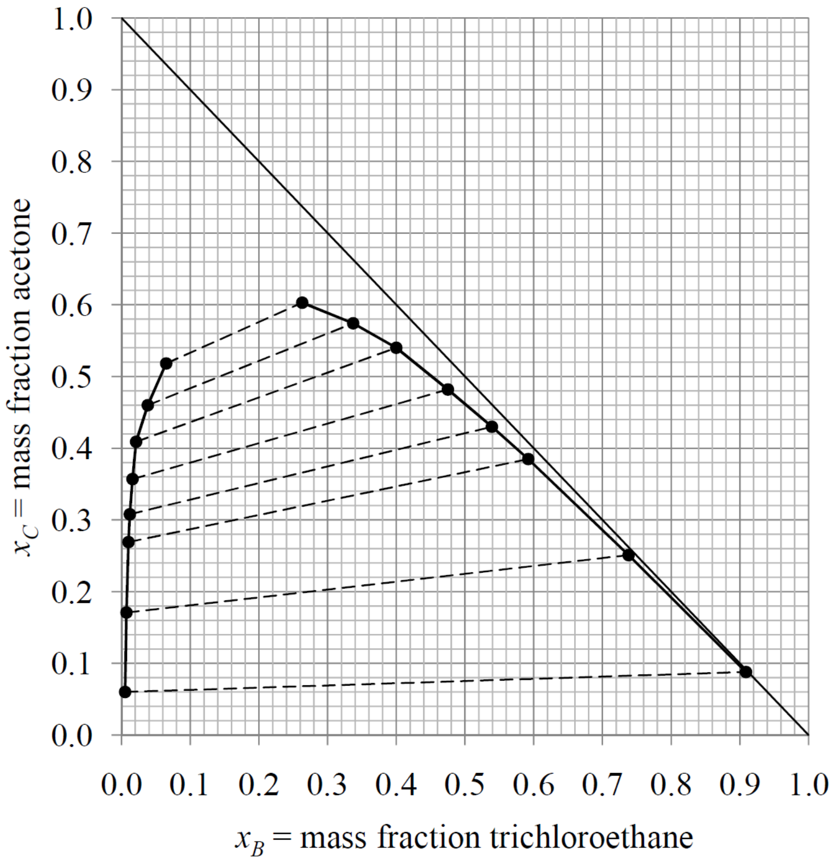


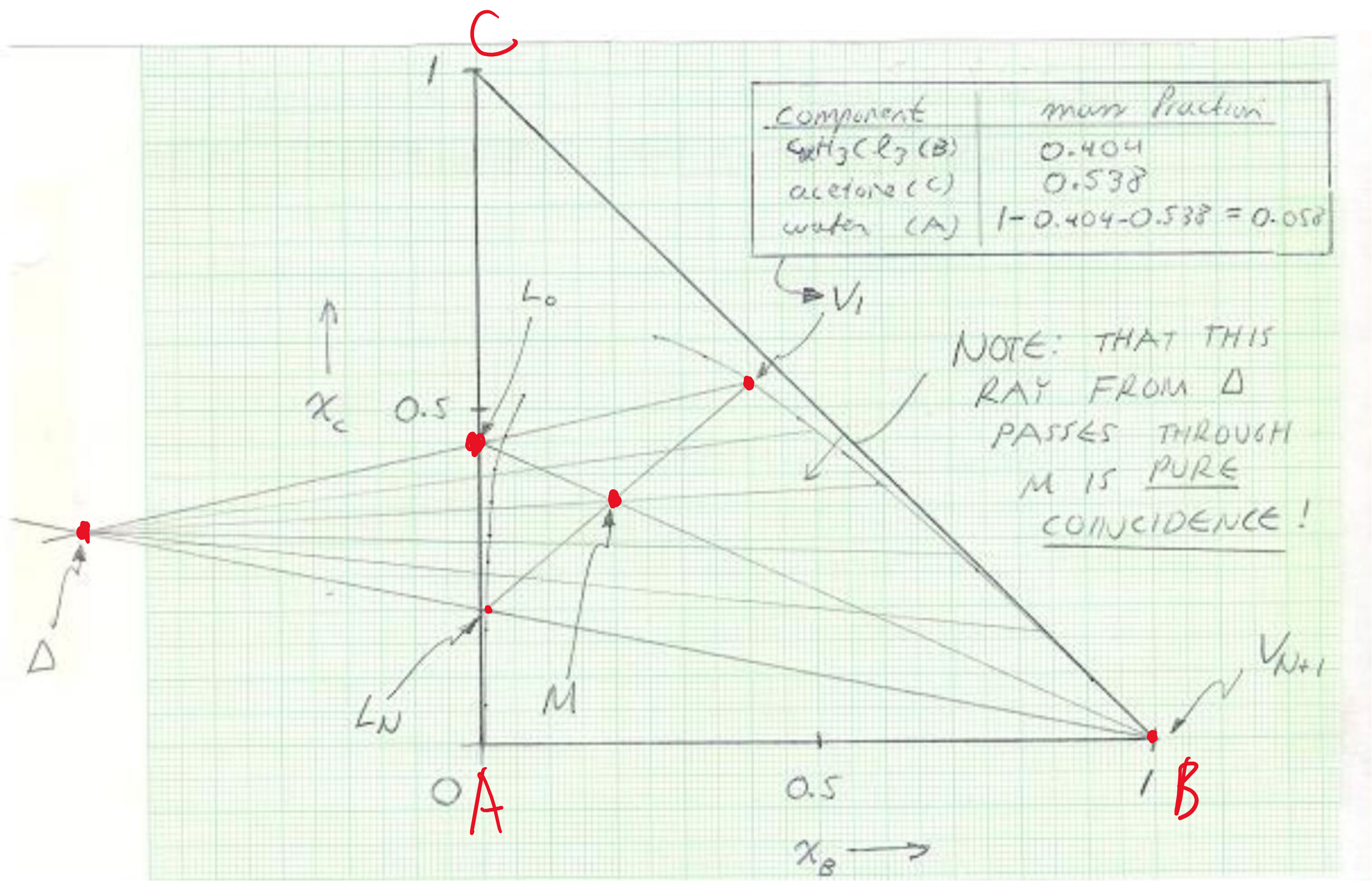


3. A 500 kg/h feed stream with composition 45 mass % acetone (solute, C) and 55 mass % water (diluent, A) is to be contacted with trichloroethane (solvent, B) in a countercurrent liquid extraction battery. Entering trichloroethane is pure. The exiting raffinate should contain 20.2 mass % acetone on a trichloroethane-free basis.

(b) Suppose that solvent trichloroethane enters at a rate of 125 kg/h. What will be the composition and flow rate of the exiting extract (V_1) stream, and how many ideal stages will be required?

mass fractions in water(A)-rich layer		mass fractions in $C_2H_3Cl_3$(B)-rich layer	
$C_2H_3Cl_3$ (B)	acetone (C)	$C_2H_3Cl_3$ (B)	acetone (C)
0.005	0.060	0.909	0.088
0.007	0.171	0.738	0.251
0.010	0.269	0.592	0.385
0.012	0.308	0.539	0.430
0.016	0.357	0.475	0.482
0.021	0.409	0.400	0.540
0.038	0.460	0.337	0.574
0.065	0.518	0.263	0.603



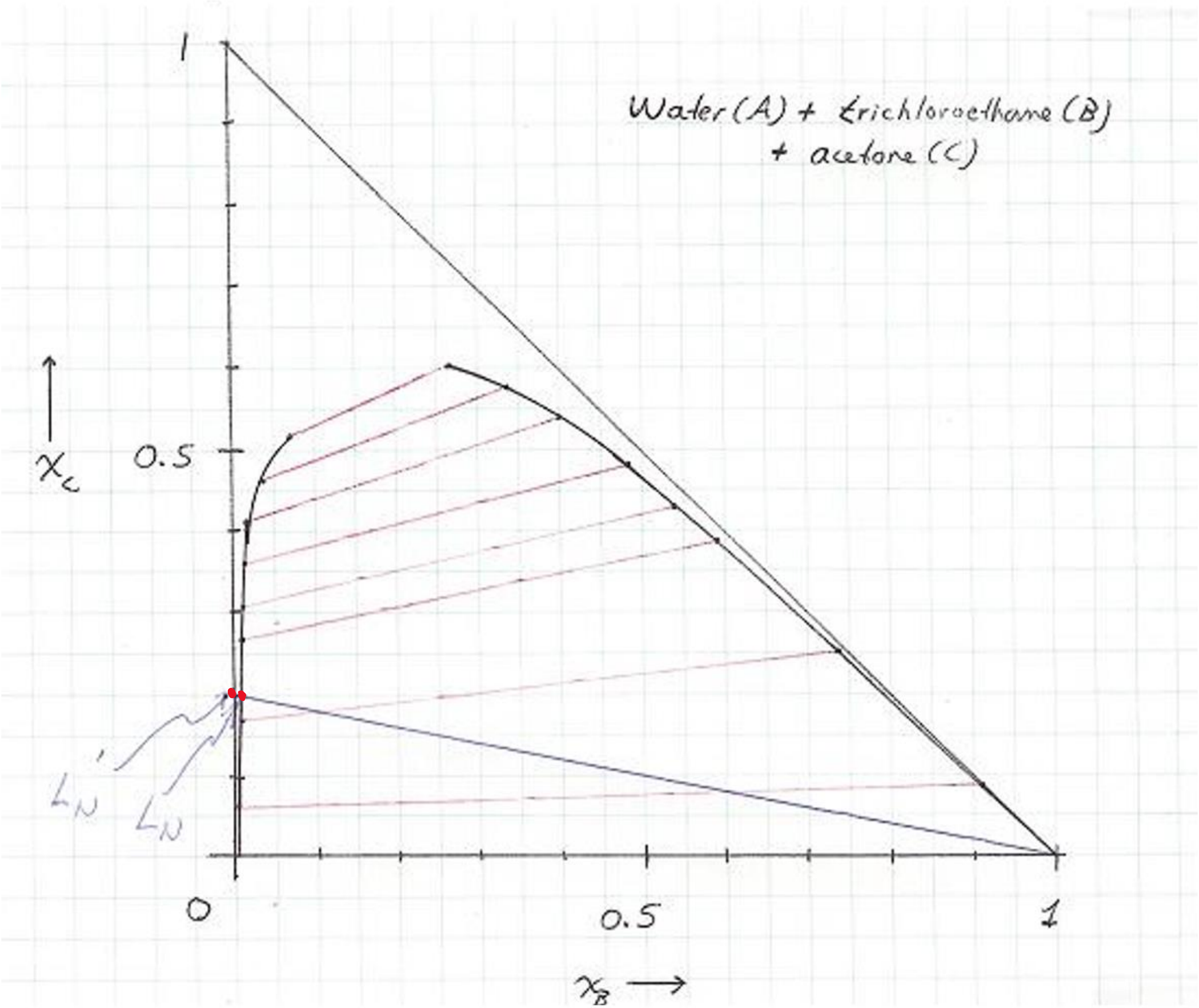


Find M :

$$x_M = \frac{L_0 \cdot x_0 + V_{N+1} \cdot y_{N+1}}{L_0 + V_{N+1}} = \frac{500 \times 0.45 + 125 \cdot 0}{500 + 125}$$

$$x_M = 0.36$$

Find Δ :


Water (A) + trichloroethane (B)
+ acetone (C)
1
0.5
x_C
L_N'
L_N
0
0.5
1
x_B

Amounts of exiting streams

$$(L_N + V_1) x_M = L_N x_N + V_1 y_1 \qquad \text{(C balance)}$$

$$L_N (x_M - x_N) = V_1 (y_1 - x_M)$$

$$\frac{L_N}{V_1} = \frac{y_1 - x_M}{x_M - x_N} = \frac{0.538 - 0.36}{0.36 - 0.20}$$

$$\boxed{L_N / V_1 = 1.11}$$

also, $\boxed{L_N + V_1 = L_0 + V_{N+1} = 500 + 125 = 625 \text{ kg/h}}$

$$\therefore \left(\frac{L_N}{V_1}\right) V_1 + V_1 = 625 \text{ kg/h}$$

1.11

$$(1.11 + 1) V_1 = 625 \text{ kg/h}$$

$$2.11\ V_1 = 625 \text{ kg/h}$$

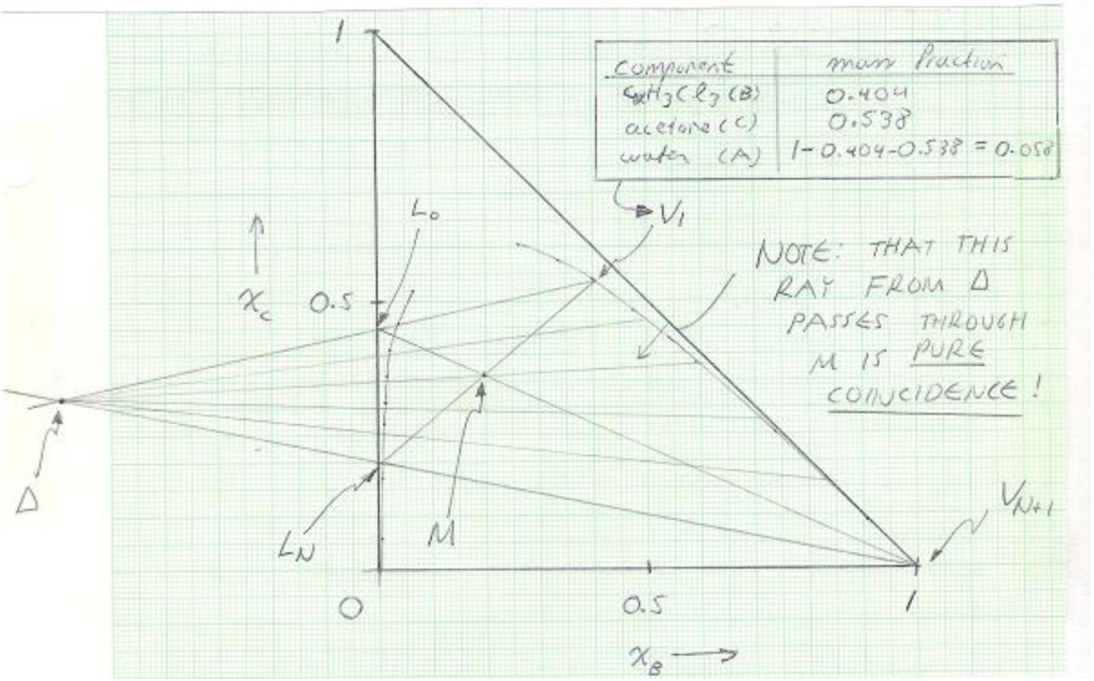
$$\boxed{V_1 = 296 \text{ kg/h} \quad \text{flow rate of exiting extract stream}}$$

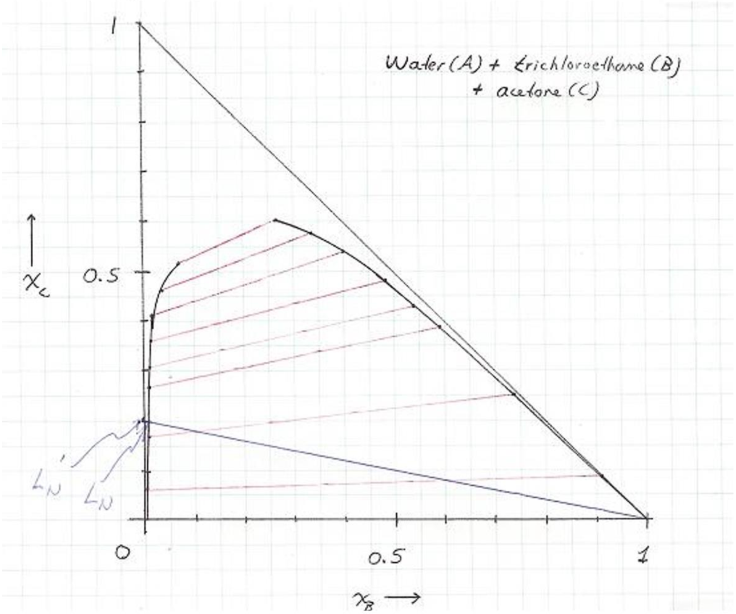
OP. LINE

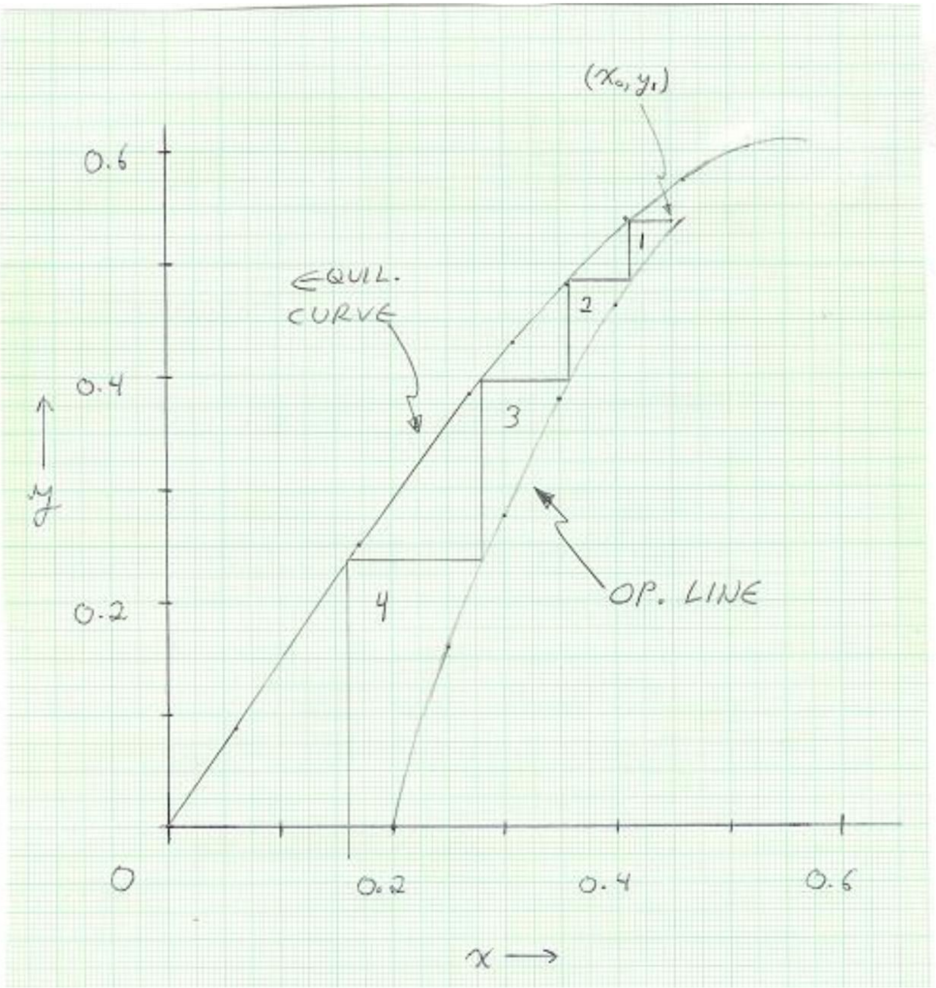
x_n	y_{n+1}
0.25	0.160
0.30	0.277
0.35	0.380
0.40	0.463

EQUIL. CURVE

x	y
0.060	0.088
0.171	0.251
0.269	0.385
0.308	0.430
0.357	0.482
0.409	0.540
0.460	0.574
0.518	0.603







From McCabe-Thiele diagram, need 4 ideal stages (or about $3\frac{2}{3}$ ideal stages).