CE407 SEPARATIONS

Lecture 03

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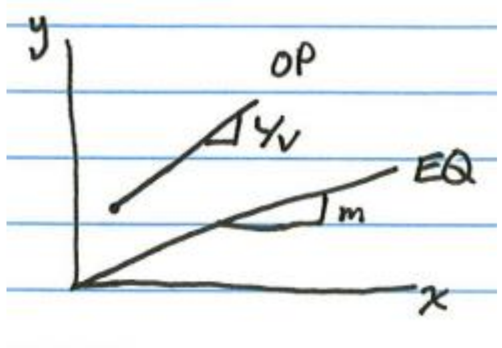
Absorption – Kremser Equation (aka Absorption Factor Method)

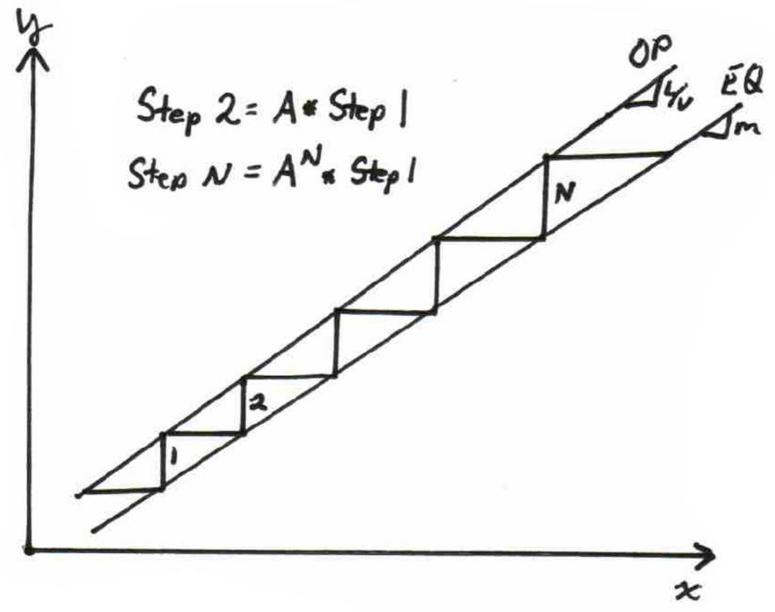
- Pages 653-659 of McCabe/Smith/Harriott cover the mathematical derivation. These notes cover the concepts
- 2 Main Conditions for the Kremser Method to be valid
  - Operating Line must be a straight line
    - This is a reasonable assumption if x and y <<1 (~ < 0.1)
  - Equilibrium Curve is a straight line
    - Raoult's or Henry's Law applies
    - Mixture of similar chemicals
    - As long as EQ curve approximates a straight line over the region of interest



- Slope of OP line $\approx \frac{L}{V}$, this is approximately constant for dilute system
- Slope of EQ line is given by equilibrium relationship (Raoult, etc) and is noted as **m**
- Absorption Factor **A**: $A = \frac{L/V}{m} = \frac{L}{mV}$ this is the ratio of the OP line slope vs the EQ line slope

Kremser Equation

- Observe: If the two lines have different slopes then each successive step will increase in size relative to the previous step.
- The increase is proportional to **A**
- If you take N steps then that step is $\mathbf{A^{N-1}}$ times the original step



- Take logarithm $\mathbf{ln(A^N) = N * ln(A)}$ this all leads to the Kremser Equation

Kremser Equation

- Various Forms, all are equivalent

$$N = \frac{ln[(y_b - y_b^*)/(y_a - y_a^*)]}{\ln[(y_b - y_a)/(y_b^* - y_a^*)]}$$ eq 20.24

$$N = \frac{ln[(y_a - y_a^*)/(y_b - y_b^*)]}{\ln[(y_b^* - y_a^*)/(y_b - y_a)]}$$ eq 20.27

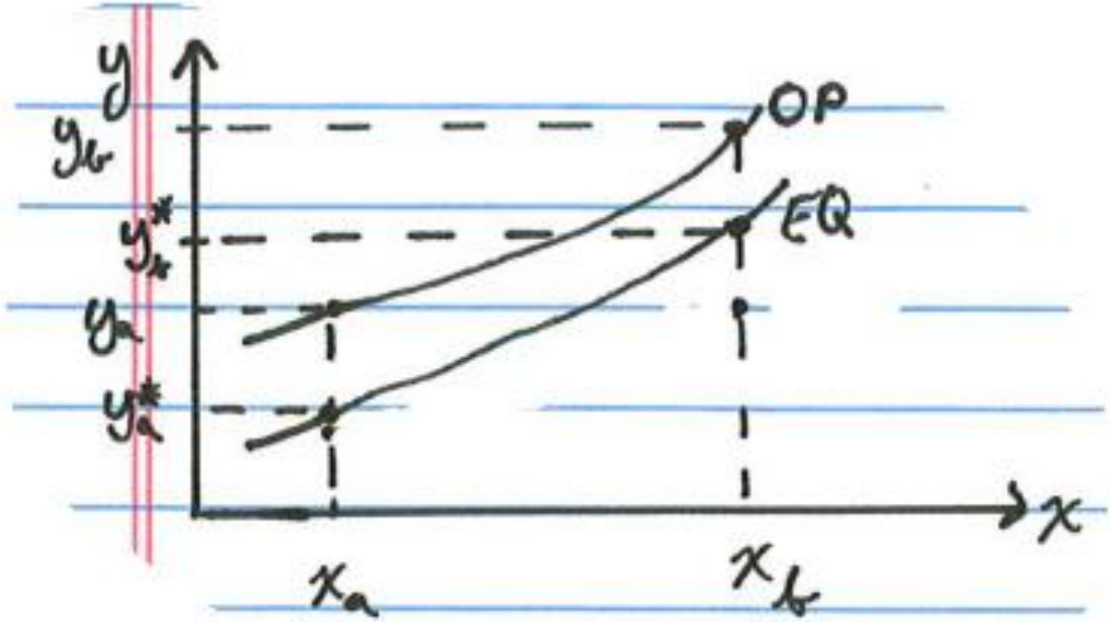
$$N = \frac{ln[(x_a - x_a^*)/(x_b - x_b^*)]}{\ln[(x_a - x_b)/(x_a^* - x_b^*)]}$$ eq 20.28

$$N = \frac{ln[(x_b - x_b^*)/(x_a - x_a^*)]}{\ln[(x_a^* - x_b^*)/(x_a - x_b)]}$$

Okay, what are those *s?

- y_a* is the value of vapor mole fraction which is in equilibrium with liquid mole fraction x_a

- y_b* is the value of vapor mole fraction which is in equilibrium with liquid mole fraction x_b



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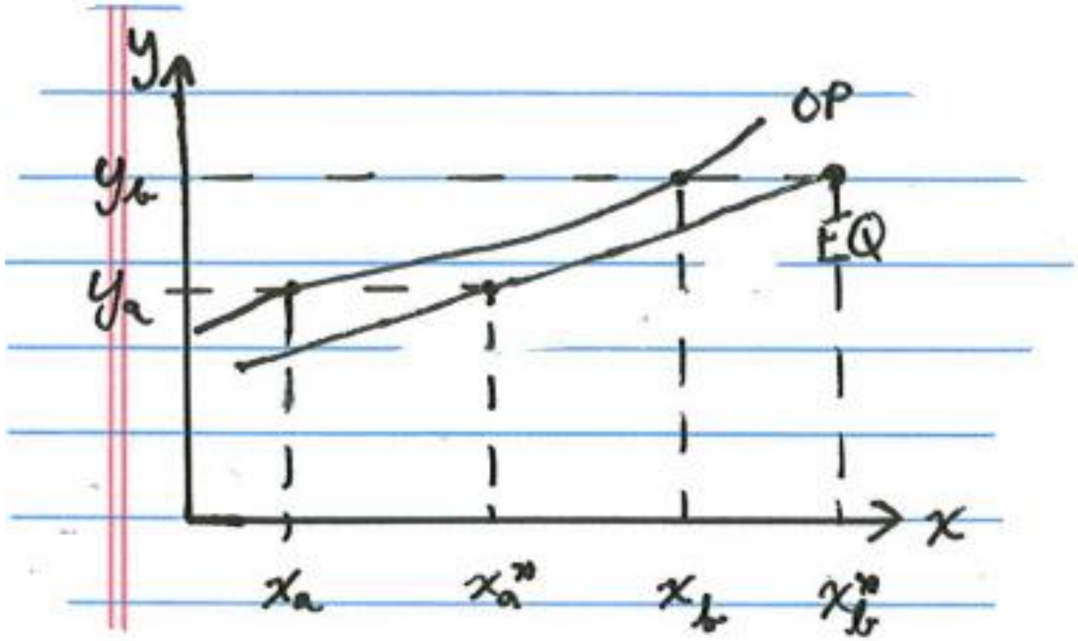


Kremser Equation

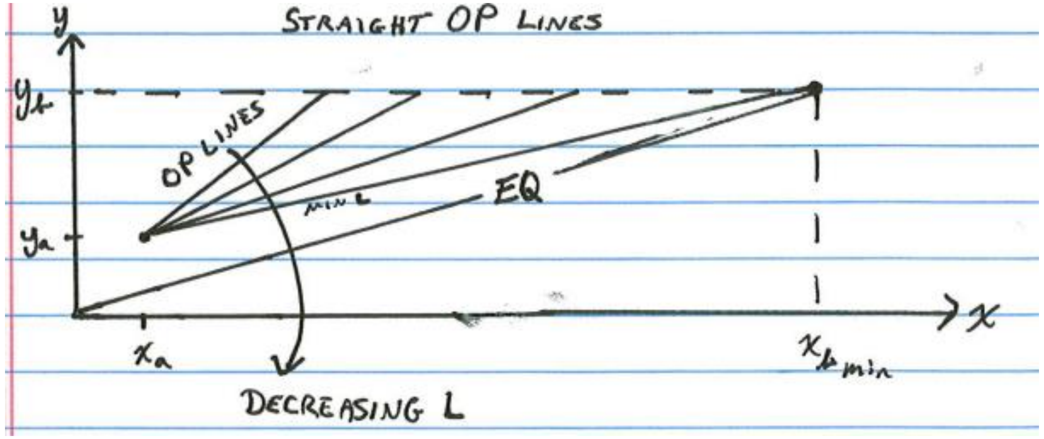
$$N = \frac{ln[(y_b - y_b^*)/(y_a - y_a^*)]}{ln[(y_b - y_a)/(y_b^* - y_a^*)]} \qquad \text{eq 20.24}$$

- **y_b – y_b*** is the difference between entering vapor mole fraction and the mole fraction that could be achieved IF we were to reach equilibrium – this is a driving force
- **y_a – y_a*** is the difference between exiting vapor mole fraction and the mole fraction that could be achieved IF we were to reach equilibrium – this is a driving force
- **y_b – y_a** is the change that we are trying to achieve
- **y_b* - y_a*** is the potential for change if the entering and exiting vapors both achieved equilibrium with their respective liquid counterparts

- Note: **N** will most likely NOT be an integer – Round Up!

Absorption Tower Minimum Liquid Flow McSH pp 577-578

- Minimum liquid flow is the value which can just achieve the desired removal of solute
    - It unfortunately leads to an infinitely tall tower…
    - However, it’s calculation is used as a basis to determine a reasonable liquid flow

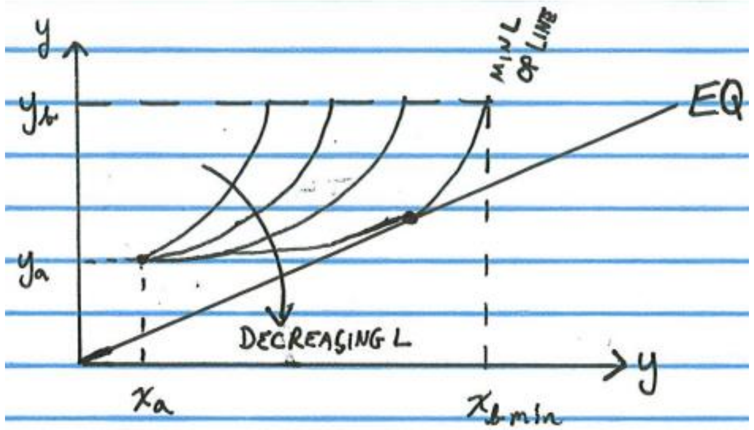


- Decreasing liquid flow leads to lower L/V and decreases the slope of OP Line
- At minimum flow the OP Line intersects the EQ Line at ($x_{b,\,min}$, y_b) and the steps between the lines become infinitely small leading to an infinite number of steps
- With a lower liquid flow you can NEVER reach y_b (where y* indicates equilibrium)

$$y_b = y^*(x_b)$$

Minimum Liquid Flow with Curved OP Line

- Minimum liquid flow is that for which OP Line first contacts (but does not cross) the EQ Curve



- $\mathbf{x_{b,\,min}}$ is the value of **x** that corresponds to $\mathbf{y_b}$ on the OP Line for minimum flow
- Also note that $$(x_b)_{min} = \frac{(L_i)_b}{(L_i)_b + (L_c)_{min}}$$
- $\mathbf{(L_i)_b}$ is the # of moles of solute exiting b end of tower – calculated by mass balance on solute
- $\mathbf{(L_c)_{min}}$ is the flow of pure liquid that leads to first contact of OP Line and EQ Line
  - i.e. minimum liquid flow of pure absorbing liquid

Recycled Absorbing Liquid

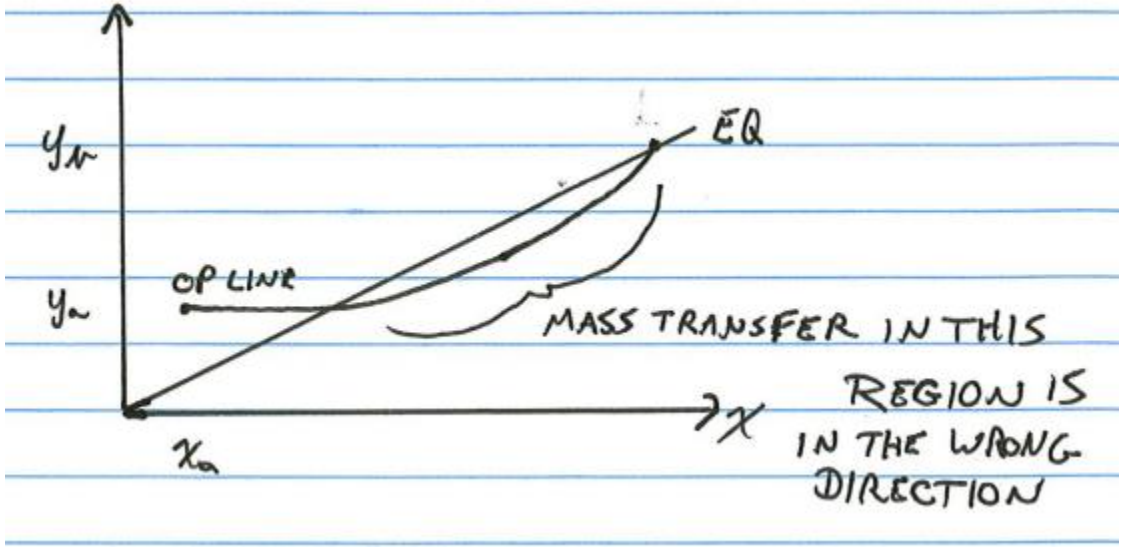
- When using recycled absorbing liquid $x_a \neq 0$

$$(L_a)_{min} * (1 - x_a) = (L_c)_{min}$$

$$(L_a)_{min} = \frac{(L_c)_{min}}{1 - x_a}$$

Do **Not** Assume that $y_b = y^*(x_b)$

- If there is curvature in the Operating Line that assumption leads to the following situation:

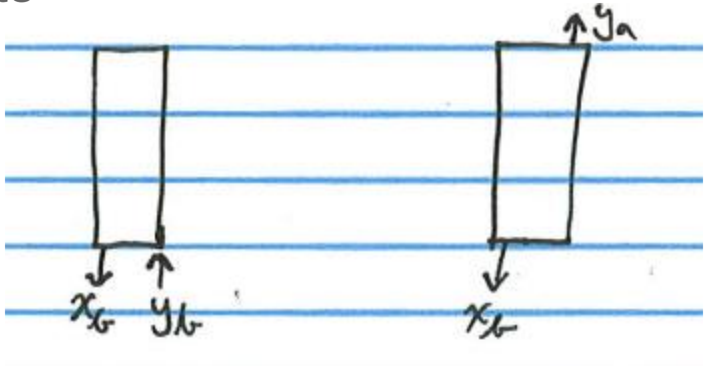


When Op Line is straight…

- When we have a dilute system with straight operating lines and can therefore use

$$x_b = x^*(y_b)$$

- it is important to note

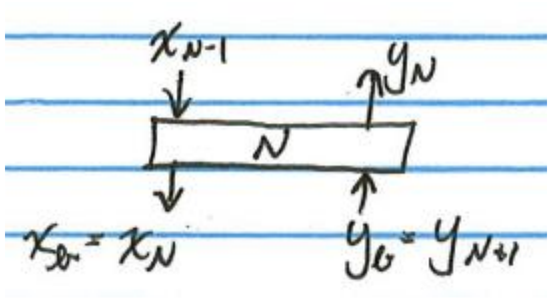


In equilibrium NOT in equilibrium

- **x_b** and **y_a** are separated by many, many stages and are never in contact!

Comment

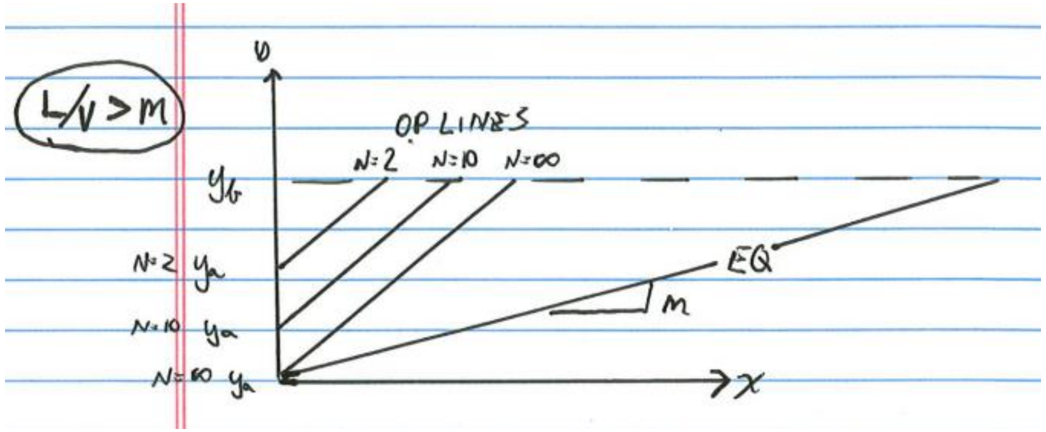
- Actually $\mathbf{x_b}$ and $\mathbf{y_b}$ are not technically in equilibrium…



- $\mathbf{x_b}$ is in equilibrium with $\mathbf{y_N}$, not $\mathbf{y_b = y_{N+1}}$
- However, for minimum liquid flow $\mathbf{y_N}$ and $\mathbf{y_{N+1}}$ are essentially the same due to the infinitely small steps

How clean can you make the vapor with a set value of L/V? L/V > m

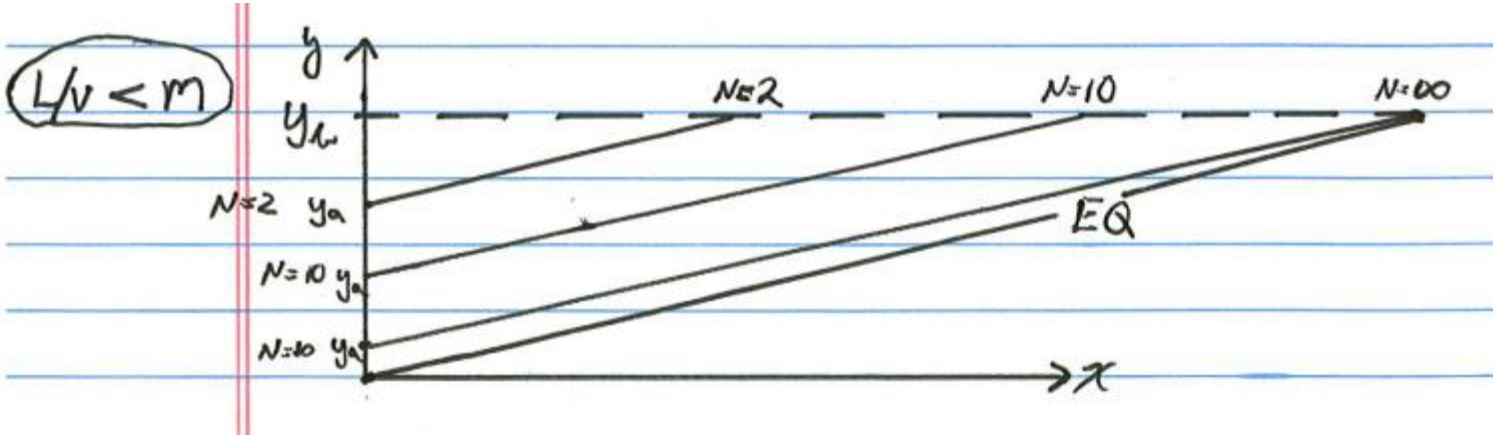
- OP Line Slope = L/V (approximately)
- EQ Curve Slope = m
  - y = mx with m being supplied perhaps by Raoult's or Henry's Laws
- N = # of stages



- By increasing the number of stages one can theoretically obtain as low a value of $\mathbf{y_a}$ as desired
- You do reach a point of diminishing returns…

How clean can you make the vapor with a set value of L/V? L/V < m

- When L/V < 1, eventually the Operating Line contacts the Equilibrium Line at **y** = $\mathbf{y_b}$ and leads to a pinch point



- Continuing to add stages will NOT lead to a lower value of $\mathbf{y_a}$

What Flow Should We Use?

- Using $\mathbf{L_{min}}$ leads to an infinite number of stages and an infinite capital cost
- Using an **L** that is extremely high reduces capital cost but leads to very high expense for absorbing liquid

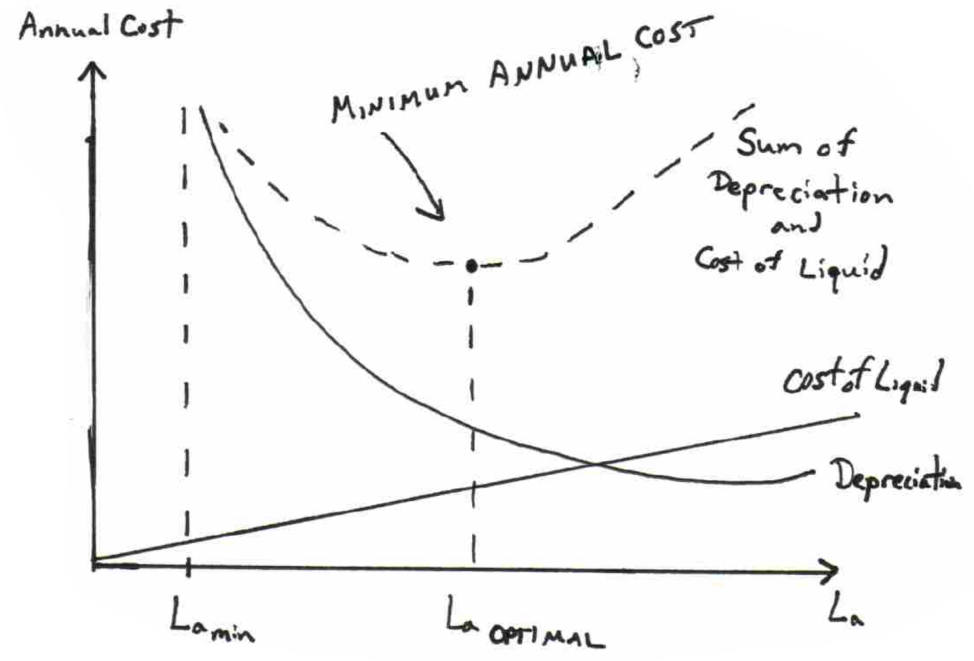
Optimize Cost

- Capital cost of building tower is spread out over the useful life of the equipment and is called depreciation

$$Simple\ Depreciation = \frac{Capital\ Cost\ of\ Equipment}{\#\ of\ years\ in\ use}$$

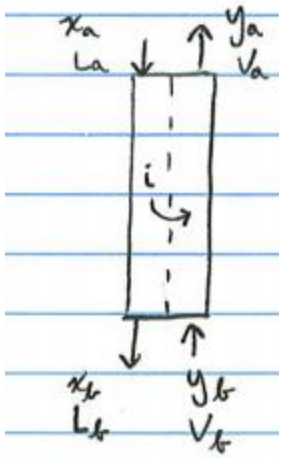
Optimized Cost

- At $\mathbf{L_{a\ min}}$ the capital cost (and therefore annual depreciation) is infinite
- As liquid flow increases the capital cost drops asymptotically
- The cost of liquid is approximately directly proportional to the amount used
- Look for minimum of the sum of capital and liquid cost
- This value is typically $L_{a,optimal} \approx 1.3 * L_{a,min}$



Stripping

- The same equipment can be used to remove solute from a liquid!
- This can be done when the conditions are such that equilibrium drives the mass transfer from liquid to vapor
- It is the reverse of absorption
- The analysis is the same…



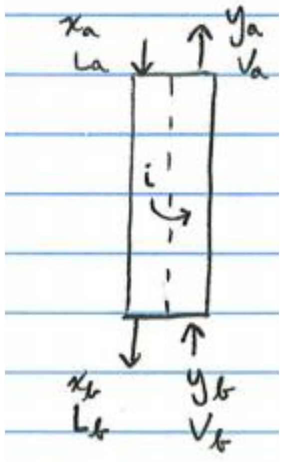
Now we know all about the entering liquid (it is what we want to treat)

- $\mathbf{L_c = (1 - x_a) * L_a}$ where $\mathbf{L_c}$ is the moles of pure liquid and is constant
- $\mathbf{(L_i)_a = x_a * L_a}$ where $\mathbf{(L_i)_a}$ is moles of solute entering in liquid stream
- We will know both $\mathbf{L_a}$ and $\mathbf{x_a}$

Stripping

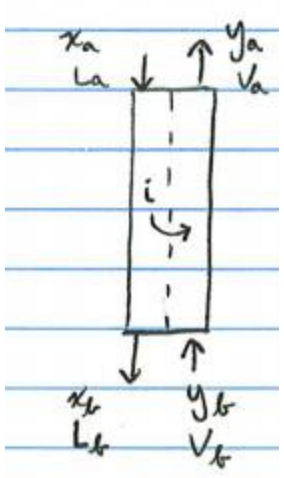
Exiting Liquid

- We will either know: Required $\mathbf{x_b}$
  - Then $\mathbf{(L_i)_b = x_b * [(L_i)_b + L_c]}$
  - And therefore $$(L_i)_b = \frac{x_b}{1 - x_b} L_C$$
- Or we will know that we are required to remove some fraction, **R**, of incoming molar solute flow
- Then $\mathbf{(L_i)_b = (1 - R) * (L_i)_a}$
- And therefore $$x_b = \frac{(L_i)_b}{(L_i)_b + L_c}$$



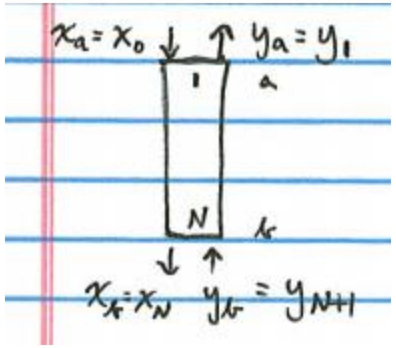
Stripping

- Entering Vapor: we will know $\mathbf{y_b}$ and $\mathbf{V_b}$ (for now…)
  - $\mathbf{(V_i)_b = y_b * V_b}$ and $\mathbf{Vc = (1 - y_b) * V_b}$
  - $\mathbf{V_c}$ is constant
- Exiting Vapor: we need to do material balance (similar to the analysis of the exiting liquid stream in an absorption process)
  - Moles of solute in exiting vapor $\mathbf{(V_i)_a = (L_i)_a + (V_i)_b - (L_i)_b}$

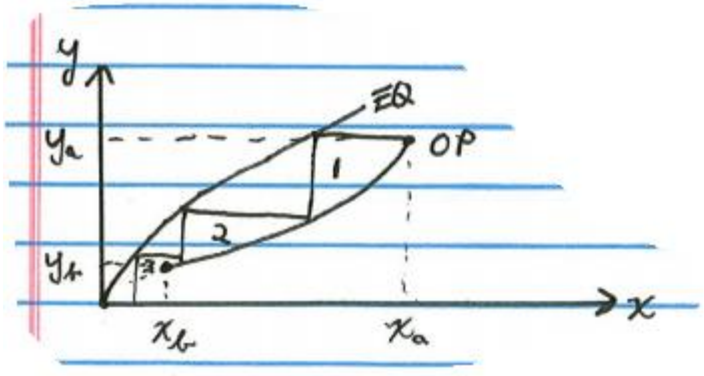


$$y_a = \frac{(V_i)_a}{(V_i)_a + V_c}$$

McCabe-Thiele and Stripping



- Now the top of the tower is the “dirty” section and bottom of tower is the “clean” section, which is the opposite of an absorption tower
- OP Line is the same equation as for absorption
- EQ Curve is the same as for absorption
- For Stripping the EQ curve is ABOVE the OP Line
  - We are transferring mass in the opposite direction as for absorption
- Now counting from top to bottom means starting in the upper right portion



Kremser Equation and Stripping

- The same equations apply
- Some consider the **x** forms to be more appropriate for stripping and the **y** forms more appropriate for absorption
- Can use any of the forms of Kremser as long as OP and EQ are straight lines in the region of interest

Minimum Gas Flow and Stripping

- This is a similar analysis as for absorption
- Straight OP Lines
  - $y_a = y^*(x_a)$

- Curved OP Lines
  - $y_{a,min} = \dfrac{(V_i)_a}{(V_i)_a + (V_c)_{min}}$
- If entering gas is not pure $(V_b)_{min} = \dfrac{(V_c)_{min}}{1 - y_b}$
- If absorbing vapor is steam then $V_b \approx 1.3 * (V_b)_{min}$
- If absorbing gas is air then $V_b \approx (3\ to\ 5) * (V_b)_{min}$
  - (The cost of steam >> the cost of air)