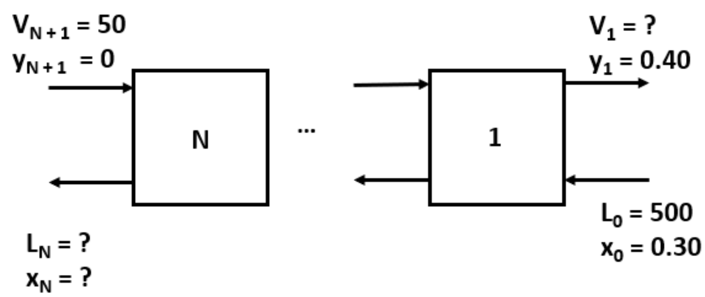
CE407 Exam 02 November 30, 2023

1. **(50 pts)** 500 kg/hr of a feed solution containing a solute (C) mass fraction of 0.30 is to be extracted using 50 kg/hr of a pure solvent stream. You are required to recover 90% of the solute in the extract stream. The mass fraction of the exiting extract is to be 0.40. Use the following equilibria data and phase diagrams on next pages to determine how many extraction stages are required. ***Use the McCabe-Thiele method to determine the required number of stages.***

Diluent Rich (Raffinate)		**Solvent Rich (Extract)**	
x (x_B)	**y (x_C)**	**x (x_B)**	**y (x_C)**
0.07	0.22	0.30	0.42
0.06	0.17	0.40	0.39
0.045	0.12	0.52	0.30
0.04	0.06	0.64	0.18
0.038	0.04	0.685	0.12
0.035	0.02	0.73	0.06

Solution

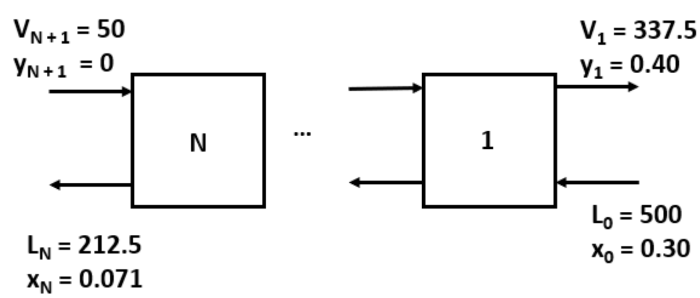
- Step 1
    - Determine the amount of solute in each stream:



-
    - Entering Feed:
        - $x_0 * L_0 = 0.30 * 500 = 150\ kg$
    - Entering Solvent
        - $y_{N+1} * V_{N+1} = 0 * 50 = 0\ kg$
    - Exiting Extract Stream
        - 90% Recovery of Solute
        - $0.90 * Solute\ in\ Feed = 0.90 * 150\ kg = 135\ kg = y_1 * V_1$

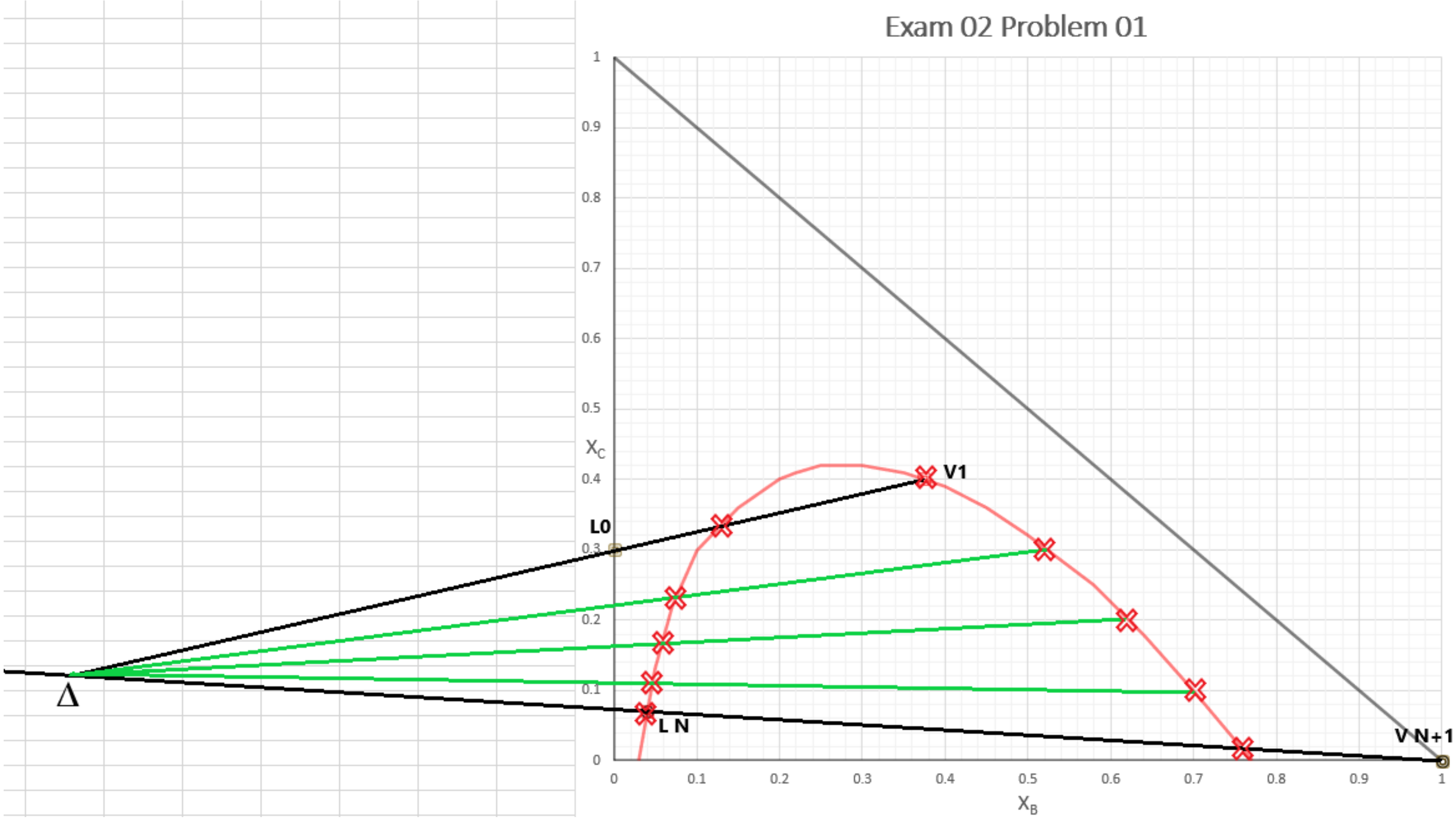
    - **Solute Balance**
    - $y_{N+1} * V_{N+1} + x_0 * L_0 = y_1 * V_1 + x_N * L_N$
    - $0 + 150 = 135 + x_N * L_N$
    - $15 = x_N * L_N$

- Step 2
    - We know that:
        - $135\ kg = y_1 * V_1$ and that $y_1 = 0.4$
        - Therefore $135\ kg = 0.4 * V_1$
        - And $V_1 = \frac{135}{0.4} = 337.5\ kg$
- Step 3
    - Solution Balance around entire battery
        - $V_{N+1} + L_0 = V_1 + L_N$
        - $50\ + 500 = 337.5 + L_N$
        - $212.5 = L_N$
- $Step\ 4$
    - Determine mass fraction solute in exiting raffinate
    - From Solute balance:
        - $15 = x_N * L_N$
        - $15 = x_N * 212.5$
        - $0.071 = x_N$



- Step 5
    - Locate $\mathbf{L_N}$, $\mathbf{L_0}$, $\mathbf{V_1}$, and $\mathbf{V_{N+1}}$ on graph
    - Draw lines $\overline{\boldsymbol{V_1L_0}}$ and $\overline{\boldsymbol{V_{N+1}L_N}}$, their intersection is the Δ point.
- Step 6
    - Anchor your rule on the delta point
    - Mark off pairs of points for the operating line. Where the ruler crosses the Raffinate (left-hand) boundary of the phase boundary you will record $\mathbf{x_c}$ as $\mathbf{x}$. Where the ruler crosses the right-hand (Extract) boundary record $\mathbf{x_c}$ as $\mathbf{y}$.
    - Choose enough points to generate a reasonably smooth operating curve.

Graph/reading op line



- **Operating Line**

x	y
0.33	0.40
0.23	0.30
0.16	0.20
0.11	0.10
0.07	0.018

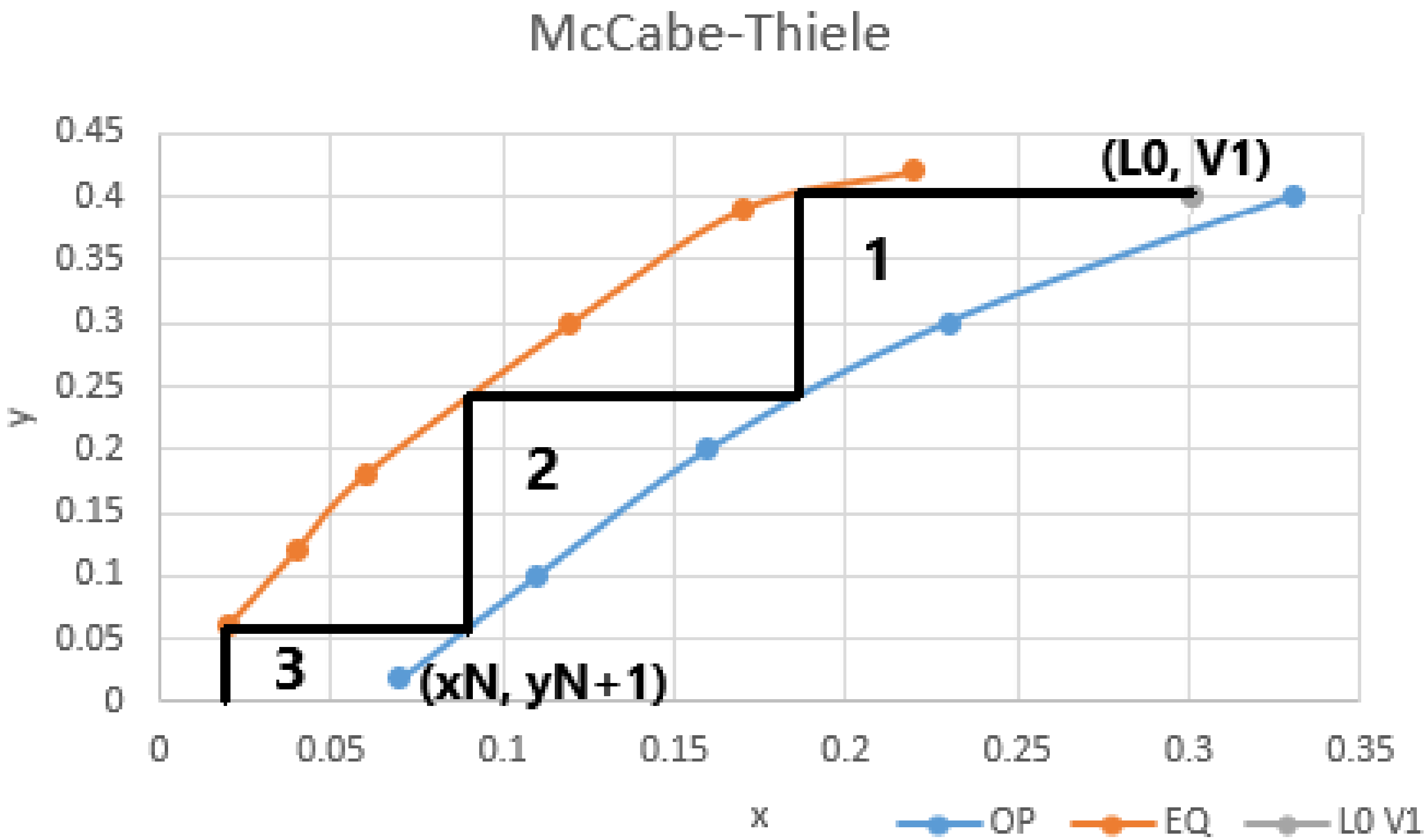
- Step 7
    - **Generate Equilibrium Curve**
    - From data given in problem statement, take the $\mathbf{x_c}$ value for Raffinate as **x** and the $\mathbf{x_c}$ value for the Extract as **y**

x (xC)	y (xC)
0.22	0.42
0.17	0.39
0.12	0.30
0.06	0.18
0.04	0.12
0.02	0.06

- Step 8
    - Plot Equilibrium and Operating Curves
    - Mark the point $(\boldsymbol{L_0}, \boldsymbol{V_1}) = (\mathbf{0.30}, \mathbf{0.40})$
    - Step off stages from $(\boldsymbol{L_0}, \boldsymbol{V_1})$ until you reach $\boldsymbol{L_N}$ which is when $\boldsymbol{x} = \mathbf{0.071}$

This separation requires 3 stages

graph



McCabe-Thiele
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
y
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
x
(L0, V1)
1
2
3
(xN, yN+1)
OP
EQ
L0 V1

2. **(50 pts)** Benzene will be stripped from a valuable oil by countercurrent contact with air in a tower packed with 1.0" plastic Pall rings. The contaminated oil (composition 96 mole % oil and 4 mole % benzene) will enter the tower at 2500 mol/hr and 95% of the entering benzene is to be removed. The flow rate of the incoming pure air will be 40,000 mol/hr. The density and viscosity of the dilute oil/benzene solution are well approximated by the properties of pure oil. The vapor phase behaves ideally. The tower will operate isothermally at 25 C and at a total pressure of 1 atm. The tower diameter shall be determined to give ΔP/ft of packing equal to 80% that of ΔP_{flood}/ft. The equilibrium curve at these conditions is **y = 0.10 x**. You may regard the operating line as being straight. Calculation of the flooding velocity should be based on flow rates at the **top** of the tower, where they are largest. The density of the vapor at the top of the column can be approximated as that of air.

Data:

Air:

MW = 28.9

Benzene:

MW = 78.11

Sc = 1.76 in air

Sc = 3500 in oil

Oil:

MW = 106

ρ = 0.83 g/cm^3

ν = 2.84 cSt

μ = 2.36 cP

Packing:

Table included in attachments

Gas Constant = 0.73024 ft^3 atm / (R lbmol)   R is degrees Rankine

Gas Constant = 82.05745 cm^3 atm / (K mol)

2.2 pounds = 1 kg

a. What is the required diameter for the tower? Use the following graph and the correlation ΔP_{flood}/ft = 0.115 $F_P^{0.7}$ inches water column per foot of packing

b. What height of packing is required? Base your solution on y – y*. Use the correlations from lecture to determine H_x and H_y.

$$H_{Oy} = H_y + m\,\frac{V}{L}\,H_x$$

c. What are N_x and N_y?

Solution: **1 hour basis**

Problem states that 95% of the Benzene will be removed from the liquid stream:

Moles Benzene in the Liquid stream at the top, a: = 0.04 * 2500 = 100 moles benzene

Moles oil (constant throughout the tower), 0.96 * 2500 = 2400 moles oil

Moles Benzene in the liquid stream at the bottom, b: = (1 - 0.95) * 100 = 5.0 moles benezene

Mole fraction of Benzene in the liquid stream at the bottom:

$$x_b = \frac{5.0}{5.0 + 2400} = 0.00208$$

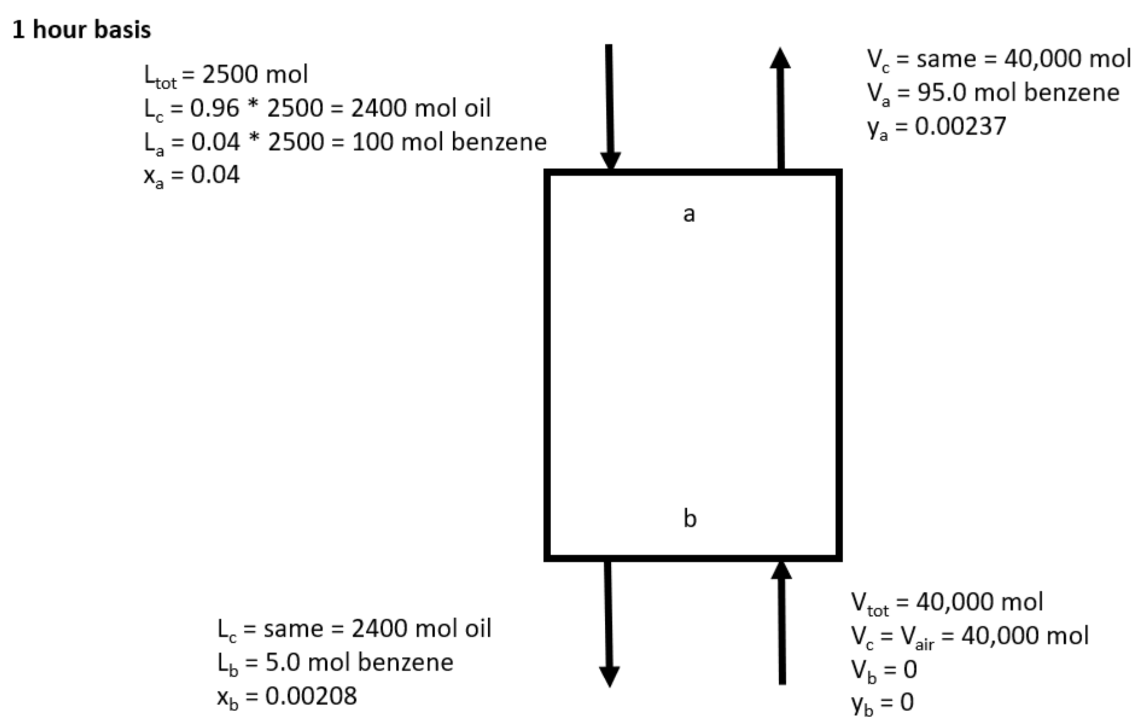
Moles of benzene in the vapor stream at top:

100 moles entering with liquid at the top + 0 moles entering with vapor at the bottom – 5.0 moles exiting with liquid at the bottom = **95.0 moles benzene exiting with vapor at the top**

Mole fraction benzene in exiting vapor:

$$y_a = \frac{95.0}{95.0 + 40{,}000} = 0.00237$$

Exam 02 Problem 02



a. Now we use the flooding correlation given:

$$\Delta P_{flood} = 0.115 * F_P^{0.7}$$

From the data table given we see that for 1" plastic Pall Rings **F_p = 55** and **f_p = 1.36**

$$\boldsymbol{\Delta P_{flood} = 0.115 * 55^{0.7} = 1.90 \frac{" H20}{ft}}$$

We are instructed to work at 80% of the flooding pressure drop, therefore we will use 1.52" water per foot of packing. (We can use the 1.5" curve directly…)

To use the attached chart we must calculate $\frac{G_x}{G_y}\sqrt{\frac{\rho_y}{\rho_x}}$

As indicated in the problem statement, we will evaluate this at the top of the tower where flooding is the most likely to happen.

Although we do not know the cross-sectional are required to calculate **G_x** or **G_y**, the ratio of them is the same as the ratio of the mass flows of the liquid and vapor.

Mass flow of vapor:

At top = 40,000 mol air/hr * 28.9 g / mol + 95.0 mol benzene/hr * 78.11 g /mol = 1163.4 kg/hr = $\boldsymbol{S * G_y}$

Mass flow of liquid:

At top = 2400 mol oil/hr * 106 g/mol + 100 mol benzene/hr * 78.11 g/mol = 262.2 kg/hr = $\boldsymbol{S * G_x}$

$$\frac{G_x}{G_y} = \frac{S * G_x}{S * G_y} = \frac{262.1}{1163.4} = 0.2254$$

The density of the liquid can be approximated as the density of oil,

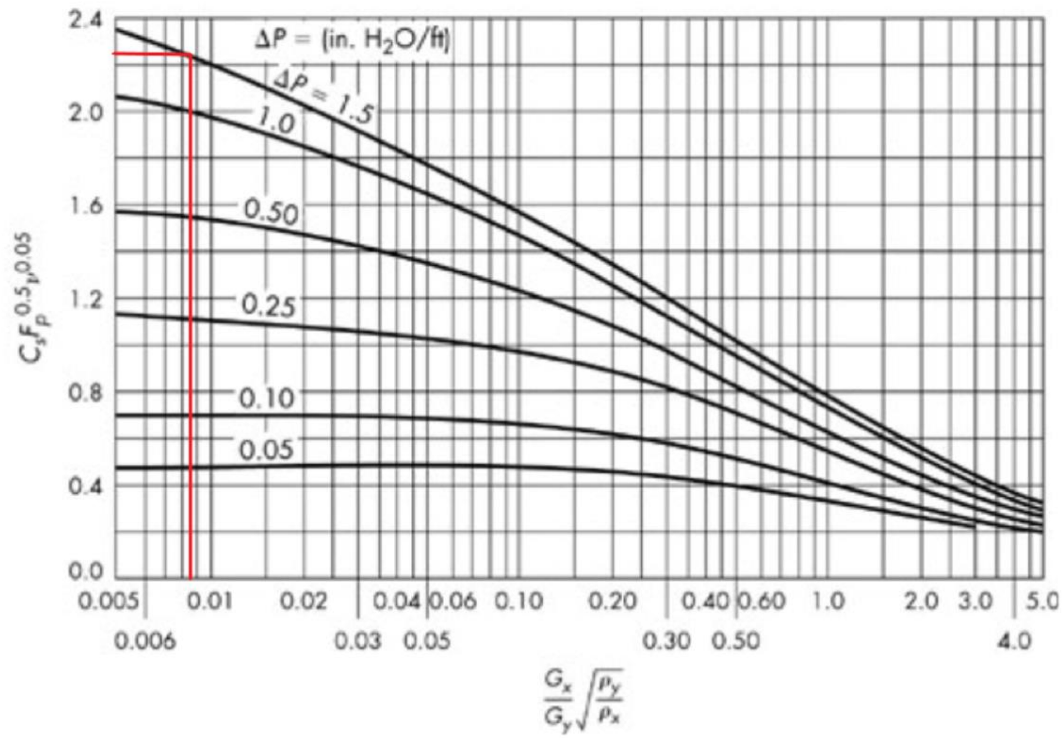
$$\rho_x = 0.83\ g/cm^3$$

The density of the vapor can be obtained using the ideal gas law:

$$\rho_y = \frac{MW\ P}{RT} = \frac{28.9 \frac{g}{mol}\ 1\ atm}{82.05745 \frac{cm^3 atm}{K\ mol}\ 298.15\ K} = 0.00118 \frac{g}{cm^3}$$

Now:

$$\frac{G_x}{G_y}\sqrt{\frac{\rho_y}{\rho_x}} = 0.2254\sqrt{\frac{0.00118}{0.83}} = 0.0085$$



From graph we can read from the line for line 1.5" water/ft that

$$C_s F_p^{0.5} \nu^{0.05} = 2.25$$

$$C_s\, 55^{\,0.5}\, 2.84^{\,0.05} = 2.25$$

$$C_s = 0.288$$

By definition:

$$C_s = u_0 \sqrt{\frac{\rho_y}{\rho_x - \rho_y}}$$

$$0.288 = u_0 \sqrt{\frac{0.00118}{0.83 - 0.00118}} = 0.0377\, u_0$$

$$u_0 = 7.64\, ft/s$$

Calculate volumetric vapor flow:

$$1163.4\, \frac{kg}{hr} * \frac{1000\, g}{kg} * \frac{cm^3}{0.00118\, g} * \frac{hr}{3600\, s} * \left(\frac{in}{2.54\, cm}\right)^3 * \left(\frac{ft}{12\, in}\right)^3 = 9.67\, \frac{ft^3}{s}$$

The required area is:

$$Area = \frac{volumetric\ flow}{linear\ velocity} = \frac{9.67\ \frac{ft^3}{s}}{7.64\ ft/s} = 1.27\ ft^2 = \frac{\pi\ D^2}{4}$$

$$D = 1.27\ ft$$

b) Now that we have the cross-sectional area we can calculate **G**$_x$ and **G**$_y$ independently:

Mass flow of vapor:

1163.4 kg/hr * (2.2 lb/kg) = 2559.5 lb/hr = $S * G_y$

Mass flow of liquid:

262.1 kg/hr * (2.2 lb/kg) = 576.6 lb/hr = $S * G_x$

$$G_x = \frac{576.6\ \frac{lb}{hr}}{1.27\ ft^2} = 454.0\ \frac{lb}{ft^2\ hr}$$

$$G_y = \frac{2559.5\ \frac{lb}{hr}}{1.27\ ft^2} = 2015.4\ \frac{lb}{ft^2\ hr}$$

We can now calculate the height of the transfer unit:

$$H_x = 0.9\ ft\left(\frac{G_x/\mu}{1500\ \frac{lb}{ft^2\ hr}\Big/0.891\ cP}\right)^{0.3}\left(\frac{S_c}{381}\right)^{0.5}\frac{1}{f_p}$$

$$H_x = 0.9\ ft\left(\frac{454.0\ \frac{lb}{ft^2\ hr}\Big/2.36\ cP}{1500\ \frac{lb}{ft^2\ hr}\Big/0.891\ cP}\right)^{0.3}\left(\frac{3500}{381}\right)^{0.5}\frac{1}{1.36} = 1.05\ ft$$

$$H_y = 1.4\ ft\left(\frac{G_y}{500\ \frac{lb}{ft^2\ hr}}\right)^{0.3}\left(\frac{1500\ \frac{lb}{ft^2\ hr}}{G_x}\right)^{0.4}\left(\frac{S_c}{0.66}\right)^{0.5}\frac{1}{f_p}$$

$$H_y = 1.4\ ft\left(\frac{2015.4}{500\ \frac{lb}{ft^2\ hr}}\right)^{0.3}\left(\frac{1500\ \frac{lb}{ft^2\ hr}}{454.0}\right)^{0.4}\left(\frac{1.76}{0.66}\right)^{0.5}\frac{1}{1.36} = 4.12\ ft$$

Now we can calculate height of overall transfer unit:

$$\boldsymbol{H_{Oy} = H_y + m\,\frac{V}{L} * H_x}$$

From problem statement we know that y = 0.10 x and therefore m = 0.10

V and L are molar flow rates in this expression

$$V/_L = \frac{40{,}000 + 95}{2500} = 16.04$$

Now:

$$\boldsymbol{H_{Oy} = 4.12\,ft + 0.10 * 16.04 * 1.05\,ft = 5.80\,ft}$$

Now we will calculate the number of transfer units:

$$\boldsymbol{N_{Oy} = \frac{y_b - y_a}{\overline{\overline{(y - y^*)_{lm}}}}}$$

$y_a = 0.00237$

$y_b = 0$

$y_a^* = 0.10 * x_a = 0.10 * 0.04 = 0.004$

$y_b^* = 0.10 * x_b = 0.10 * 0.00208 = 0.000208$

$y_b - y_a = 0 - 0.00237 = -0.00237$

$y_a - y_a^* = 0.00237 - 0.004 = -0.00163$

$y_b - y_b^* = 0 - 0.000208 = -0.000208$

$$\overline{\boldsymbol{(y - y^*)_{lm}}} = \frac{(y_a - y_a^*) - (y_b - y_b^*)}{\ln\frac{y_a - y_a^*}{y_b - y_b^*}} = \frac{-0.00163 - (-0.000208)}{\ln\frac{-0.00163}{-0.000208}} = \frac{-0.00142}{2.05880} = -0.000690$$

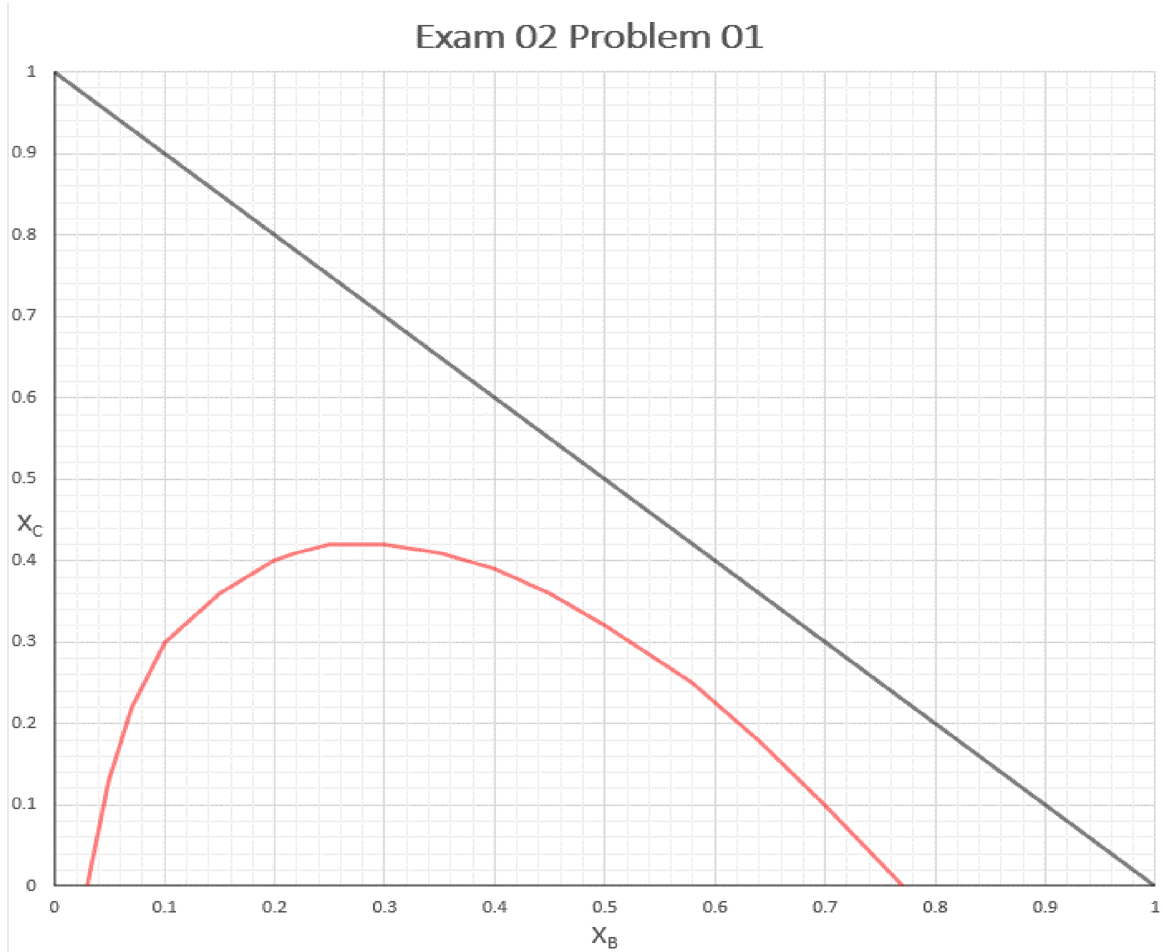
$$\boldsymbol{N_{Oy} = \frac{y_b - y_a}{\overline{\overline{(y - y^*)_{lm}}}} = \frac{-0.00237}{-0.000690} = 3.44}$$

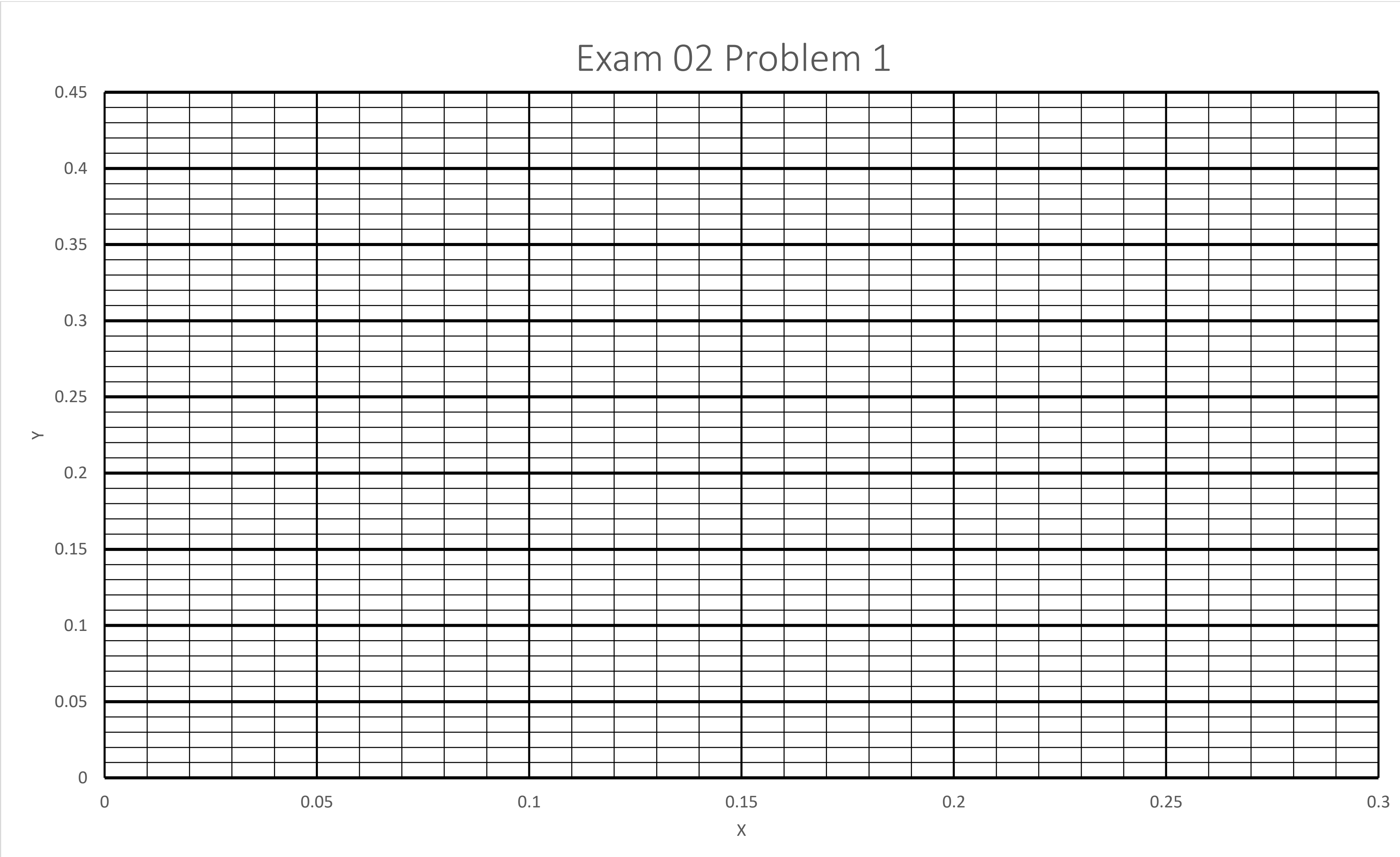
The required height of the packing can be calculated as:

$$\boldsymbol{Z_t = H_{Oy} * N_{Oy} = 5.80\,ft * 3.44 = 20.0\,ft}$$

c) $\boldsymbol{Z_t = H_x * N_x = H_y * N_y = H_{Oy} * N_{Oy} = 20\,ft}$

$\boldsymbol{N_x = 19.0}$ $\boldsymbol{N_y = 4.9}$

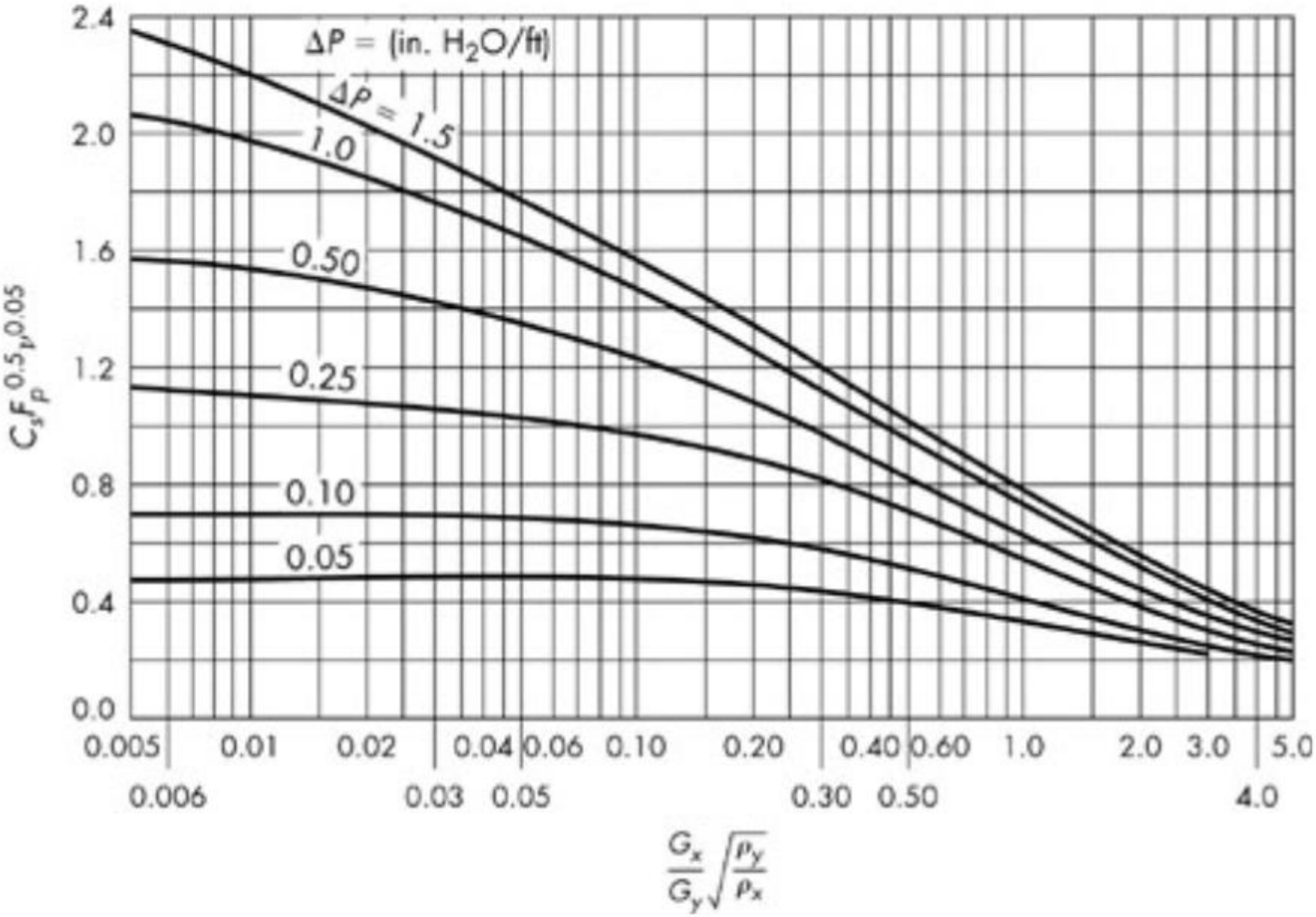

Exam 02 Problem 01
1
0.9
0.8
0.7
0.6
0.5
XC
0.4
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
XB



Exam 02 Problem 1
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
Y
0
0.05
0.1
0.15
0.2
0.25
0.3
X

TABLE 18.1
Characteristics of dumped tower packings[12,15b,27]

Type	Material	Nominal size, in.	Bulk density,† lb/ft^3	Total area,† ft^2/ft^3	Porosity ε	Packing factors‡ F_p	f_p
Raschig rings	Ceramic	$\frac{1}{2}$	55	112	0.64	580	1.52§
		1	42	58	0.74	155	1.36§
		$1\frac{1}{2}$	43	37	0.73	95	1.0
		2	41	28	0.74	65	0.92§
Pall rings	Metal	1	30	63	0.94	56	1.54
		$1\frac{1}{2}$	24	39	0.95	40	1.36
		2	22	31	0.96	27	1.09
	Plastic	1	5.5	63	0.90	55	1.36
		$1\frac{1}{2}$	4.8	39	0.91	40	1.18
Berl saddles	Ceramic	$\frac{1}{2}$	54	142	0.62	240	1.58§
		1	45	76	0.68	110	1.36§
		$1\frac{1}{2}$	40	46	0.71	65	1.07§
Intalox saddles	Ceramic	$\frac{1}{2}$	46	190	0.71	200	2.27
		1	42	78	0.73	92	1.54
		$1\frac{1}{2}$	39	59	0.76	52	1.18
		2	38	36	0.76	40	1.0
		3	36	28	0.79	22	0.64



ΔP = (in. H2O/ft)
ΔP = 1.5
1.0
0.50
0.25
0.10
0.05
2.4
2.0
1.6
1.2
0.8
0.4
0.0
$C_s F_p^{0.5} \nu^{0.05}$
0.005
0.006
0.01
0.02
0.03
0.04
0.05
0.06
0.10
0.20
0.30
0.40
0.50
0.60
1.0
2.0
3.0
4.0
5.0
$\frac{G_x}{G_y}\sqrt{\frac{\rho_y}{\rho_x}}$