

CE407 SEPARATIONS

Lecture 02

Instructor: David Courtemanche



Counter Current Absorption

- **a** refers to top of tower, **b** refers to bottom

- Entering Gas

V_b = the # of moles of total gas entering at b end of column

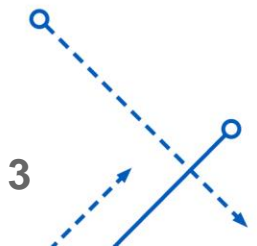
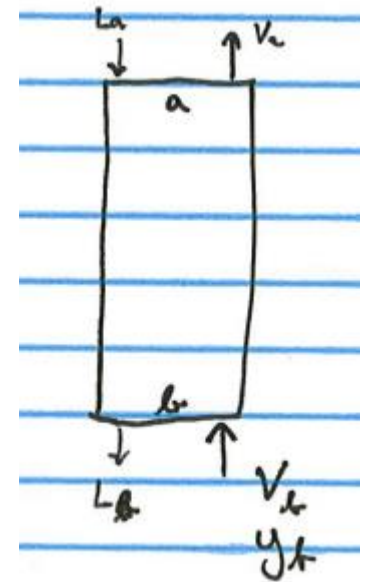
y_b = the mole fraction of solute in gas entering at b end

Note: No subscript i is needed for the mole fraction. It is understood that the y_b refers to the solute.

$(V_i)_b = y_b * V_b$ = the # of moles of SOLUTE entering at b end

$V_c = (1 - y_b) * V_b$ = the # of moles of INERT GAS entering at b end

- Note that V_c is constant. (That's what the "c" stands for...)
- If entering gas is presented as a volumetric flow, use the Ideal Gas Law to convert to molar flow



Counter Current Absorption

- **Exiting Gas**

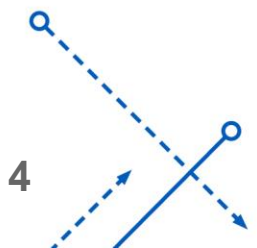
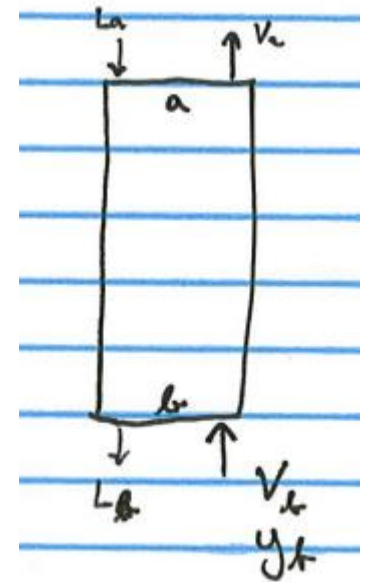
- Often specified by mole fraction of solute in exiting vapor stream
- V_a = the # of moles of total gas exiting at a end of column
- y_a = the mole fraction of solute in gas exiting at a end
- Note: No subscript i is needed for the mole fraction. It is understood that the y_a refers to the solute.

- $(V_i)_a = y_a * V_a = y_a * ((V_i)_a + V_c) =$ the # of moles of SOLUTE exiting at a end

- This can be rearranged to obtain

$$(V_i)_a = \frac{y_a}{1 - y_a} V_c$$

- Note that V_c is constant and therefore equal to value calculated at b end of tower.



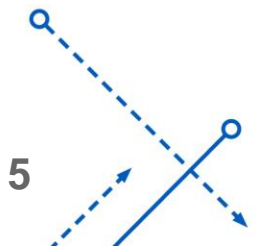
Counter Current Absorption

- Exiting gas may be specified by % of solute removed
 - “R % of incoming solute is removed”
 - Therefore the rest of the solute remains in the exiting stream and the number of moles of solute in the exiting vapor is

$$(V_i)_a = \left(\frac{100 - R}{100} \right) (V_i)_b$$

- The mole fraction of the exiting vapor can be calculated as

$$y_a = \frac{(V_i)_a}{(V_i)_a + V_c}$$



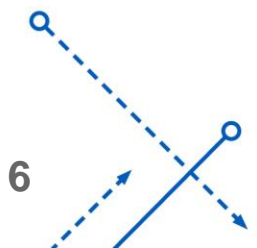
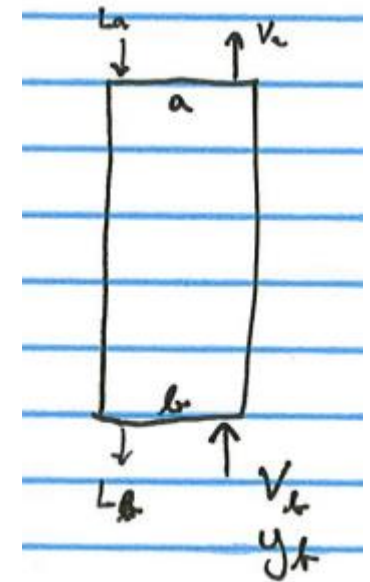
Counter Current Absorption

- L_a = total # of moles of liquid entering at **a** end
 - This is a given (for now)
- x_a = mole fraction of solute in entering liquid at **a** end
 - This will be given (and is often equal to 0)
- L_c = # of moles of pure absorbing liquid

$$L_c = (1 - x_a) * L_a$$

- This is a constant through the tower under the “Usual Assumptions”
- $(L_i)_a$ = # of moles of solute entering with the liquid stream at **a** end

$$(L_i)_a = x_a * L_a$$



Counter Current Absorption

- Process specifications define the three streams vapor in, vapor out, and liquid in
- We will use a mass balance on the solute to determine the liquid out stream conditions

$$\text{Solute In} = \text{Solute Out}$$

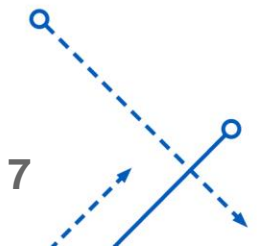
$$(V_i)_b + (L_i)_a = (V_i)_a + (L_i)_b$$

- From this we see that: $(L_i)_b = (V_i)_b - (V_i)_a + (L_i)_a$

- We can compute exiting liquid mole fraction as:

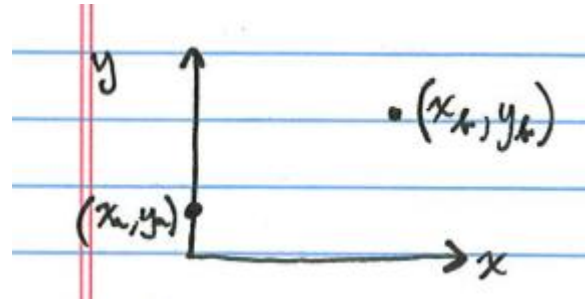
$$x_b = \frac{(L_i)_b}{(L_i)_b + L_c} = \frac{\text{moles solute exiting in liquid}}{\text{total moles exiting in liquid}}$$

- Note: This mass balance is telling us what the exiting liquid stream must be if we are to achieve the specified change to the vapor stream – we have not looked at what tower design is required to accomplish this change!

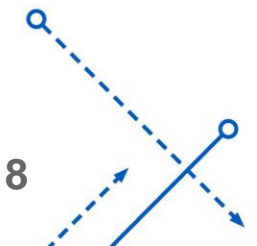


Operating Line

- A plot of mole fraction of vapor versus mole fraction of liquid throughout the tower



- Flow rate of vapor decreases as we go up higher in the tower (we are removing solute...)
- Flow rate of liquid increases as we go lower in the tower (we are absorbing solute...)
- V_c and L_c are assumed constant
- We know the points at the **a** and **b** ends of the tower – how do we determine the intermediate points?



Operating Line

- In order to determine the relationship of y to x at any given height in the tower, we establish a control volume from the top of the tower to an arbitrary height
- At the top:

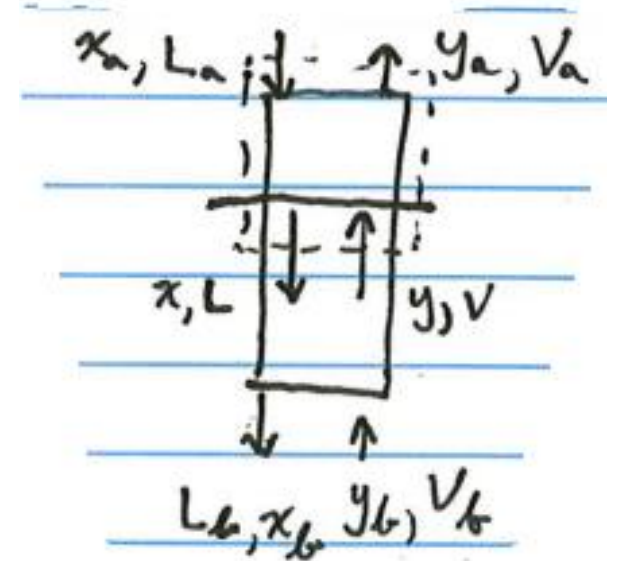
$$L_c = (1 - x_a) * L_a \text{ therefore } L_a = \frac{L_c}{1 - x_a}$$

$$V_c = (1 - y_a) * V_a \text{ therefore } V_a = \frac{V_c}{1 - y_a}$$

- At a generic height:

$$L_c = (1 - x) * L \text{ therefore } L = \frac{L_c}{1 - x}$$

$$V_c = (1 - y) * V \text{ therefore } V = \frac{V_c}{1 - y}$$



- We have now defined the total molar flow of vapor and liquid at both ends of the control volume.

Operating Line

- Total Molar Balance Across the Control Volume

Total Moles In = Total Moles Out

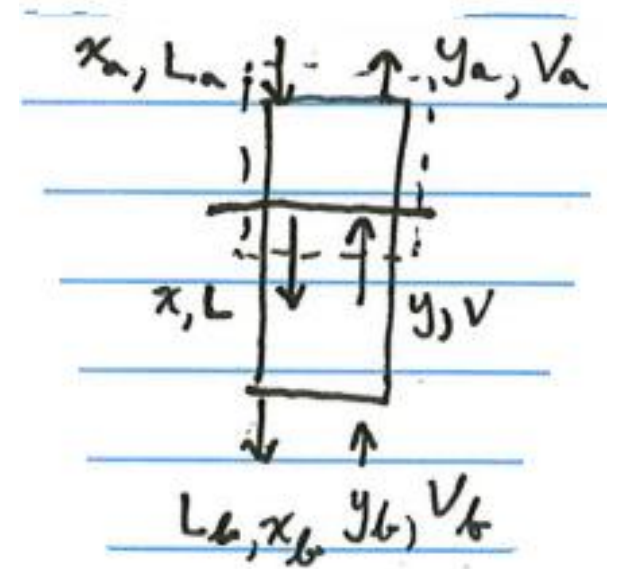
$$V + L_a = V_a + L$$

$$\frac{V_c}{1-y} + \frac{L_c}{1-x_a} = \frac{V_c}{1-y_a} + \frac{L_c}{1-x}$$

- Rearrange algebraically:

$$\star \quad y = 1 - \left[\frac{1}{1-y_a} + \frac{L_c}{V_c} \left(\frac{1}{1-x} - \frac{1}{1-x_a} \right) \right]^{-1} \quad \star$$

- This is called the **Operating Line**. It is actually somewhat curved. If the system is dilute it is fairly straight.



Operating Line

- What if we had done a solute balance instead?

Solute In = Solute out

$$V * y + L_a * x_a = V_a * y_a + L * x$$

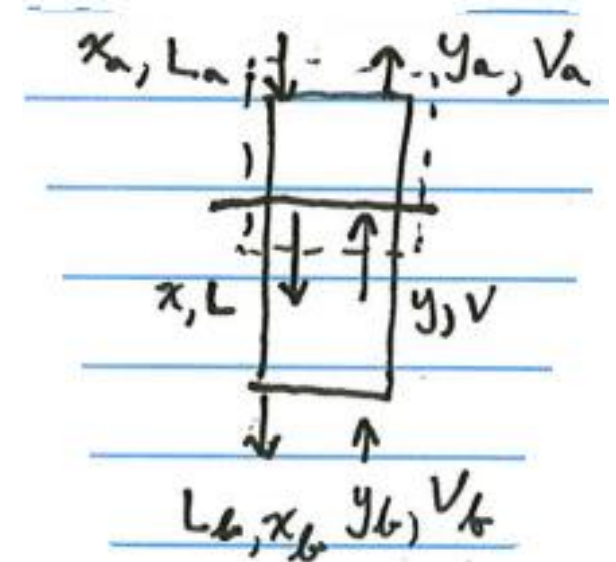
- If the system is dilute (say, x and $y < 0.1$ or 0.05)
 - Then $V_a \approx V_b \approx V$ and $L_a \approx L_b \approx L$
- Now solute balance is approximately

$$V * y + L * x_a = V * y_a + L * x$$

and therefore

$$y = y_a + \frac{L}{V} * (x - x_a)$$

- This is an approximation – USE THE BOXED EQUATION ON PREVIOUS SLIDE!!!



L / V

- L / V is a common parameter when describing an absorption tower.
- For a dilute system it is relatively constant, but what value do we use?
- 3 Reasonable Choices:

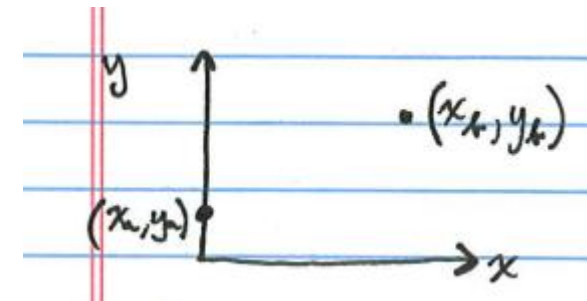
- $$\frac{L}{V} = \frac{\frac{1}{2}(L_a + L_b)}{\frac{1}{2}(V_a + V_b)}$$

- $$\frac{L}{V} = \frac{1}{2} \left(\frac{L_a}{V_a} + \frac{L_b}{V_b} \right)$$

- $$\frac{L}{V} = \frac{y_b - y_a}{x_b - x_a}$$

Preferred Method

- $y = y_a + \frac{L}{V} * (x - x_a)$ - Approximate OP Line for dilute cases



Operating Line

- Choose several values of x between x_a and x_b
- Use operating line equation (boxed equation on slide 10) to calculate the corresponding values of y

- Plot:

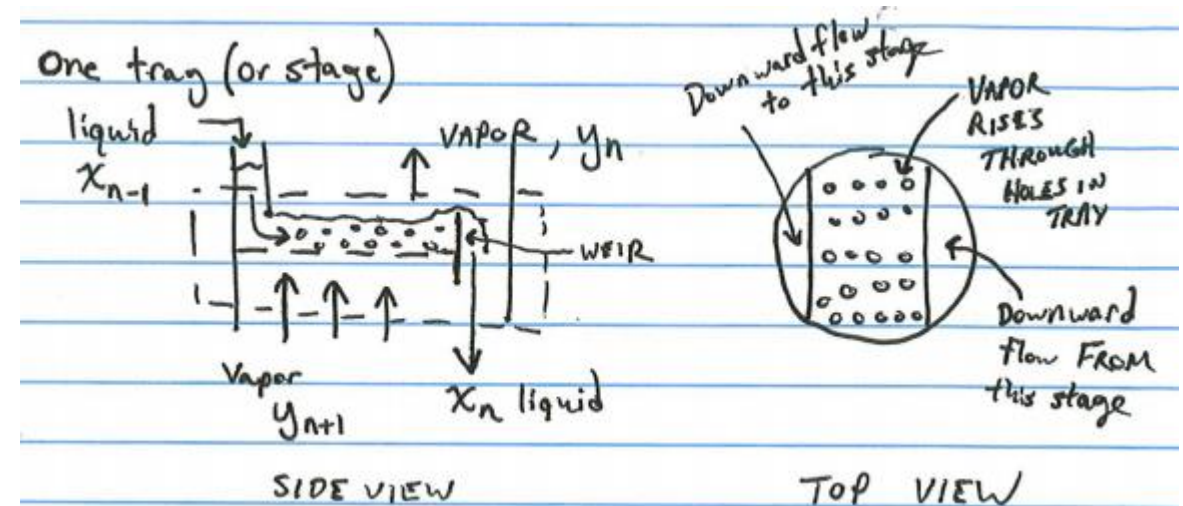


- The Operating Line is just a MOLE BALANCE (I will sometimes use the term mass balance, as well...) (If there are no chemical reactions taking place then a mole balance and a mass balance are equivalent.)



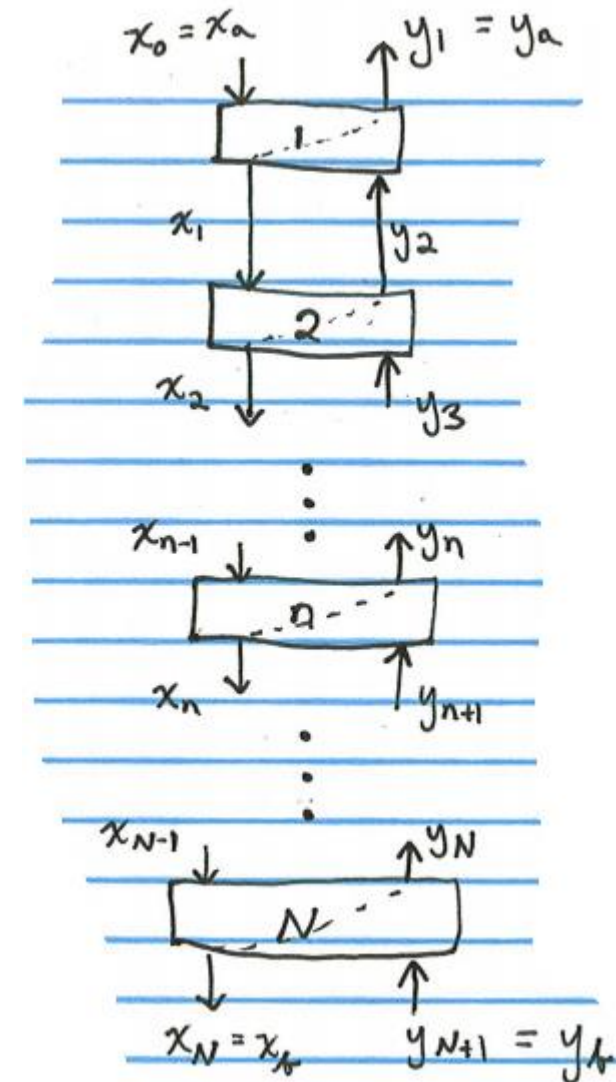
Absorption Tower Analysis

- A tower has many stages (or trays)
- Liquid flows down channel on left hand side of this drawing
- Liquid flows across the tray
 - Level is set by weir
- Vapor flows up through holes in tray
- Mass transfer occurs between the bubbles and the liquid
- Liquid flows down the channel on right hand side of this drawing
- **ASSUME THAT VAPOR LEAVING A GIVEN STAGE IS IN EQUILIBRIUM WITH THE LIQUID LEAVING THAT SAME STAGE**



Absorption Tower Analysis

- Stages are numbered from top to bottom
- n is an arbitrary stage location
- N is the bottom stage
- Subscripts for x and y refer to the stage from which that flow originates (i.e. which stage did it come from)
- For stage n :
 - Vapor enters from stage $n+1$ and has mole fraction y_{n+1}
 - Liquid enters from stage $n-1$ and has mole fraction x_{n-1}
 - Vapor leaving stage n has mole fraction y_n
 - Liquid leaving stage n has mole fraction x_n
- x_n and y_n are related via Equilibrium Relations
- x_n and y_{n+1} are related via the Operating Line
 - Note that x_n and y_{n+1} cross the control volume we used to analyze the operating line



Equilibrium Relationships

- Raoult's Law $y_i P = x_i P_i^{sat}(T)$

$$y_n = \frac{P_i^{sat}(T)}{P} * x_n$$

- Henry's Law $y_i P = x_i \gamma_i^\infty P_i^{sat}(T)$

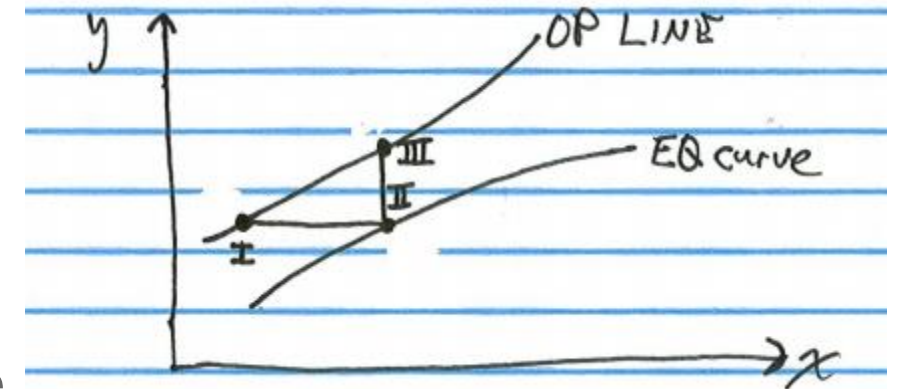
$$y_n = \frac{k_i(T)}{P} x_n$$

- Note that subscript **n** refers to stage number – it is understood that it refers to the solute
- Vapor Liquid Equilibria (VLE) data supplied
 - Be cautious of the units!
 - They will often be mass ratios, mass fractions, partial pressures, etc
 - You **MUST** convert to mole fractions

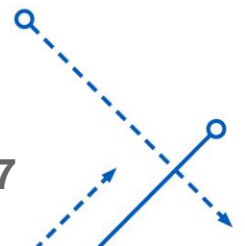


McCabe-Thiele Method

- From notes titled “Counter Current Absorption”, scheme 3 we can see that even a two stage calculation is very complex. We want to solve for **N** stages...
- McCabe-Thiele is a graphical solution
- Plot both the Operating Line (mass balance) and the Equilibrium Curve on the same graph.
- Point I is (x_a, y_a) which is (x_0, y_1)
- Point II has the same value of y as Point I and is therefore $y = y_1$. As Point II lies on the equilibrium curve $x = x_1$. (Remember x_n and y_n are in equilibrium!)
- Point III has the same value of x as Point II and is therefore $x = x_1$. Because Point III is on the Operating Line we know that $y = y_2$. (Remember that (x_n, y_{n+1}) are on the Operating Line!)



- We have just determined the relationship from $y = y_1$ to $y = y_2$: we have moved one stage!



McCabe-Thiele

- Continue until we reach (x_b, y_b)
- Note that it is unlikely that any stage will land exactly on (x_b, y_b)
- This looks like about 3.6 stages
 - Obviously there is no physical meaning of a partial stage
 - Report this on homework or in exam as “3.6 stages, to be rounded up to 4 stages”
- Stepping **ALWAYS** begins and finishes on the Operating Line!
- Later in the course we will learn how to determine the physical height of each stage
 - Rough estimate: 6” for small flows and 2’-3’ for large flows

