# **CE407 SEPARATIONS**

Lecture 08

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#### Continuous Distillation with Reflux – Nearly Pure Products McSH pp 692-694

• What if the purity required is  $\mathbf{x}_{\mathbf{D}} = 0.9999?$ 



- You simply cannot draw the triangle small enough
- Not even if you blow up the section near **x**<sub>D</sub>
- What if the purity required is  $\mathbf{x}_{\mathbf{D}} = 0.999999?$ 
  - The width of your pen stroke doesn't allow you to distinguish between  $\mathbf{x}_{\mathbf{D}} = 0.9999$  and  $\mathbf{x}_{\mathbf{D}} = 0.999999$





## What to do for Very High Purity cases?

- In a region a short distance from  $\mathbf{x}_{D} = 0.9999$  the equilibrium curve is fairly straight and  $\alpha$  is approximately constant
- According to constant molal overflow assumption, the Operating Line is straight
- Those are the two assumptions of the **Kremser Equation**!!!
- We cannot use the Kremser Equation across the entire column because the EQ Curve has so much curvature
- We cannot use the McCabe-Thiele method across the entire column because the EQ Curve and the OP Line converge so closely that the steps become too small as we approach x<sub>p</sub> values extremely close to 1
- We will split the column into two sections with a cutoff value of **x**, **x**<sub>cutoff</sub>
- x<sub>cutoff</sub> is chosen so that we can successfully use McCabe-Thiele below the cutoff (i.e. the triangles are not too small) and that the EQ curve can be considered straight between x<sub>cutoff</sub> and x<sub>D</sub> so that Kremser is appropriate





# **Kremser Region**

#### Equilibrium "Line"

- We need to approximate the EQ Curve with a straight line in this region
- Obviously the (1, 1) must be on the EQ Line
- We need one more point to define the EQ Line
  - $\mathbf{y}^* = \mathbf{y}^*(\mathbf{x}_{\text{cutoff}})$
- In other words, use VLE data or relationship to calculate what value of **y** is in equilibrium with **x**<sub>cutoff</sub>
  - Note that in Excel one can use Goalseek to find the temperature leading to x = x<sub>cutoff</sub> and then read off y\* = y\*(x<sub>cutoff</sub>)
- Solve for  $\mathbf{y} = \mathbf{m}\mathbf{x} + \mathbf{b}$  that satisfies both (1, 1) and  $(\mathbf{x}_{cutoff}, \mathbf{y}^*(\mathbf{x}_{cutoff}))$

#### **Operating Line**

• We already have that from  $y_{n+1} = \frac{R}{R+1} x_n + \frac{x_D}{R+1}$ 

Caution: We are trying to determine a mole fraction that has many significant digits

• Carry several more digits in each term of the calculation than the number in the specification



## High Purity – Kremser Section

- Kremser Equation  $N = \frac{\ln[(y_b y_b^*)/(y_a y_a^*)]}{\ln[(y_b y_a)/(y_b^* y_a^*)]}$
- a is at  $\mathbf{x} = \mathbf{x}_{\mathbf{D}}$ 
  - $\mathbf{X}_{a} = \mathbf{X}_{D}$
  - $y_a = x_D$
  - $y_a^* = y^*(x_D)$  is calculated from the the EQ "Line"  $y = m^* x_D + b$ 
    - Note that the "b" has nothing to do with position b, it is the intercept in the EQ "Line" equation
- b is at **x** = **x**<sub>cutoff</sub>
  - $\mathbf{X}_{\mathbf{b}} = \mathbf{X}_{\mathbf{cutoff}}$
  - $y_b = \frac{R}{R+1} x_{cutoff} + \frac{x_D}{R+1}$
  - $y_b^* = y^*(x_{cutoff})$  is the value of y in equilibrium with  $x_{cutoff}$ . This is the value that we used to determine the EQ "Line". You could calculate it with the EQ "Line" equation, but why bother? You already have it.







# **High Purity**

- Plug the values for  $y_a$ ,  $y_b$ ,  $y_a^*$ , and  $y_b^*$  into Kremser equation to obtain  $N_{Kremser}$
- Use McCabe-Thiele to determine the number of stages between  $x_B$  and  $x_{cutoff}$
- This gives you **N**graphical
  - In the graph at the right,  $N_{graphical} = 2 + Reboiler$
- N = N<sub>Kremser</sub> + N<sub>graphical</sub>
- As always, round up for partial stages







# High Purity with Non-ideal Stages

- We need to calculate effective equilibrium values for  $y_a^*$  and  $y_b^*$
- $y_a^{*'} = y_a + \eta_M (y_a^* y_a)$
- $y_b^{*'} = y_b + \eta_M (y_b^* y_b)$
- $N = \frac{\ln[(y_b y_b^{*\prime})/(y_a y_a^{*\prime})]}{\ln[(y_b y_a)/(y_b^{*\prime} y_a^{*\prime})]}$
- Do the same procedure used before for McCabe-Thiele and  $\eta_{M}$  to determine  $\mathbf{N}_{\text{graphical}}$







Enthalpy Balances McSH pp 694-701 (also Eq. 21-1 and 21.2 on pp 679 and 682)

• We want to develop expressions for

$H_x(T, x)$	and	$H_y(T,y)$
Liquid Enthalpy		Vapor Enthalpy

- We will determine enthalpy as a function of temperature, composition and phase
- Enthalpy is a state function and is relative to a reference state. We can choose any state as our zero point units are energy/mole
  - We will choose to use: Each pure component as a liquid at the boiling temperature of the lighter component *this choice will prove to simplify later calculations…*
- Declare:  $H_{x,1}(T_{nb1}) = 0$  and  $H_{x,2}(T_{nb1}) = 0$
- $H_{x,1}$  is the enthalpy of pure component 1, liquid form
- $H_{x,2}$  is the enthalpy of pure component 2, liquid form
- T<sub>nb1</sub> is the Normal Boiling Temperature of pure Component 1
- T<sub>nb2</sub> is the Normal Boiling Temperature of pure Component 2





# Enthalpy of Pure Liquids

- $H_{xi}(T)$  is the enthalpy of pure component *i* in liquid form at temperature **T**
- $H_{xi}(T) = H_{xi}(T_{nb1}) + C_{pi}^{L} * (T T_{nb1})$ 
  - $C_{pi}^{L}$  is the Liquid Heat Capacity of Pure Component i
  - This assumes a constant value for the heat capacity
    - Units are  $\frac{energy}{mol \, ^{\circ}\mathrm{C}}$  eg  $\frac{cal}{mol \, ^{\circ}\mathrm{C}}$
  - Because we have chosen a reference of  $H_{xi}(T_{nb1}) = 0$

$$H_{xi}(T) = C_{pi}^L * (T - T_{nb1})$$



# Enthalpy of Pure Vapors, light component

- $H_{y1}(T)$  is the enthalpy of pure component **1** in vapor form at temperature **T**
- $H_{y1}(T) = H_{x1}(T_{nb1}) + \Delta H_1^{vap}(T_{nb1}) + C_{p1}^V * (T T_{nb1})$ 
  - $C_{p1}^{V}$  is the Vapor Heat Capacity of Pure Component 1
  - This assumes a constant value for the heat capacity
    - Units are  $\frac{energy}{mol \, ^{\circ}\mathrm{C}}$  eg  $\frac{cal}{mol \, ^{\circ}\mathrm{C}}$
  - $\Delta H_1^{vap}(T_{nb1})$  is the heat of vaporization of pure component 1 at its normal boiling temperature
  - Because we have chosen a reference of  $H_{xi}(T_{nb1}) = 0$

$$H_{y1}(T) = \Delta H_1^{vap}(T_{nb1}) + C_{p1}^V * (T - T_{nb1})$$



#### Enthalpy of Pure Vapors, heavy component

 $H_{y2}(T)$  is the enthalpy of pure component **2** in vapor form at temperature **T** 

$$H_{y2}(T) = H_{x2}(T_{nb1}) + C_{p2}^{L} * (T_{nb2} - T_{nb1}) + \Delta H_{2}^{vap}(T_{nb2}) + C_{p2}^{V} * (T - T_{nb2})$$

reference state raise liquid temp to  $T_{nb2}$  convert to vapor at  $T_{nb2}$  superheat vapor to final temp

- $\Delta H_2^{vap}(T_{nb2})$  is the heat of vaporization of pure component 2 at its normal boiling temperature
- It will prove useful to express  $C_{p2}^{V} * (T T_{nb2})$  as

$$C_{p2}^{V} * (T - T_{nb2}) = C_{p2}^{V} * (T - T_{nb1} + T_{nb1} - T_{nb2})$$
  
=  $C_{p2}^{V} * (T_{nb1} - T_{nb2}) + C_{p2}^{V} * (T - T_{nb1})$ 

- This allows all four terms  $(H_{x1}, H_{x2}, H_{y1}, H_{y2})$  to be expressed in terms of  $(T T_{nb1})$
- Because we have chosen a reference of  $H_{xi}(T_{nb1}) = 0$

$$H_{y2}(T) = C_{p2}^{L} * (T_{nb2} - T_{nb1}) + \Delta H_{2}^{vap}(T_{nb2}) + C_{p2}^{V} * (T_{nb1} - T_{nb2}) + C_{p2}^{V} * (T - T_{nb1})$$

0



## Example

• Benzene (1) and Toluene (2)

Component	C <sub>pi</sub> <sup>L</sup> cal/molC	C <sub>pi</sub> <sup>v</sup> cal/molC	▲ H <sub>i</sub> <sup>vap</sup> cal/mol	T <sub>nbi</sub> C
1	33	23	7360	80.1
2	40	33'	7960	110.6

- $H_{x1}(T) = 0 + 33 * (T 80.1) = 33 * (T 80.1)$
- $H_{x2}(T) = 0 + 40 * (T 80.1) = 40 * (T 80.1)$
- $H_{y1}(T) = 0 + 7360 + 23 * (T 80.1) = 7360 + 23 * (T 80.1)$
- $H_{y2}(T) = 0 + 40 * (110.6 80.1) + 7960 + 33' * (80.1 110.6) + 33' * (T 80.1)$ = 8174 + 33' \* (T - 80.1)



Txy Phase Diagram Benzene and Toluene







## **Enthalpy of Mixtures**

- Liquid Mixtures
  - $H_x(T,x) = x * H_{x1}(T) + (1 x) * H_{x2}(T) + \Delta H^{mix}(T,x)$
  - For an ideal mixture  $\Delta H^{mix}(T, x) = 0$
- Vapor Mixtures
  - $H_y(T, y) = y * H_{y1}(T) + (1 y) * H_{y2}(T)$
  - For a mixture of vapors  $\Delta H^{mix}(T, x) = 0$





#### Enthalpy of Mixtures, example

• 
$$H_x(t,x) = x * 33 * (T - 80.1) + (1 - x) * 40 * (T - 80.1)$$
  
=  $[33x + 40(1 - x)] * (T - 80.1)$ 

= (40 - 7x) \* (T - 80.1)

•  $H_y(T,y) = y * [7360 + 23(T - 80.1)] + (1 - y) * [8174 + 33(T - 80.1)]$ = 8174 - 814y + (33 - 10y) \* (T - 80.1)

