Maximum Achievable Capacity in Airborne MIMO Communications with Arbitrary Alignments of Linear Transceiver Antenna Arrays

Weifeng Su, *Member, IEEE,* John D. Matyjas, *Senior Member, IEEE,* Michael J. Gans, *Fellow, IEEE,* and Stella Batalama, *Senior Member, IEEE*

***Abstract*—In this paper, the capacity of airborne multiple-input-multiple-output (MIMO) wireless communication systems with arbitrary alignments of linear transmit and receive antenna arrays is systematically analyzed and the maximum achievable capacity is determined. Based on a general three-dimensional (3D) airborne MIMO communication model, we are able to approximate the airborne MIMO capacity as a function of the transmit and receive antenna array geometry in the 3D space. The capacity approximation is asymptotically tight as the distance between the transmit and receive antenna arrays large compared to their size. Based on the asymptotically tight capacity approximation, we derive an upper bound as well as a lower bound of the airborne MIMO capacity. Interestingly, both the upper and lower bounds are achievable. We also derive a necessary and sufficient condition for airborne MIMO communication systems to achieve the capacity upper bound for any given 3D transceiver antenna array geometry. The necessary and sufficient condition allows us to properly select the system parameters and design airborne MIMO communication systems that reach the best possible performance in terms of system capacity. We prove that when the distance between the transmit and receive antenna arrays is within a certain range, there exists a set of system parameter values (e.g. antenna element separation) for which the capacity of the MIMO communication system achieves the theoretical upper bound and this capacity value is larger than the average capacity of the corresponding conventional MIMO communication system under Rayleigh fading. Finally, we prove that the airborne MIMO capacity converges to the capacity lower bound when the distance between the transmit and receive antenna arrays goes to infinity. Extensive numerical studies included in this paper illustrate and validate our theoretical developments.**

***Index Terms*—Airborne multiple-input-multiple-output (MIMO) communications, capacity, lower bound, upper bound, free-space MIMO communications, Rayleigh fading.**

Manuscript received November 8, 2012; revised April 19, 2013; accepted September 4, 2013. The associate editor coordinating the review of this paper and approving it for publication was C.-B. Chae.

This work was supported in part by the U.S. National Research Council (NRC) Research Fellowship Program and by the U.S. Air Force Research Laboratory (AFRL) under Grants FA87501110015 and FA87501110016. The findings and conclusions expressed in this material are those of the authors and do not necessarily reflect the views of NRC and AFRL. Security clearance approval for public release (reference number: 88ABW-2011-5507).

W. Su and S. Batalama are with the Department of Electrical Engineering, State University of New York (SUNY) at Buffalo, Buffalo, NY 14260, USA (e-mail: {weifeng, batalama}@buffalo.edu).

J. D. Matyjas and M. J. Gans are with the Air Force Research Laboratory/RIT, Rome, NY 13441, USA (e-mail: {John.Matyjas, Michael.Gans}@us.af.mil).

Digital Object Identifier 10.1109/TWC.2013.101613.121746

I. Introduction

MULTIPLE-INPUT-multiple-output (MIMO) wireless communications can provide substantial capacity increase in rich scattering and reflection environments compared to conventional single-input-single-output (SISO) systems [1]–[4]. In such environments, often characterized as Rayleigh fading, wireless channel between each transmit antenna and each receive antenna exhibits random fading which leads to non-singular MIMO channels that allow decomposition into multiple equivalent independent channels over which multiple independent data steams are transmitted. More specifically, the capacity of MIMO systems under Rayleigh fading increases linearly with the number of transmit antennas provided that the number of receive antennas is not less than that of transmit antennas [3], [4]. Bell Laboratory Layered Space-Time (BLAST) communication systems [5] verified the MIMO theory and revealed that the capacity of the MIMO architecture is indeed significantly greater than that of the SISO architecture. Since then, extensive work has been carried out to analyze MIMO communication systems and design practical space-time codes and modulation schemes to achieve the potential MIMO capacity (see, for example, [6]–[12] and the references therein).

MIMO techniques have been widely used for ground or near-ground wireless cellular and local area networks such as 4G LTE and WiFi networks which operate in rich scattering environments. The feasibility of applying the MIMO concept to airborne ad-hoc networks was studied in [13], where aircrafts or unmanned-ariel-vehicles (UAV) communicate with each other through multiple antennas carried within each aircraft. Key challenges in airborne or free-space MIMO wireless communications are: (i) the absence of rich scattering and reflections; and (ii) the fact that the link between each transmit antenna and each receive antenna is essentially a line-of-sight Gaussian channel. Consequently, airborne MIMO channels may be highly correlated in which case they induce a singular MIMO channel matrix, and thus may not offer the promising capacity increase compared to the conventional gound/near-ground MIMO wireless communications. In [13], the capacity of airborne MIMO channels between two F-35 jet airplanes (each equipped with 12 antenna elements) was evaluated, and it was shown that when the distance between the two airplanes is within a certain range, the airborne MIMO channels do provide significant capacity increase compared to the single-antenna case. In fact, it was shown that, in some cases, the capacity may even exceed the conventional ergodic Rayleigh MIMO capacity. When the distance is long, the capacity of the airborne MIMO channel between two airplanes decreases dramatically. We note that in [14], an experiment of free-space near-ground MIMO link was performed in a parking lot at Bell Labs in Crawford Hill, New Jersey. Link capacity was measured and showed dependence on antenna array geometry, distance, and electric field polarization.

In this paper, we systematically analyze the capacity of airborne MIMO wireless communication systems with linear transmit and receive antenna arrays and arbitrary three-dimensional (3D) alignments (a 3D model is necessary for two flying aircrafts with transceiver antenna arrays). We develop a necessary and sufficient condition for the airborne MIMO systems to achieve maximum capacity for any given transceiver antenna array geometry in the 3D space. First, we develop a general three-dimensional model for airborne MIMO communications that is able to accommodate arbitrary alignments of the transmit and receive antenna arrays. Second, we approximate the airborne MIMO capacity as a function of the transmit and receive antenna array geometry. The capacity approximation is (asymptotically) tight when the distance between the transmit and receive antenna arrays is large compared to their size, which is true in practical airborne MIMO communication scenarios. Third, based on the asymptotically tight capacity approximation, we determine an upper and a lower bounds for the airborne MIMO capacity. Interestingly, both bounds are achievable. Fourth, we derive a necessary and sufficient condition for airborne MIMO communication systems to achieve the capacity upper bound. We note that the capacity of airborne MIMO channels was also investigated in [15]–[17] under the term of line-of-sight MIMO channels. In particular, [15] proposed a condition on optimal antenna element separation for the case where the transmitter and receiver antenna arrays are parallel, while [16], [17] considered the more general non-parallel/arbitrary 3D alignment case. The necessary and sufficient condition developed in this paper shows that the conditions presented in [15]–[17] are sufficient, but *not* necessary. In addition, the condition presented in [16], [17] for the selection of system parameters (e.g. antenna element separation) to reach the best possible capacity value for any given transceiver antenna array geometry in the 3D space is true *only when* the zenith angle of the linear transmit and receive antenna arrays is zero, i.e. the transmitter and receiver antenna arrays are aligned on the same plane which is a rare event to happen with two flying aircrafts as the transceiver antenna arrays are carried within two different aircrafts respectively. In this paper, we develop a necessary and sufficient condition on system parameters to achieve the best possible capacity for any given transceiver antenna array geometry in the 3D space. The necessary and sufficient condition allows us to design airborne MIMO communication systems by selecting proper system parameters to reach the best possible capacity and it is worth noting that the capacity value is larger than the average capacity of the corresponding conventional MIMO communication system under Rayleigh fading. Finally, we prove that the airborne MIMO capacity converges to the capacity lower bound when the distance between the transmit and receive antenna arrays goes to infinity. Extensive numerical studies are provided to illustrate and validate our theoretical developments.

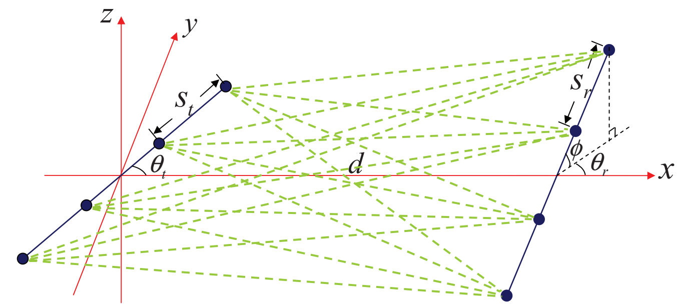


Fig. 1. System model of airborne MIMO wireless communications.

The rest of the paper is organized as follows. In Section II, we develop the general model for airborne MIMO wireless communication systems with arbitrary antenna array alignment. In Section III, we first determine the lower and upper bounds for the capacity of the airborne MIMO systems. Then we develop a necessary and sufficient condition for the airborne MIMO communication systems to achieve the capacity upper bound, and also discuss under what conditions the airborne MIMO capacity reaches the capacity lower bound. Extensive numerical studies are carried out in Section IV. Finally, some conclusions are drawn in Section V.

II. SYSTEM MODEL

We consider an airborne (free-space) MIMO wireless communication system with M_t transmit antennas and M_r receive antennas, as shown in Fig. 1. The transmit antennas are equally spaced with separation s_t meters while the receive antennas are equally spaced with separation s_r meters. Let us consider a three-dimensional Cartesian coordinate system and, without loss of generality, assume that the transmit antenna array is in the (x, y)-plane with its center located at the origin. The transmit antenna array is assumed to have an angle $\theta_t \in [0, 2\pi)$ from the x-axis. We further assume that the center of the receive antenna array is on the x-axis, and the distance between the centers of the two antenna arrays is d meters. The receive antenna array is assumed to be at an angle $\theta_r \in [0, 2\pi)$ from the (x, z)-plane and at a *zenith* angle $\phi \in [-\pi, \pi)$ from the (x, y)-plane.

With the above model, the coordinates of the m-th transmit antenna element, $m = 1, 2, \cdots, M_t$, can be given by

$$[x_t(m), y_t(m), z_t(m)] = \frac{(M_t - 2m + 1)s_t}{2}[\cos\theta_t,\, \sin\theta_t,\, 0], \tag{1}$$

and the coordinates of the n-th receive antenna element, $n = 1, 2, \cdots, M_r$, can be given by

$$[x_r(n), y_r(n), z_r(n)] = [d,\, 0,\, 0] + \frac{(M_r - 2n + 1)s_r}{2} \times [\cos\phi\cos\theta_r,\, \cos\phi\sin\theta_r,\, \sin\phi]. \tag{2}$$

In airborne or free-space environment, the channel coefficient between the m-th transmit antenna and the n-th receive antenna can be modeled as [18]

$$h_{m,n} = \frac{\lambda}{r_{m,n}} e^{-j2\pi \frac{r_{m,n}}{\lambda}}, \quad 1 \le m \le M_t, 1 \le n \le M_r, \quad (3)$$

where λ is the carrier wavelength and $r_{m,n}$ is the distance between the m-th transmit antenna and the n-th receive antenna, i.e. $r_{m,n} = \sqrt{(x_t - x_r)^2 + (y_t - y_r)^2 + (z_t - z_r)^2}$. From (1) and (2), the square of the distance $r_{m,n}$ can be calculated as

$$\begin{aligned} r_{m,n}^2 &= a_m^2 + b_n^2 - 2a_m b_n \cos\phi \cos(\theta_t - \theta_r) \\ &\quad - 2d(a_m \cos\theta_t - b_n \cos\phi \cos\theta_r) + d^2, \quad (4) \end{aligned}$$

where

$$\begin{aligned} a_m &= \frac{(M_t - 2m + 1)s_t}{2}, \quad m = 1, 2, \cdots, M_t, \quad (5) \\ b_n &= \frac{(M_r - 2n + 1)s_r}{2}, \quad n = 1, 2, \cdots, M_r. \quad (6) \end{aligned}$$

The received signal of the airborne MIMO communication system can be modeled as

$$Y = \sqrt{P}\, XH + N, \quad (7)$$

where $Y = [y_1\ y_2\ \cdots\ y_{M_r}]$ is the received signal vector, P is the transmission power, $X = [x_1\ x_2\ \cdots\ x_{M_t}]$ is the transmitted signal vector with a normalized average power constraint $E|x_m|^2 = \frac{1}{\sqrt{M_t}}$, $1 \le m \le M_t$, and H is the channel coefficient matrix given by

$$H = \begin{bmatrix} h_{1,1} & h_{1,2} & \cdots & h_{1,M_r} \\ h_{2,1} & h_{2,2} & \cdots & h_{2,M_r} \\ \vdots & \vdots & \ddots & \vdots \\ h_{M_t,1} & h_{M_t,2} & \cdots & h_{M_t,M_r} \end{bmatrix}. \quad (8)$$

In (7), $N = [n_1\ n_2\ \cdots\ n_{M_r}]$ is an additive noise vector whose elements are assumed to be i.i.d. complex Gaussian random variables with zero mean and variance $\mathcal{N}_0 = 3.52 \times 10^{-21}$ watts/Hz, (i.e. -174.5dBm at atmospheric temperature 255K [19]). Then, the signal-to-noise ratio (SNR) at the n-th receive antenna, $n = 1, 2, \cdots, M_r$, is

$$SNR_n = \frac{P\lambda^2}{M_t \mathcal{N}_0} \sum_{m=1}^{M_t} \frac{1}{r_{m,n}^2}. \quad (9)$$

The capacity of the airborne MIMO communications is [13]

$$C_{\text{airborne}} = \log_2 \left[\det \left(I_{M_t} + \frac{P}{M_t \mathcal{N}_0} HH^{\mathcal{H}} \right) \right], \quad (10)$$

where I_{M_t} is the identity matrix of size M_t and $\mathcal{H}$ stands for the Hermitian operator.

III. Maximum and Minimum Capacity of Airborne MIMO Communications

In this section, we first determine a lower bound and an upper bound on the capacity of the airborne MIMO system. Then, we develop a necessary and sufficient condition for such a system to achieve the capacity upper bound. Finally, we show that the capacity converges to the lower bound when the distance between the transmit and receive antenna arrays goes to infinity.

Let us denote

$$HH^{\mathcal{H}} \triangleq \begin{bmatrix} g_{1,1} & g_{1,2} & \cdots & g_{1,M_t} \\ g_{2,1} & g_{2,2} & \cdots & g_{2,M_t} \\ \vdots & \vdots & \ddots & \vdots \\ g_{M_t,1} & g_{M_t,2} & \cdots & g_{M_t,M_t} \end{bmatrix}, \quad (11)$$

then according to (3) and (8), each element $g_{k,l}$ of the above matrix in (11), $1 \le k, l \le M_t$, can be specified as

$$g_{k,l} = \sum_{n=1}^{M_r} h_{k,n} h_{l,n}^* = \sum_{n=1}^{M_r} \frac{\lambda^2}{r_{k,n} r_{l,n}} e^{-j2\pi \frac{r_{k,n} - r_{l,n}}{\lambda}}. \quad (12)$$

For simplicity of notation, for any m, n such that $1 \le m \le M_t$ and $1 \le n \le M_r$, we rewrite the square of the distance $r_{m,n}$ in (4) as

$$r_{m,n}^2 = A_{m,n} + B_{m,n} d + d^2, \quad (13)$$

where

$$\begin{aligned} A_{m,n} &\triangleq a_m^2 + b_n^2 - 2a_m b_n \cos\phi \cos(\theta_t - \theta_r), \quad (14) \\ B_{m,n} &\triangleq -2a_m \cos\theta_t + 2b_n \cos\phi \cos\theta_r. \quad (15) \end{aligned}$$

Then, for any k, l, n such that $1 \le k \le M_t$, $1 \le l \le M_t$ and $1 \le n \le M_r$, we have

$$\begin{aligned} r_{k,n} - r_{l,n} &= \sqrt{A_{k,n} + B_{k,n} d + d^2} - \sqrt{A_{l,n} + B_{l,n} d + d^2} \\ &= d \times F\left(\frac{1}{d}\right), \quad (16) \end{aligned}$$

where

$$F(x) \triangleq \sqrt{A_{k,n} x^2 + B_{k,n} x + 1} - \sqrt{A_{l,n} x^2 + B_{l,n} x + 1}. \quad (17)$$

If we consider the second-order Taylor expansion of $F(x)$, i.e.

$$F(x) \approx F(0) + F'(0)x + \frac{1}{2} F''(0) x^2, \quad (18)$$

where $F(0) = 0$ and

$$\begin{aligned} F'(0) &= \frac{1}{2}(B_{k,n} - B_{l,n}) = -(a_k - a_l)\cos\theta_t, \\ F''(0) &= (A_{k,n} - A_{l,n}) - \frac{1}{4}(B_{k,n}^2 - B_{l,n}^2) \\ &= (a_k^2 - a_l^2)\sin^2\theta_t - 2(a_k - a_l) b_n \cos\phi \sin\theta_t \sin\theta_r, \end{aligned}$$

then the approximation in (18) is tight when x is small. In other words, when the distance d is large (or equivalently $\frac{1}{d}$ is small), we may approximate $r_{k,n} - r_{l,n}$ in (16) as

$$\begin{aligned} r_{k,n} - r_{l,n} &\approx -(a_k - a_l)\cos\theta_t \\ &\quad + \frac{(a_k^2 - a_l^2)\sin^2\theta_t - 2(a_k - a_l) b_n \cos\phi \sin\theta_t \sin\theta_r}{2d}. \quad (19) \end{aligned}$$

Note that for any n, such that $1 \le n \le M_r$, b_n can be written as $b_n = b_1 + (n-1)s_r$, so the approximation in (19) can be rewritten as

$$\begin{aligned} r_{k,n} - r_{l,n} &\approx (r_{k,1} - r_{l,1}) \\ &\quad - \frac{(a_k - a_l)(n-1)s_r \cos\phi \sin\theta_t \sin\theta_r}{d}, \quad (20) \end{aligned}$$

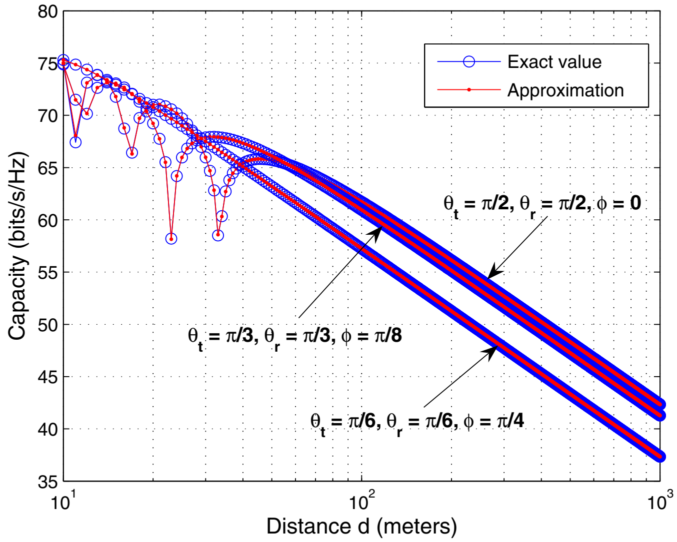


Fig. 2. Comparisons of the exact value and the approximation of the airborne MIMO capacity ($M_t = M_r = 2$).

for any $n = 1, 2, \cdots, M_r$. Therefore, for any k, l such that $1 \leq k \leq M_t$ and $1 \leq l \leq M_t$, each element $g_{k,l}$ of the matrix in (11) can be tightly approximated as follows:

$$\begin{aligned} g_{k,l} &\approx \sum_{n=1}^{M_r} \frac{\lambda^2}{r_{k,n} r_{l,n}} e^{-j2\pi \frac{r_{k,1}-r_{l,1}}{\lambda}} \\ &\qquad \times e^{j2\pi \frac{(a_k - a_l)(n-1)s_r \cos\phi \sin\theta_t \sin\theta_r}{\lambda d}} \\ &\approx \frac{\lambda^2}{d^2} e^{-j2\pi \frac{r_{k,1}-r_{l,1}}{\lambda}} \sum_{n=1}^{M_r} e^{j2\pi \frac{(l-k)(n-1)s_t s_r \cos\phi \sin\theta_t \sin\theta_r}{\lambda d}} \\ &\triangleq \tilde{g}_{k,l}, \end{aligned} \tag{21}$$

where the second approximation is due to the fact that $r_{k,n} \approx d$ and $r_{l,n} \approx d$ for large d.

If we denote

$$\tilde{G} \triangleq \begin{bmatrix} \tilde{g}_{1,1} & \tilde{g}_{1,2} & \cdots & \tilde{g}_{1,M_t} \\ \tilde{g}_{2,1} & \tilde{g}_{2,2} & \cdots & \tilde{g}_{2,M_t} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{g}_{M_t,1} & \tilde{g}_{M_t,2} & \cdots & \tilde{g}_{M_t,M_t} \end{bmatrix}, \tag{22}$$

then (11) and (21) imply that the capacity of the airborne MIMO communication system in (10) can be approximated by

$$\tilde{C}_{\text{airborne}} = \log_2 \left[\det \left(I_{M_t} + \frac{P}{M_t \mathcal{N}_0} \tilde{G} \right) \right]. \tag{23}$$

The approximation of the airborne MIMO capacity is tight for large d, i.e. when the distance between the transmit antenna array and the receive antenna array is large compared to the sizes of the antenna arrays, which is true in practical airborne MIMO communication scenarios. The tightness of the capacity approximation in shown in Fig. 2 for different transceiver antenna array alignments. In particular, we compare the exact value of the capacity C_{airborne} in (10) and the capacity approximation $\tilde{C}_{\text{airborne}}$ in (23) with varying distance d. In this example, the airborne MIMO system has two transmit antennas ($M_t = 2$) and two receive antennas ($M_r = 2$), and the antenna separation is $s_t = s_r = 1$m. We assume that the system operates at 10GHz band ($\lambda = 0.03$m) and the transmission power is $P = -10$dBm (watts/Hz). We consider three different transceiver antenna array alignments: (i) $\theta_t = \frac{\pi}{2}, \theta_r = \frac{\pi}{2}, \phi = 0$; (ii) $\theta_t = \frac{\pi}{3}, \theta_r = \frac{\pi}{3}, \phi = \frac{\pi}{8}$; and (iii) $\theta_t = \frac{\pi}{6}, \theta_r = \frac{\pi}{6}, \phi = \frac{\pi}{4}$. We observe that with distance varying, the eigenvalues of the corresponding airborne MIMO channel matrix in (11) change which may result in airborne MIMO capacity fluctuation. From the figure, we can see that the difference between the exact value and the approximation of the capacity is almost indistinguishable. In the rest of this paper, we will analyze the capacity based on the tight approximation $\tilde{C}_{\text{airborne}}$ in (23).

Let us denote the eigenvalues of the matrix $\tilde{G}$ in (22) as $\lambda_1, \lambda_2, \cdots, \lambda_{M_t}$. Then, the airborne MIMO capacity $\tilde{C}_{\text{airborne}}$ in (23) can be calculated as

$$\tilde{C}_{\text{airborne}} = \log_2 \prod_{i=1}^{M_t} \left(1 + \frac{P}{M_t \mathcal{N}_0} \lambda_i \right). \tag{24}$$

Since the eigenvalues $\lambda_1, \lambda_2, \cdots, \lambda_{M_t}$ are nonnegative and $\lambda_1 + \lambda_2 + \cdots + \lambda_{M_t} = trace(\tilde{G}) = M_t M_r \frac{\lambda^2}{d^2}$, then a lower bound of the airborne MIMO capacity can be determined as follows:

$$\begin{aligned} \tilde{C}_{\text{airborne}} &\geq \log_2 \left(1 + \frac{P}{M_t \mathcal{N}_0} \sum_{i=1}^{M_t} \lambda_i \right) \\ &= \log_2 \left(1 + M_r \frac{P\lambda^2}{\mathcal{N}_0 d^2} \right). \end{aligned} \tag{25}$$

On the other hand, since the geometric mean of nonnegative numbers is less than or equal to the arithmetic mean, we have

$$\begin{aligned} \tilde{C}_{\text{airborne}} &\leq \log_2 \left[\frac{1}{M_t} \sum_{i=1}^{M_t} \left(1 + \frac{P}{M_t \mathcal{N}_0} \lambda_i \right) \right]^{M_t} \\ &= M_t \log_2 \left(1 + \frac{M_r}{M_t} \frac{P\lambda^2}{\mathcal{N}_0 d^2} \right). \end{aligned} \tag{26}$$

The above upper bound is valid for arbitrary M_t and M_r. When $M_r < M_t$, a sharper upper bound can be obtained by rewriting the MIMO capacity as $C_{\text{airborne}} = \log_2 \left[\det \left(I_{M_r} + \frac{P}{M_t \mathcal{N}_0} H^{\mathcal{H}} H \right) \right]$. Then, following similar reasoning as above, an upper bound can be obtained as

$$\tilde{C}_{\text{airborne}} \leq M_r \log_2 \left(1 + \frac{P\lambda^2}{\mathcal{N}_0 d^2} \right). \tag{27}$$

By combining (25), (26) and (27), the capacity of airborne MIMO wireless communication systems can be bounded as follows:

$$\begin{aligned} \log_2 \left(1 + M_r \frac{P\lambda^2}{\mathcal{N}_0 d^2} \right) &\leq \tilde{C}_{\text{airborne}} \\ &\leq \min(M_t, M_r) \log_2 \left(1 + \frac{\max(M_t, M_r)}{M_t} \frac{P\lambda^2}{\mathcal{N}_0 d^2} \right). \end{aligned} \tag{28}$$

The airborne MIMO capacity upper bound in (28) is achievable. In the following section, we identify conditions under which the airborne MIMO systems reach the above capacity upper and lower bounds.

A. Maximum Achievable Capacity

Theorem 1 below shows a necessary and sufficient condition for airborne MIMO communication systems to achieve the capacity upper bound in (28). The necessary and sufficient condition reveals the optimal alignment of the transceiver antenna arrays to achieve the maximum capacity.

Theorem 1: An airborne MIMO communication system achieves the maximum capacity, i.e.

$$\tilde{C}_{\text{airborne}} = \min(M_t, M_r)\log_2\left(1+\frac{\max(M_t,M_r)}{M_t}\frac{P\lambda^2}{\mathcal{N}_0 d^2}\right), \tag{29}$$

if and only if the following two conditions are satisfied:

(i) There exists a non-zero integer p such that

$$s_t s_r \cos\phi \sin\theta_t \sin\theta_r = \frac{p\lambda d}{\max(M_t, M_r)}. \tag{30}$$

(ii) For any $m = 1, 2, \cdots, \min(M_t, M_r)-1$, there does not exist any integer q_m such that

$$s_t s_r \cos\phi \sin\theta_t \sin\theta_r = \frac{q_m \lambda d}{m}. \tag{31}$$

Proof: First, we will prove the theorem for the case where $M_t \le M_r$. Since the matrix $\tilde{G}$ in (22) is positive semi-definite, the matrix $I_{M_t} + \frac{P}{M_t\mathcal{N}_0}\tilde{G}$ is positive definite. According to the Hadamard inequality ([24], p.477), we have

$$\det\left(I_{M_t} + \frac{P}{M_t\mathcal{N}_0}\tilde{G}\right) \le \prod_{i=1}^{M_t}\left(1+\frac{P}{M_t\mathcal{N}_0}\tilde{g}_{i,i}\right). \tag{32}$$

In (32), the equality holds if and only if $I_{M_t} + \frac{P}{M_t\mathcal{N}_0}\tilde{G}$ is diagonal [24], which happens only when the matrix $\tilde{G}$ is diagonal. From (21), we know that $\tilde{g}_{i,i} = M_r\frac{\lambda^2}{d^2}$ for any $i = 1, 2, \cdots, M_t$. Thus, we conclude that the airborne MIMO communication system achieves the maximum capacity

$$\tilde{C}_{\text{airborne}} = M_t \log_2\left(1+\frac{M_r}{M_t}\frac{P\lambda^2}{\mathcal{N}_0 d^2}\right) \tag{33}$$

i.e. (32) holds with equality, if and only if the matrix $\tilde{G}$ is diagonal.

From (21), we can see that for any k and l, $1 \le k \ne l \le M_t$, if $e^{j2\pi\frac{(l-k)s_t s_r \cos\phi\sin\theta_t\sin\theta_r}{\lambda d}} \ne 1$, then

$$\tilde{g}_{k,l} = \frac{\lambda^2}{d^2}e^{-j2\pi\frac{r_{k,1}-r_{l,1}}{\lambda}}\frac{1-e^{j2\pi\frac{(l-k)M_r s_t s_r\cos\phi\sin\theta_t\sin\theta_r}{\lambda d}}}{1-e^{j2\pi\frac{(l-k)s_t s_r\cos\phi\sin\theta_t\sin\theta_r}{\lambda d}}}. \tag{34}$$

Thus, when $e^{j2\pi\frac{(l-k)M_r s_t s_r\cos\phi\sin\theta_t\sin\theta_r}{\lambda d}} = 1$, the off-diagonal element $\tilde{g}_{k,l} = 0$. Therefore, we conclude that the matrix $\tilde{G}$ is diagonal if and only if for any k and l, $1 \le k \ne l \le M_t$,

$$e^{j2\pi\frac{(l-k)M_r s_t s_r\cos\phi\sin\theta_t\sin\theta_r}{\lambda d}} = 1, \tag{35}$$

$$e^{j2\pi\frac{(l-k)s_t s_r\cos\phi\sin\theta_t\sin\theta_r}{\lambda d}} \ne 1. \tag{36}$$

It is not difficult to check that the requirement in (35) is satisfied if and only if there exists a non-zero integer p such that

$$s_t s_r \cos\phi\sin\theta_t\sin\theta_r = \frac{p\lambda d}{M_r}, \tag{37}$$

which is the condition (i) of the theorem. Similarly, the constraint in (36) is equivalent to the condition (ii) of the theorem, i.e. for any $m = 1, 2, \cdots, M_t - 1$, there does not exist any integer q_m such that

$$s_t s_r\cos\phi\sin\theta_t\sin\theta_r = \frac{q_m\lambda d}{m}. \tag{38}$$

So far, we proved the theorem for the case of $M_t \le M_r$.

When $M_t > M_r$, we may rewrite the MIMO capacity as

$$C_{\text{airborne}} = \log_2\left[\det\left(I_{M_r} + \frac{P}{M_t\mathcal{N}_0}H^{\mathcal{H}}H\right)\right]. \tag{39}$$

Then, following the above discussion we can similarly prove that

$$\tilde{C}_{\text{airborne}} \le M_r\log_2\left(1+\frac{P\lambda^2}{\mathcal{N}_0 d^2}\right), \tag{40}$$

and the equality is achieved if and only if for any k and l, $1 \le k \ne l \le M_r$,

$$e^{j2\pi\frac{(l-k)M_t s_t s_r\cos\phi\sin\theta_t\sin\theta_r}{\lambda d}} = 1, \tag{41}$$

$$e^{j2\pi\frac{(l-k)s_t s_r\cos\phi\sin\theta_t\sin\theta_r}{\lambda d}} \ne 1. \tag{42}$$

We can see that the requirement in (41) is satisfied if and only if there exists a non-zero integer p such that

$$s_t s_r\cos\phi\sin\theta_t\sin\theta_r = \frac{p\lambda d}{M_t}, \tag{43}$$

which is the condition (i) of the theorem, while the constraint in (42) is satisfied when for any $m = 1, 2, \cdots, M_r - 1$, there does not exist any integer q_m such that

$$s_t s_r\cos\phi\sin\theta_t\sin\theta_r = \frac{q_m\lambda d}{m}, \tag{44}$$

which is the condition (ii) of the theorem. Therefore, we have the results in the theorem for the case of $M_t > M_r$, which completes the proof. □

We note that [16], [17] presented a condition on optimal antenna element separation to achieve the best possible line-of-sight MIMO capacity for any given transceiver antenna array geometry in the 3D space. In terms of the notation used in this paper, the condition of [16], [17] is given by

$$s_t s_r\sin\theta_t\sin\theta_r = \frac{\lambda d}{\max(M_t, M_r)}. \tag{45}$$

However, expression (30) of Theorem 1 indicates that the antenna element separation condition in (45) is optimal *only when* the zenith angle of the transceiver antenna arrays is zero ($\phi = 0$), i.e. when the transmitter and receiver antenna arrays are aligned on the same plane. When the zenith angle of the transceiver antenna arrays is not zero ($\phi \ne 0$), the condition in (45) is not optimal. For example, when $M_t = M_r = 2$, $\phi = \frac{\pi}{3}$ and $\theta_t = \theta_r = \frac{\pi}{2}$, the antenna element separation condition in (45) becomes $s_t s_r = \frac{\lambda d}{2}$, and the corresponding MIMO capacity is

$$\begin{aligned}\tilde{C}_0 &= \log_2\left\{\det\left(I_2 + \frac{P\lambda^2}{2\mathcal{N}_0 d^2}\right.\right.\\ &\quad\times\left.\left.\begin{bmatrix} 2 & (1+j)e^{-j2\pi\frac{r_{1,1}-r_{2,1}}{\lambda}} \\ (1-j)e^{-j2\pi\frac{r_{2,1}-r_{1,1}}{\lambda}} & 2\end{bmatrix}\right)\right\}\\ &= \log_2\left\{1+2\frac{P\lambda^2}{\mathcal{N}_0 d^2}+\frac{1}{2}\left(\frac{P\lambda^2}{\mathcal{N}_0 d^2}\right)^2\right\}.\end{aligned} \tag{46}$$

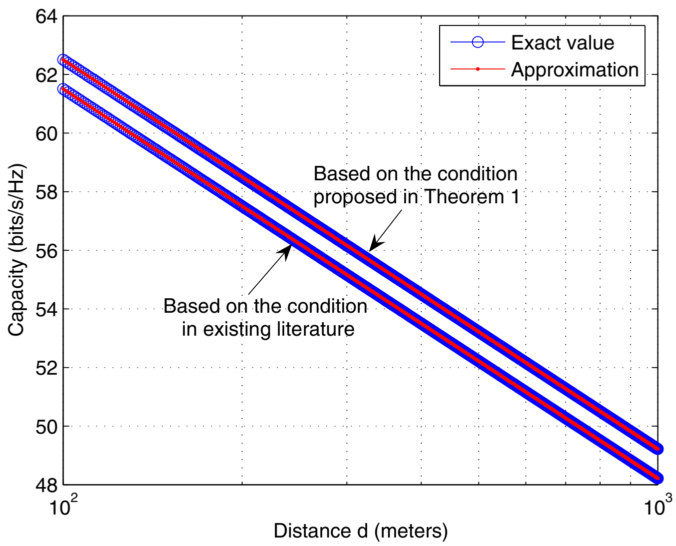


Fig. 3. Capacity of airborne MIMO systems with different antenna element separations ($\theta_t = \theta_r = \frac{\pi}{2}$, $\phi = \frac{\pi}{3}$, and $M_t = M_r = 2$).

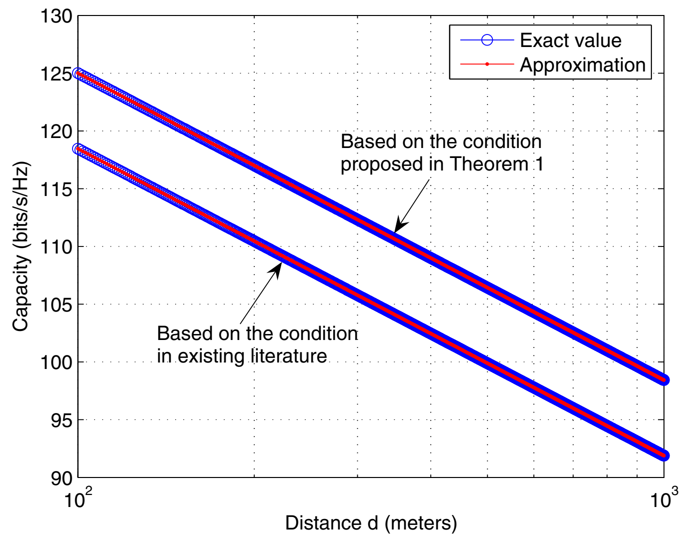


Fig. 4. Capacity of airborne MIMO systems with different antenna element separations ($\theta_t = \theta_r = \frac{\pi}{2}$, $\phi = \frac{\pi}{3}$, and $M_t = M_r = 4$).

However, based on Theorem 1, the optimal antenna element separation is $s_t s_r = \frac{\lambda d}{2\cos\phi} = \lambda d$ (or $p\lambda d$ for any odd integer p), and the corresponding MIMO capacity is

$$\tilde{C}_{\text{airborne}} = \log_2 \left[1 + \frac{P\lambda^2}{\mathcal{N}_0 d^2}\right]^2, \tag{47}$$

which is the maximum achievable capacity. We can see that $\tilde{C}_0 < \tilde{C}_{\text{airborne}}$, i.e. the capacity resulting from the element separation condition in (45) is less than the maximum achievable capacity. In Fig. 3, we plot the resulting airborne MIMO capacity based on the antenna element separation presented in [16], [17] and the optimal antenna separation we propose in this paper (Theorem 1). For each case, we plot both the exact and the approximated values which are indistinguishable. In particular, we assume that the system operates at 10GHz band ($\lambda = 0.03$m) and the transmission power is $P = -10$dBm (watts/Hz). We observe that when the zenith angle between the transceiver antenna arrays is not zero ($\phi = \frac{\pi}{3}$ in Fig. 3), the resulting MIMO capacity based on the condition presented in [16], [17] (i.e. $s_t s_r = \frac{\lambda d}{2}$) does not achieve the maximum capacity given by Theorem 1 (the corresponding optimal antenna element separation should be $s_t s_r = \lambda d$ in this case). The performance gap increases as the sizes of the antenna arrays increase, which is shown in Fig. 4 where we perform the same study as in Fig. 3 for $M_t = M_r = 4$.

We can also show that when the transmitter and receiver antenna arrays are aligned on the same plane, i.e. the zenith angle of the transceiver antenna arrays is zero ($\phi = 0$), the optimal antenna separation condition presented in [16], [17] is sufficient, *but not necessary*. For example, when $\phi = 0$, and $M_t = M_r = 3$, the antenna separation condition in [16], [17] becomes

$$s_t s_r \sin\theta_t \sin\theta_r = \frac{\lambda d}{3}. \tag{48}$$

The condition in (48) is sufficient to achieve the maximum MIMO capacity value. In fact, based on Theorem 1 (for $\phi = 0$, and $M_t = M_r = 3$), the following condition

$$s_t s_r \sin\theta_t \sin\theta_r = \frac{2\lambda d}{3}, \tag{49}$$

which is different from (48), also achieves the maximum MIMO capacity value.

In summary, in this section we proposed a necessary and sufficient condition (Theorem 1) to achieve the best possible capacity for any given 3D geometry described by θ_t, θ_r, and ϕ of two linear transceiver antenna arrays on board two flying aircrafts where θ_t, θ_r, ϕ depend on the flying patterns of the aircrafts. From an implementation point of view, Theorem 1 suggests, for example, that if we employ adaptively activated antenna elements that adjust their separation according to (29)–(31) we can achieve the best possible capacity for any given value of θ_t, θ_r and ϕ. For example, if $M_t = M_r$, then a simple sufficient (but not necessary condition) to achieve maximum capacity is

$$s_t s_r \cos\phi \sin\theta_t \sin\theta_r = \frac{\lambda d}{M_t}, \tag{50}$$

which can be obtained from Theorem 1 for $p = 1$.

B. Minimum Capacity

In the following, we show, in Theorem 2, that the lower bound of the airborne MIMO capacity in (28) is reached when the distance between the transmit antenna array and the receive antenna array goes to infinity. The proof of Theorem 2 needs the following lemma.

Lemma 1: For any integer $K > 0$ and real number $t \geq 0$, define $T_K(t)$ as follow

$$T_K(t) \triangleq \begin{bmatrix} t & 1 & \cdots & 1 \\ 1 & t & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & t \end{bmatrix}_{K\times K}. \tag{51}$$

Then the determinant of the matrix $T_K(t)$ is

$$\det\left(T_K(t)\right) = (t-1)^{K-1}(t+K-1). \qquad (52)$$

Proof: We prove the lemma by induction. When $K=2$, it is easy to calculate that $\det\left(T_2(t)\right) = t^2-1$, which validates the expression in (52). Let us assume that the expression in (52) is valid for any integer $K \geq 2$, then the determinant of $T_{K+1}(t)$ can be calculated as follows

$$\begin{aligned}
&\det\left(T_{K+1}(t)\right) \\
&= \det\left(\frac{t-1}{t}\underbrace{\begin{bmatrix} \frac{t^2}{t-1} & 0 & 0 & \cdots & 0 \\ 0 & t+1 & 1 & \cdots & 1 \\ 0 & 1 & t+1 & \cdots & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 1 & 1 & \cdots & t+1 \end{bmatrix}}_{(K+1)\times(K+1)}\right) \\
&= \left(\frac{t-1}{t}\right)^{K+1}\frac{t^2}{t-1}\det\left(T_K(t+1)\right) \\
&= \left(\frac{t-1}{t}\right)^{K+1}\frac{t^2}{t-1}t^{K-1}(t+K) \\
&= (t-1)^{(K+1)-1}\left[t+(K+1)-1\right],
\end{aligned}$$

in which the first equality results from the diagonalizing the matrix $T_{K+1}(t)$ such that all off-diagonal entries in the first row and in the first column are zeros. The above derivations show that the expression in (52) is also valid for $K+1$. By induction, we have the result in Lemma 1. □

Theorem 2: When the distance between the transmit antenna array and the receive antenna array goes to infinity, the capacity of the airborne MIMO communication system converges to its minimum, i.e.

$$\tilde{C}_{\text{airborne}} \to \log_2\left(1 + M_r\frac{P\lambda^2}{\mathcal{N}_0 d^2}\right), \quad (d\to\infty). \qquad (53)$$

Proof: We prove the theorem first for the case when $M_t \leq M_r$. In this case, when the distance d between the transceiver antenna arrays goes to infinity, then (21) implies that for any k and l, $1 \leq k \neq l \leq M_t$, the term $\frac{(l-k)(n-1)s_t s_r \cos\phi \sin\theta_t \sin\theta_r}{\lambda d}$ goes to zero, i.e,

$$e^{j2\pi\frac{(l-k)(n-1)s_t s_r \cos\phi\sin\theta_t\sin\theta_r}{\lambda d}} \to 1.$$

Thus, $g_{k,l}$ in (21) converges to the following

$$g_{k,l} \to M_r\frac{\lambda^2}{d^2}e^{-j2\pi\frac{r_{k,1}-r_{l,1}}{\lambda}}, \quad \text{as } d\to\infty. \qquad (54)$$

So, when $d\to\infty$, we have

$$\det\left[I_{M_t} + \frac{P}{M_t\mathcal{N}_0}HH^{\mathcal{H}}\right] \to \det\left[I_{M_t} + \frac{M_r}{M_t}\frac{P\lambda^2}{\mathcal{N}_0 d^2}\mathbf{1}_{M_t}\right],$$

where $\mathbf{1}_{M_t}$ is an all-one matrix of size $M_t\times M_t$. Let $\gamma \triangleq \frac{M_r}{M_t}\frac{P\lambda^2}{\mathcal{N}_0 d^2}$, according to Lemma 1, we have

$$\begin{aligned}
\det\left[I_{M_t} + \frac{P}{M_t\mathcal{N}_0}HH^{\mathcal{H}}\right] &\to \det\left[\gamma T_{M_t}\left(1+\frac{1}{\gamma}\right)\right] \\
&= \gamma^{M_t}\left(\frac{1}{\gamma}\right)^{M_t-1}\left(\frac{1}{\gamma}+M_t\right) \\
&= 1+\gamma M_t.
\end{aligned}$$

Thus, from (10), the airborne MIMO capacity converges to $\log_2\left(1+\gamma M_t\right) = \log_2\left(1 + M_r\frac{P\lambda^2}{\mathcal{N}_0 d^2}\right)$ which is the lower bound of the airborne MIMO capacity shown in (28).

When $M_t > M_r$, we can rewrite the airborne MIMO capacity as

$$C_{\text{airborne}} = \log_2\left[\det\left(I_{M_r} + \frac{P}{M_t\mathcal{N}_0}H^{\mathcal{H}}H\right)\right], \qquad (55)$$

and similarly denote $H^{\mathcal{H}}H \triangleq \{g_{k,l} : 1 \leq k,l \leq M_r\}$, where each element of the matrix can be specified as

$$\begin{aligned}
g_{k,l} &= \sum_{n=1}^{M_t} h^*_{n,k}h_{n,l} \\
&\approx \frac{\lambda^2}{d^2}e^{-j2\pi\frac{r_{1,k}-r_{1,l}}{\lambda}} \\
&\quad\times\sum_{n=1}^{M_t} e^{j2\pi\frac{(l-k)(n-1)s_t s_r\cos\phi\sin\theta_t\sin\theta_r}{\lambda d}}. \qquad (56)
\end{aligned}$$

When the distance d goes to infinity, then for any k and l, $1 \leq k \neq l \leq M_r$, we have

$$g_{k,l} \to M_t\frac{\lambda^2}{d^2}e^{-j2\pi\frac{r_{1,k}-r_{1,l}}{\lambda}}, \quad \text{as } d\to\infty. \qquad (57)$$

In this case, let $\gamma \triangleq \frac{P\lambda^2}{\mathcal{N}_0 d^2}$, according to Lemma 1, we have

$$\begin{aligned}
\det\left[I_{M_r} + \frac{P}{M_t\mathcal{N}_0}H^{\mathcal{H}}H\right] &\to \det\left[\gamma T_{M_r}\left(1+\frac{1}{\gamma}\right)\right] \\
&= \gamma^{M_r}\left(\frac{1}{\gamma}\right)^{M_r-1}\left(\frac{1}{\gamma}+M_r\right) \\
&= 1+\gamma M_r.
\end{aligned}$$

Therefore, for $M_t > M_r$, the airborne MIMO capacity converges to $\log_2\left(1+\gamma M_r\right) = \log_2\left(1 + M_r\frac{P\lambda^2}{\mathcal{N}_0 d^2}\right)$, which is again the minimum capacity in (53). □

Theorem 2 implies that when the distance between the transceiver antenna arrays goes to infinity, the resulting capacity is the same as the capacity of a system with M_r receive antennas and only one transmit antenna, i.e. long distances between transmit and receive antenna arrays make the receiver perceive a transmitter of one virtual transmit antenna. Equivalently, this can be viewed as a system with single input (i.e. one transmit antenna) and single output with a directional array of M_r antennas at the receiver side (i.e. one receiver output port). Moreover, we note that the condition in (53) of Theorem 2 for reaching minimum capacity is sufficient, but not necessary. The airborne MIMO communication system also reach its minimum capacity in other scenarios as shown in the following two examples.

- **Example 1:** When $\phi = \frac{\pi}{2}$, i.e. the transmit antenna array and the receive antenna array are perpendicular, the airborne MIMO communication system reaches its minimum capacity. In this case, for any k and l, $1 \leq k \neq l \leq M_t$, $g_{k,l}$ in (21) is

$$g_{k,l} = M_r \frac{\lambda^2}{d^2} e^{-j2\pi \frac{r_{k,1} - r_{l,1}}{\lambda}}.$$

Thus,

$$\begin{aligned} C_{\text{airborne}} &= \log_2 \det \left[I_{M_t} + \frac{P}{M_t \mathcal{N}_0} H H^{\mathcal{H}} \right] \\ &= \log_2 \det \left[\gamma T_{M_t} \left(1 + \frac{1}{\gamma} \right) \right], \end{aligned}$$

where matrix $T_{M_t}(\cdot)$ is defined in Lemma 1 and $\gamma = \frac{M_r}{M_t} \frac{P\lambda^2}{\mathcal{N}_0 d^2}$. Applying the determinant expression of Lemma 1, we obtain

$$C_{\text{airborne}} = \log_2 \left(1 + M_r \frac{P\lambda^2}{\mathcal{N}_0 d^2} \right),$$

which is the minimum airborne MIMO capacity. We note that when $\phi = \frac{\pi}{2}$, the capacity of the airborne MIMO system reaches its minimum capacity regardless of the values of the angles θ_t and θ_r.
- **Example 2:** When $\theta_t = 0$ or $\theta_r = 0$, i.e. the transmit antenna array points to the center of the receive antenna array or vice versa, the airborne MIMO communication system also reaches the minimum capacity. In this case, for any k and l, $1 \leq k \neq l \leq M_t$, $g_{k,l}$ in (21) is $g_{k,l} = M_r \frac{\lambda^2}{d^2} e^{-j2\pi \frac{r_{k,1} - r_{l,1}}{\lambda}}$. Thus, following the same discussion as in Example 1, we have $C_{\text{airborne}} = \log_2 \left(1 + M_r \frac{P\lambda^2}{\mathcal{N}_0 d^2} \right)$, which is the minimum airborne MIMO capacity. We note that when $\theta_t = 0$, the capacity of the airborne MIMO system reaches its minimum capacity regardless of the value of the angle θ_r. Similarly, when $\theta_r = 0$, the capacity is the same for any angle θ_t.

IV. Numerical Results and Comparisons

In this section, we provide numerical studies to illustrate the capacity of airborne MIMO communication systems with different transceiver antenna array alignments and different antenna element separations. We assume that the airborne MIMO communication system operates at a 10GHz band (i.e. $\lambda = 0.03$m). The transmit antenna separation s_t is within 0 and $s_{t,\max}$ and the receive antenna separation s_r is within 0 and $s_{r,\max}$, where $s_{t,\max} = s_{r,\max} = 1$m in the numerical studies. We consider three different alignments of transmit and receive antenna arrays: (i) $\theta_t = \frac{\pi}{2}, \theta_r = \frac{\pi}{2}, \phi = 0$; (ii) $\theta_t = \frac{\pi}{3}, \theta_r = \frac{\pi}{3}, \phi = \frac{\pi}{8}$; and (iii) $\theta_t = \frac{\pi}{6}, \theta_r = \frac{\pi}{6}, \phi = \frac{\pi}{4}$. To obtain better insight from the comparisons, we normalize the effect of path loss. That is, for any given distance d between the transmit and receive antenna arrays, we adjust the transmission power P such that the average SNR at each receive antenna is fixed at 10dB, i.e. $SNR_n = 10$dB.

In our studies, we also include the capacity of conventional near-ground MIMO communications with rich scattering and reflection environment where dynamic channels are typically

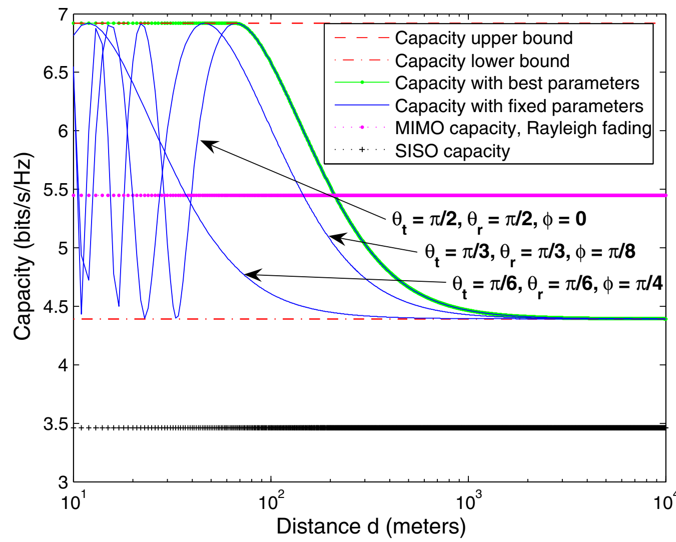


Fig. 5. Capacity of airborne MIMO communications, $M_t = M_r = 2$.

characterized as Rayleigh fading [1], [18], [20]. Assuming that the channels are independent complex Gaussian random variables with zero-mean and unit-variance, the average capacity of a Rayleigh fading MIMO communication system with M_t transmit antennas and M_r receive antennas is given by [20]–[23]

$$\begin{aligned} C_{\text{Rayleigh}} &= M_t \log_2 \left[1 + \rho \frac{M_r}{M_t} - \mathcal{F}(\rho) \right] \\ &\quad + M_r \log_2 \left[1 + \rho - \mathcal{F}(\rho) \right] \\ &\quad - \frac{M_t}{\rho \ln 2} \mathcal{F}(\rho), \end{aligned} \tag{58}$$

where

$$\mathcal{F}(\rho) = \frac{1 + \rho \frac{M_t + M_r}{M_t} - \sqrt{1 + 2\rho \frac{M_t + M_r}{M_t} + \rho^2 \frac{(M_t - M_r)^2}{M_t^2}}}{2} \tag{59}$$

and ρ is the average SNR at each receive antenna. When $M_t = M_r$, the function in (59) is reduced to $\mathcal{F}(\rho) = \frac{1 + 2\rho - \sqrt{1 + 4\rho}}{2}$ and the corresponding ergodic Rayleigh fading MIMO capacity is

$$\begin{aligned} C_{\text{Rayleigh}} &= M_t \log_2 \frac{1 + 2\rho + \sqrt{1 + 4\rho}}{2} \\ &\quad - \frac{M_t}{\ln 2} \left(1 + \frac{1 - \sqrt{1 + 4\rho}}{2\rho} \right) \\ &\approx M_t \log_2 (1 + \rho) - \frac{M_t}{\ln 2}, \end{aligned} \tag{60}$$

in which the capacity approximation is tight for enough high SNR ρ. Apparently, the ergodic Rayleigh fading MIMO capacity is less than the maximum airborne MIMO capacity which is $C_{\text{airborne}} = M_t \log_2 (1 + \rho)$ in this case. The difference between the Rayleigh fading MIMO capacity and the maximum airborne MIMO capacity increases with the number of antennas, which can be observed in the figures 5–7 for systems with $M_t = M_r = 2$, 3 and 4, respectively.

In Figs. 5–7, we plot the capacity of the airborne MIMO communication systems for $M_t = M_r = 2$, 3 and 4, respectively, with the transceiver antenna array alignments

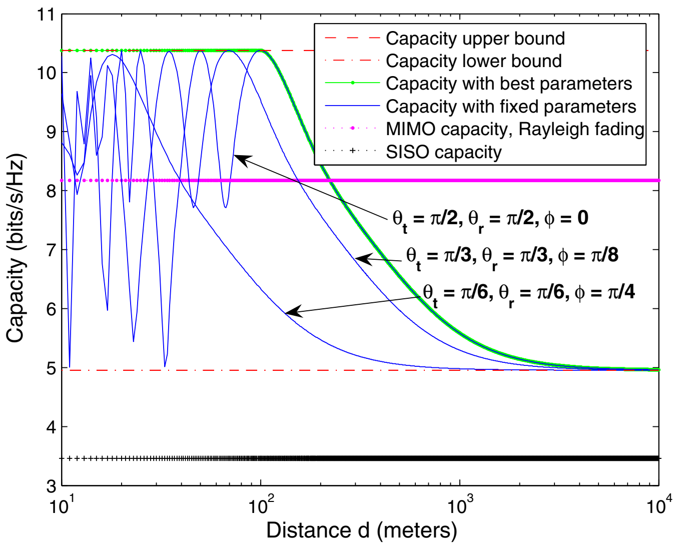


Fig. 6. Capacity of airborne MIMO communications, $M_t = M_r = 3$.

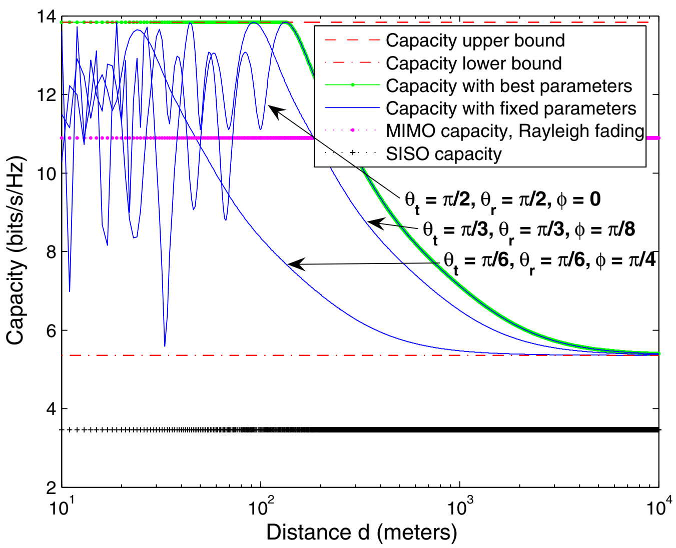


Fig. 7. Capacity of airborne MIMO communications, $M_t = M_r = 4$.

(i)-(iii). We compare the exact capacity (solid lines) with the capacity upper bound in (29) (dashed line) and the capacity lower bound in (53) (dotted dashed line). From the figures, we observe that for different alignments of the transmit and receive antenna arrays, the airborne MIMO capacity may reach its upper bound when the distance d is relatively small and it converges to its lower bound when the distance becomes large ($\to \infty$), which is consistent with the analytical result in Theorem 2. We also observe that the airborne MIMO capacity periodically reaches its lower bound even for small distance d with the fixed antenna parameters. However, based on Theorem 2, we can adjust the antenna element separation such as $s_t = s_r = \sqrt{\frac{\lambda d}{\cos\phi \sin\theta_t \sin\theta_r \max(M_t,M_r)}}$ to achieve the best possible capacity value. In the numerical studies, we set the antenna separation as $s_t = \min\left(\sqrt{\frac{\lambda d}{\cos\phi \sin\theta_t \sin\theta_r \max(M_t,M_r)}}, s_{t,\max}\right)$ and $s_r = \min\left(\sqrt{\frac{\lambda d}{\cos\phi \sin\theta_t \sin\theta_r \max(M_t,M_r)}}, s_{r,\max}\right)$ to take into account airborne aircraft space limitation. We can see that with the proper selection of antenna element separation, the airborne MIMO capacity can achieve its maximum value (solid line with '·') when the distance d is within a certain range which is determined by the antenna separation limits $s_{t,\max}$ and $s_{r,\max}$. Furthermore, when we compare the airborne MIMO capacity with the ergodic MIMO capacity with Rayleigh fading (shown by dotted lines with '·'), we can see that the airborne MIMO capacity may exceed the ergodic Rayleigh fading MIMO capacity when the distance d is within a certain range, while it is below the ergodic Rayleigh fading MIMO capacity when the distance is large. For comparison purposes, the airborne SISO capacity (dotted lines with '+') is also included in the figures. We see that the airborne SISO capacity is below the lower bound of the airborne MIMO capacity in each figure, which means that the airborne MIMO architecture guarantees larger capacity than its SISO counterpart.

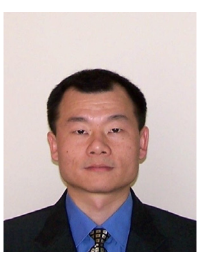
V. Conclusions

In this paper, we analyzed the capacity of airborne MIMO wireless communication systems with arbitrary linear antenna array alignment. With a general three-dimensional model and a second-order Taylor expansion, we developed an asymptotically tight approximation for the airborne MIMO capacity, which enables us to determine an upper bound and a lower bound of the capacity. Then, we derived a necessary and sufficient condition for the airborne MIMO communication system to achieve the capacity upper bound. The necessary and sufficient condition allows us to properly select the system parameters of the airborne MIMO communication system in order to achieve the best possible capacity. It turns out that the optimal antenna separation condition proposed in [15], [16], [17] is sufficient, but not necessary. Finally, we discussed airborne MIMO communication scenarios that reach the capacity lower bound. Extensive numerical studies validated our theoretical developments. When the distance between the transmit and receive antenna arrays is within a certain range, we are able to design airborne MIMO communication systems such that their capacity exceeds the average capacity of conventional MIMO communications over Rayleigh fading channels. When the distance is large, the capacity of an airborne MIMO communication system converges to its minimum value which is still larger than its SISO counterpart.

References

[1] A. Goldsmith, *Wireless Communications.* Cambridge University Press, 2005.
[2] E. Biglieri, R. Calderbank, A. Constantinides, A. Goldsmith, A. Paulraj, and H. V. Poor, *MIMO Wireless Communications.* Cambridge University Press, 2007.
[3] I. E. Telatar, "Capacity of multi-antenna Gaussian channels," AT&T Bell Labs., Tech. Rep., 1995.
[4] G. J. Foschini and M. J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Personal Commun.*, vol. 6, pp. 311-335, Mar. 1998.
[5] G. J. Foschini, "Layered space-time architecture for wireless communication in a fading environment when using multiple antennas," *Bell Labs Tech. J.*, vol. 1, no. 2, pp. 41–59, 1996.
[6] V. Tarokh, N. Seshadri, and A. R. Calderbank, "Space-time codes for high data rate wireless communication: performance criterion and code construction," *IEEE Trans. Inf. Theory*, vol. 44, no. 2, pp. 744-765, 1998.

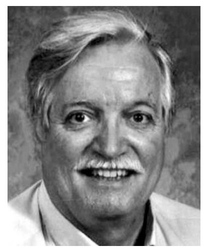
[7] S. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 16, no. 8, pp. 1451-1458, 1998.
[8] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-time block codes from orthogonal designs," *IEEE Trans. Inf. Theory*, vol. 45, pp. 1456-1467, July 1999.
[9] B. M. Hochwald and T. L. Marzetta, "Unitary space-time modulation for multiple-antenna communication in Rayleigh flat fading," *IEEE Trans. Inf. Theory*, vol. 46, no. 2, pp. 543–564, 2000.
[10] W. Su and X.-G. Xia, "Signal constellations for quasi-orthogonal space-time block codes with full diversity," *IEEE Trans. Inf. Theory*, vol. 50, no. 10, pp. 2331-2347, Oct. 2004.
[11] W. Su, X.-G. Xia, and K. J. R. Liu, "A systematic design of high-rate complex orthogonal space-time block codes," *IEEE Commun. Lett.*, vol. 8, no. 6, pp. 380–382, June 2004.
[12] W. Su, Z. Safar, and K. J. R. Liu, "Full-rate full-diversity space-frequency codes with optimum coding advantage," *IEEE Trans. Inf. Theory*, vol. 51, no. 1, pp. 229–249, Jan. 2005.
[13] M. J. Gans, "Aircraft free-space MIMO communications," in *Proc. 2009 Asilomar Conf. Signals, Syst., Comput.*, pp. 663–666.
[14] P. Kyritsi and D. Chizhik, "Capacity of multiple antenna systems in free space and above perfect ground," *IEEE Commun. Lett.*, vol. 6, no. 8, pp. 325–327, Aug. 2002.
[15] D. Gesbert, H. Bolcskei, D. A. Gore, and A. Paulraj, "Outdoor MIMO wireless channels: models and performance prediction," *IEEE Trans. Commun.*, vol. 50, no. 12, pp. 1926–1934, Dec. 2002.
[16] F. Bohagen, P. Orten, and G. E. Oien, "Design of optimal high-rank line-of-sight MIMO channels," *IEEE Trans. Wireless Commun.*, vol. 6, no. 4, pp. 1420–1425, Apr. 2007.
[17] F. Bohagen, P. Orten, and G. E. Oien, "On spherical vs. plane wave modeling of line-of-sight MIMO channels," *IEEE Trans. Commun.*, vol. 57, no. 3, pp. 841–849, Mar. 2009.
[18] T. Rappaport, *Wireless Communications: Principles and Practice*, 2nd ed. Prentice Hall, 2002.
[19] "Federal Standard 1037C," Ed. 1996, U.S. National Telecommunications and Information Administration.
[20] S. Verdu and S. Shamai, "Spectral efficiency of CDMA with random spreading," *IEEE Trans. Inf. Theory*, vol. 45, no. 2, pp. 622-640, Mar. 1999.
[21] P. Rapajic and D. Popescu, "Information capacity of a random signature multiple-input multiple-output channel," *IEEE Trans. Commun.*, vol. 48, no. 8, pp. 1245–1248, Aug. 2000.
[22] A. Lozano, "Capacity-approaching rate function for layered multi-antenna architectures," *IEEE Trans. Wireless Commun.*, vol. 2, no. 4, pp. 616–620, July 2003.
[23] M. J. Gans, "Channel capacity between antenna arrays—part I: sky noise dominates," *IEEE Trans. Commun.*, vol. 54, no. 9, pp. 1586–1592, Sept. 2006.
[24] R. A. Horn and C. R. Johnson, *Matrix Analysis*. Cambridge University Press, 1985.



Weifeng Su (M'03) received the Ph.D. degree in Electrical Engineering from the University of Delaware, Newark in 2002. He received his B.S. and Ph.D. degrees in Mathematics from Nankai University, Tianjin, China, in 1994 and 1999, respectively. His research interests span a broad range of areas from signal processing to wireless communications and networking, including space-time coding and modulation for MIMO wireless communications, MIMO-OFDM systems, and cooperative communications for wireless networks.

Dr. Su joined the Department of Electrical Engineering at the State University of New York (SUNY) at Buffalo in 2005, where he is currently an Associate Professor. From June 2002 to March 2005, he was a Postdoctoral Research Associate with the Department of Electrical and Computer Engineering and the Institute for Systems Research, University of Maryland, College Park. Dr. Su is the recipient of the 2010 IEEE International Conference on Communications (ICC) Best Paper Award. He received the Invention of the Year Award from the University of Maryland in 2005. Dr. Su has been serving as Editor for several IEEE Journals including IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS, IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY, and IEEE SIGNAL PROCESSING LETTERS. He also co-organized two Special Issues for IEEE journals in the field of cooperative communications and networking. Dr. Su co-authored the book *Cooperative Communications and Networking* published by Cambridge University Press in 2009.

John D. Matyjas (M'02, SM'13) received his Ph.D. degree in electrical engineering from the State University of New York at Buffalo in 2004. He has since been employed by the Air Force Research Laboratory in Rome, NY, serving as the Connectivity Core Technical Competency Lead. His research interests are in the areas of wireless multiple-access communications and networking, cognitive radio, statistical signal processing and optimization, and neural networks. Additionally, he serves as an adjunct faculty in the Department of Electrical Engineering at the State University of New York Institute of Technology at Utica/Rome. Dr. Matyjas is the recipient of the Mohawk Valley "Engineer of the Year" Award and, as a co-author, the 2010 IEEE International Conference on Communications (ICC) "Best Paper Award." He is a member of the IEEE Communications, Information Theory, Computational Intelligence, and Signal Processing Societies; chair of the IEEE Mohawk Valley Chapter Signal Processing Society; and a member of the Tau Beta Pi and Eta Kappa Nu engineering honor societies.



Michael J. Gans (S'60, M'66, SM'84, F'93) received the B.S. degree from the University of Notre Dame, Notre Dame, IN, in 1956, and the M. S. and Ph.D. degrees in electrical engineering from the University of California at Berkeley, in 1961 and 1965, respectively.

He has been on the staff of the Air Force Research Laboratory, Information Directorate, Rome, NY, since 2003. He had been at Bell Laboratories from 1966 to 2001. His primary technical areas include mobile radio, antennas, satellites, fiber optics, and infrared communications.

Stella Batalama (S'91, M'94, SM'13) received the Diploma degree in computer engineering and science (5-year program) from the University of Patras, Greece in 1989 and the Ph.D. degree in electrical engineering from the University of Virginia, Charlottesville, VA, in 1994.

In 1995 she joined the Department of Electrical Engineering, State University of New York at Buffalo, Buffalo, NY, where she is presently a Professor. From July 2009 to June 2011 she served as the Associate Dean for Research of the School of Engineering and Applied Sciences and since 2010, she is serving as the Chair of the Electrical Engineering Department. During the summers of 1997-2002 she was Visiting Faculty in the U.S. Air Force Research Laboratory (AFRL), Rome, NY. From Aug. 2003 to July 2004 she served as the Acting Director of the AFRL Center for Integrated Transmission and Exploitation (CITE), Rome NY.

Her research interests include small-sample-support adaptive filtering and receiver design, cooperative and cognitive communications and networks, covert communications and steganography, robust spread-spectrum communications and adaptive multiuser detection, compressive sampling. Dr. Batalama was an associate editor for the IEEE COMMUNICATIONS LETTERS (2000-2005) and the IEEE TRANSACTIONS ON COMMUNICATIONS (2002-2008).