Optimal Power Assignment for Minimizing the Average Total Transmission Power in Hybrid-ARQ Rayleigh Fading Links

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***Abstract*—We address the fundamental problem of identifying the optimal power assignment sequence for hybrid automatic-repeat-request (H-ARQ) communications over quasi-static Rayleigh fading channels. For any targeted H-ARQ link outage probability, we find the sequence of power values that minimizes the average total expended transmission power. We first derive a set of equations that describe the optimal transmission power assignment and enable its exact recursive calculation. To reduce calculation complexity, we also develop an approximation to the optimal power sequence that is close to the numerically calculated exact result. The newly founded power allocation solution reveals that conventional equal-power H-ARQ assignment is far from optimal. For example, for targeted outage probability of 10^{-3} with a maximum of two transmissions, the average total transmission power with the optimal assignment is 9 dB lower than the equal-power protocol. The difference in average total power cost grows further when the number of allowable retransmissions increases (for example, 11 dB gain with a cap of 5 transmissions) or the targeted outage probability decreases (27 dB gain with outage probability 10^{-5} and transmissions capped at 5). Interestingly, the optimal transmission power assignment sequence is neither increasing nor decreasing; its form depends on given total power budget and targeted outage performance levels. Extensive numerical and simulation results are presented to illustrate the theoretical development.**

***Index Terms*—Hybrid automatic-repeat-request (H-ARQ) protocol, optimum power allocation, outage probability, Rayleigh fading.**

I. INTRODUCTION

AUTOMATIC-repeat-request (ARQ) communication protocols, in which a receiver requests retransmission when a packet is not correctly received, are commonly used in data link control to enable reliable data packet transmissions [1]–[7]. In a basic/simplest ARQ protocol, a receiver decodes an information packet based only on the received signal in each transmission round [1], [2]. Advanced ARQ schemes, in which a receiver may decode an information packet by combining received signals from all previous transmission rounds, have been known as hybrid ARQ (H-ARQ) protocols [3]–[7]. Since the receiver needs to save previously received signals, H-ARQ communication protocols require more memory at the receiver side compared to the basic ARQ protocols. However, the performance of the H-ARQ protocols is substantially better than that of the basic ARQ protocols and the performance improvement is worth the memory increase at the receiver side [2], [6], [7], especially with today's cheapest and smallest memory chips.

In wireless links formed by wireless devices with limited power resources, power efficiency is a key research matter in the optimization of ARQ retransmission protocols [8]–[12]. In [8], a power control scheme was proposed for ARQ retransmissions in down-link cellular systems in order to minimize the total transmission power of multiple users where each user uses constant transmission power. In [9], the power efficiency of various ARQ protocols was discussed by taking into account the energy consumed by the transmitting and receiving electronic circuitry in ARQ retransmissions. Note that in both [8] and [9], the power efficiency of ARQ protocols was examined under the assumption of the same transmission power level in each retransmission round. In [10], the transmission power in each retransmission round was optimized for a variety of ARQ protocols by assuming that channel state information (CSI) is available at the transmitter side and CSI takes values from a prescribed finite set of values. In [11], by assuming that partial CSI is available, optimal transmission power in each retransmission round was determined for an H-ARQ protocol by a linear programming method that selects a power value from a set of discrete power levels. Recently in [12], without assuming CSI available at the transmitter side, an optimal power transmission strategy was identified for a basic ARQ protocol where the receiver decodes based only on the received signal in each transmission round. It was assumed that the channel changes independently in each retransmission round. A necessary and sufficient condition for the optimal transmission power sequence was found which indicates that power must be increasing in every retransmission. We note that this result is not valid to slowly fading channels. More recently in [13], without a priori CSI at the transmitter, the authors maximized the average transmission rate for an incremental redundancy H-ARQ protocol where the transmitter sends out different encoded redundant parity symbols in each retransmis-

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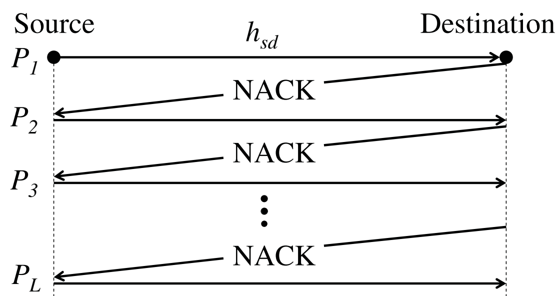

Fig. 1. Illustration of a hybrid-ARQ protocol with transmission power P_l in the lth (re-)transmission round, $1 \leq l \leq L$.

sion round. The average transmission rate maximization under optimal power assignment was also formulated and numerical results were presented for an incremental redundancy H-ARQ protocol with one maximum retransmission.

In this work, we consider advanced H-ARQ transmission protocols in which a destination node may decode an information packet by combining all received signals from previous (re-)transmission rounds to increase detection reliability. We assume that the source-destination channel experiences *quasi-static* Rayleigh fading, i.e. the channel does not change during retransmissions of the same information packet and it may change independently when transmitting a new information packet. Our goal is to find the optimal power assignment strategy that minimizes the average total transmission power for any given targeted outage probability. First, we derive a set of equations that describe the optimal transmission power values in H-ARQ retransmission rounds. Then, a simple recursive algorithm is developed to exactly calculate the optimal transmission power level for each retransmission round. Interestingly, it turns out that the optimal transmission power assignment sequence is neither increasing nor decreasing; its form depends on given total power budget and targeted outage performance levels. This is fundamentally different from the case in [12] that the optimal transmission power must be increasing in retransmissions in the fast fading scenario (i.e. the channel changes independently in each retransmission round). To reduce calculation complexity and obtain more insight understanding of the optimal power assignment strategy, we also develop an approximation to the optimal power sequence that is close to the numerically calculated exact result. The tight approximation shows that the optimal transmission power in each retransmission round is a function of P_1 (the transmission power in the first round) in a polynomial form. The optimal power assignment values also reveal that the conventional equal-power assignment (using the same transmission power in all retransmission rounds) is far from optimal. As an example, for a targeted outage probability of 10^{-3} and maximum number of transmissions $L = 2$, the average total transmission power based on the optimal power assignment is 9 dB less than that of using the common equal-power scheme. We also observe that the larger the maximum number of retransmissions allowed in the H-ARQ protocol or the lower the required outage probabilities, the more power savings the optimal power assignment strategy offers. Substantial numerical and simulation results are presented to illustrate the theoretical development.

The rest of the paper is organized as follows. In Section II, we review briefly the H-ARQ transmission scheme and formulate the power assignment optimization problem. In Section III, we find the optimal power assignment strategy for the H-ARQ protocol and present an exact recursive calculation algorithm. In Section IV, we develop a simple approximation of the optimal power assignment sequence and compare it with the exact calculation result. Numerical and simulation studies are carried out in Section V to compare the performance of the equal and optimal power assignment strategies. Finally, some conclusions are drawn in Section VI.

II. System Model and Problem Formulation

We consider an H-ARQ transmission protocol implemented between a source node and a destination node as illustrated in Fig. 1. Assume that L is the number of retransmission rounds allowed in the H-ARQ protocol. The H-ARQ transmission scheme operates as follows. First, the source transmits an information packet to the destination and the destination indicates success or failure of receiving the packet by feeding back a single bit of acknowledge (ACK) or negative-acknowledgement (NACK), respectively. The feedback channel is assumed error-free. Then, if a NACK is received by the source and the maximum number of retransmissions L is not reached, the source retransmits the packet at a potentially different transmission power to be determined/optimized. If an ACK is received by the source or the maximum retransmission number L is reached, the source begins transmission of a new information packet. In each retransmission round, the destination attempts to decode an information packet by combining received signals from all previous transmission rounds by the standard maximal-ratio-combining (MRC) technique [14]. If the destination still cannot decode an information packet after L (re)transmission rounds, then an outage is declared which means that the signal-to-noise ratio (SNR) of the combined received signals at the destination is below a required SNR.

The H-ARQ transmission scheme can be modeled as follows. With L maximum retransmission rounds allowed in the H-ARQ protocol, the base-band received signal $y_{sd,l}$ at the destination at the lth transmission round can be written as

$$y_{sd,l} = \sqrt{P_l}\ h_{sd}\ x_s + \eta_{sd,l}, \quad l = 1, 2, \cdots, L, \tag{1}$$

where x_s is the transmitted information symbol from the source, P_l is the transmission power used by the source at the lth transmission round, h_{sd} is the source-destination channel coefficient, and $\eta_{sd,l}$ is additive noise at the lth round. The channel coefficient h_{sd} is modeled as zero-mean complex Gaussian random variable with variance σ_{sd}^2. The channel is assumed to be *quasi-static*, i.e. the channel does not change during retransmissions of the same information packet and it may change independently when a new information packet is transmitted. The source-destination channel coefficient is assumed to be known at the receiver side, but unknown at the transmitter side. The additive noise contribution $\eta_{sd,l}$ is modeled as a zero-mean complex Gaussian random variable with variance $\mathcal{N}_0$.

At the destination side, the receiving node combines the received signals from all previous retransmission rounds and jointly decodes the information packet based on the MRC combining technique [14]. Note that the MRC combining is applied over base-band symbol-level signals in (1) before decoding an entire information packet. With the assumption that the channel does not change in retransmissions of the same information packet, the SNR of the combined signal at the destination at the lth $(1 \leq l \leq L)$ retransmission round can be given as [14], [15]

$$\gamma_{sd,l} = \frac{\sum_{i=1}^{l} P_i |h_{sd}|^2 |x_s|^2}{\mathcal{N}_0}. \tag{2}$$

Without loss of generality, let us assume the average power of the transmitted information symbol is 1, then we have $\gamma_{sd,l} = \frac{\sum_{i=1}^{l} P_i |h_{sd}|^2}{\mathcal{N}_0}$. Since $|h_{sd}|$ follows a Rayleigh distribution with mean zero and variance σ_{sd}^2, so for any targeted SNR γ_0, the probability of the event that the destination cannot decode correctly after l transmission rounds can be calculated as

$$p^{out,l} = \Pr\left[\gamma_{sd,l} < \gamma_0\right] = 1 - e^{-\frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 \sum_{i=1}^{l} P_i}}. \tag{3}$$

Set $p^{out,0} = 1$. Then, the probability that the H-ARQ protocol stops successfully at the lth, $1 \leq l < L$, transmission round is $p^{out,l-1} - p^{out,l}$, which means the destination cannot decode correctly at the $(l-1)$th round, but succeeds at the lth round.

Our goal is to find an optimal power assignment sequence $\mathbf{P} = [P_1, P_2, ..., P_L]$ for the H-ARQ protocol such that under a targeted outage probability p_0, the average total transmission power for the protocol to deliver an information packet is minimized. Since the probability that the protocol succeeds exactly at the lth $(1 \leq l \leq L-1)$ round is $p^{out,l-1} - p^{out,l}$ and the corresponding total transmission power is $P_1 + P_2 + \cdots + P_l$, so the average total transmission power of the H-ARQ protocol can be expressed as

$$\bar{P} = \sum_{l=1}^{L-1} \left(p^{out,l-1} - p^{out,l}\right) \sum_{i=1}^{l} P_i + p^{out,L-1} \sum_{i=1}^{L} P_i. \tag{4}$$

Note that the last term in (4) is due to the fact that the protocol stops retransmissions after the Lth round no matter whether decoding at the Lth round is successful or not. For the H-ARQ protocol with a targeted outage probability p_0, the problem of finding optimal power assignment can be formulated as follows:

$$\begin{aligned} \min \quad & \bar{P} \quad \text{with respect to } P_1, P_2, \cdots, P_L \geq 0 \\ \text{subject to} \quad & p^{out,L} \leq p_0 \end{aligned} \tag{5}$$

where $\bar{P}$ is specified in (4).

III. Optimal Transmission Power Assignment

In this section, we investigate the optimal power assignment strategy for the H-ARQ protocol to minimize the average total transmission power. We obtain a set of equations that describe the optimal transmission power values, and then develop a recursive algorithm to exactly calculate the optimal transmission power level for each retransmission round.

The average total transmission power in (4) can be rewritten by switching the summation order (between the indices l and i) as follows

$$\bar{P} = \sum_{i=1}^{L} P_i \left[\sum_{l=i}^{L-1} \left(p^{out,l-1} - p^{out,l}\right) + p^{out,L-1}\right] \tag{6}$$

where we first consider the summation by enumerating the index i from 1 to L, then consider the summation index l $(i \leq l \leq L-1)$. Since for each i, $\sum_{l=i}^{L-1} (p^{out,l-1} - p^{out,l}) = p^{out,i-1} - p^{out,L-1}$, so the average total transmission power can be represented as

$$\bar{P} = P_1 + \sum_{l=2}^{L} P_l \ p^{out,l-1}. \tag{7}$$

Moreover, the constraint in (5) means that with a targeted SNR γ_0, the outage probability of the H-ARQ protocol with L retransmissions should not be larger than the specified outage probability value p_0, i.e.

$$p^{out,L} = 1 - e^{-\frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 \sum_{i=1}^{L} P_i}} \leq p_0. \tag{8}$$

Denote $P_0 \triangleq \frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 \ln \frac{1}{1-p_0}}$, then the constraint is equivalent to

$$\sum_{l=1}^{L} P_l \geq P_0 \tag{9}$$

and the optimization problem in (5) can be further specified as

$$\begin{aligned} \min_{P_1,\cdots,P_L \geq 0} \quad & \bar{P} = P_1 + \sum_{l=2}^{L} P_l \left[1 - e^{-\frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 \sum_{i=1}^{l-1} P_i}}\right] \\ \text{subject to} \quad & \sum_{l=1}^{L} P_l \geq P_0. \end{aligned} \tag{10}$$

Next, we relax temporarily the non-negative condition on P_l, $l = 1, 2, \cdots, L$, and consider the sum-power constraint in (10) with equality in order to consider a Lagrange multiplier method to solve the optimization problem. We will prove later that the obtained solution is indeed optimal under the constraint in (10) and it satisfies the non-negative condition. Let us form a Lagrangian objective function as

$$\mathcal{L}\left(\mathbf{P}, \lambda\right) = P_1 + \sum_{l=2}^{L} P_l \left[1 - e^{-\frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 \sum_{i=1}^{l-1} P_i}}\right] + \lambda \left[\sum_{l=1}^{L} P_l - P_0\right]. \tag{11}$$

Taking the derivative of $\mathcal{L}\left(\mathbf{P}, \lambda\right)$ with respect to λ and setting it equal to zero, we have the power constraint as $\sum_{l=1}^{L} P_l - P_0 = 0$. The derivatives of $\mathcal{L}\left(\mathbf{P}, \lambda\right)$ with respect to P_k are

$$\frac{\partial \mathcal{L}}{\partial P_1} = 1 - \sum_{l=2}^{L} \frac{P_l \gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 \left(\sum_{i=1}^{l-1} P_i\right)^2} \, e^{-\frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 \sum_{i=1}^{l-1} P_i}} + \lambda, \tag{12}$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial P_k} = & \left[1 - e^{-\frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 \sum_{i=1}^{k-1} P_i}}\right] \\ & - \sum_{l=k+1}^{L} \frac{P_l \gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 \left(\sum_{i=1}^{l-1} P_i\right)^2} \, e^{-\frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 \sum_{i=1}^{l-1} P_i}} + \lambda, \\ & \qquad k = 2, 3, \cdots, L-1, \end{aligned} \tag{13}$$

$$\frac{\partial \mathcal{L}}{\partial P_L} = 1 - e^{-\frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 \sum_{i=1}^{L-1} P_i}} + \lambda. \tag{14}$$

Based on $\frac{\partial \mathcal{L}}{\partial P_1} = 0$ and $\frac{\partial \mathcal{L}}{\partial P_2} = 0$, we have

$$\frac{\partial \mathcal{L}}{\partial P_1} - \frac{\partial \mathcal{L}}{\partial P_2} = \left[1 - \frac{P_2 \gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 P_1^2}\right] e^{-\frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 P_1}} = 0, \quad (15)$$

which implies

$$P_2 = \frac{\sigma_{sd}^2 P_1^2}{\gamma_0 \mathcal{N}_0}. \quad (16)$$

For any $k = 3, 4, \cdots, L$, according to $\frac{\partial \mathcal{L}}{\partial P_{k-1}} = 0$ and $\frac{\partial \mathcal{L}}{\partial P_k} = 0$, we have

$$\begin{aligned} 0 &= \frac{\partial \mathcal{L}}{\partial P_{k-1}} - \frac{\partial \mathcal{L}}{\partial P_k} \\ &= -\, e^{-\frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 \sum_{i=1}^{k-2} P_i}} \\ &\quad + \left[1 - \frac{P_k \gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 \left(\sum_{i=1}^{k-1} P_i\right)^2}\right] e^{-\frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 \sum_{i=1}^{k-1} P_i}} \end{aligned} \quad (17)$$

which means

$$P_k = \frac{\sigma_{sd}^2 (\sum_{i=1}^{k-1} P_i)^2}{\gamma_0 \mathcal{N}_0} \left[1 - e^{-\frac{P_{k-1} \gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 \left(\sum_{i=1}^{k-1} P_i\right)\left(\sum_{i=1}^{k-2} P_i\right)}}\right], \quad (18)$$

for any $k = 3, 4, \cdots, L$. We can easily verify that the Lagrangian solutions $P_2, P_3, \cdots, P_L$ in (16) and (18) are positive.

In the following, we would like to show that the average total transmission power $\bar{P}$ cannot be further minimized with strict inequality in (10). If there exists a power sequence $P_1^*, P_2^*, \cdots, P_L^*$ such that $P_1^* + P_2^* + \cdots + P_L^* > P_0$ and the average total transmission power $\bar{P}$ is minimized, then let us consider another power sequence

$$\tilde{P}_k = r P_k^*, \qquad k = 1, 2, \cdots, L, \quad (19)$$

in which r is an arbitrary number satisfying

$$r \geq \frac{P_0}{P_1^* + P_2^* + \cdots + P_L^*}. \quad (20)$$

We can see that the new power sequence $\tilde{P}_1, \tilde{P}_2, \cdots, \tilde{P}_L$ satisfies the power constraint in (10). With the new power sequence, the resulting average total transmission power is

$$\begin{aligned} f(r) &\triangleq \tilde{P}_1 + \sum_{l=2}^{L} \tilde{P}_l \left[1 - e^{-\frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 \sum_{i=1}^{l-1} \tilde{P}_i}}\right] \\ &= r P_1^* + \sum_{l=2}^{L} r P_l^* \left[1 - e^{-\frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 \sum_{i=1}^{l-1} r P_i^*}}\right], \end{aligned} \quad (21)$$

which is a function of r. Taking derivative of $f(r)$ with respect to r, we have

$$\frac{\partial f(r)}{\partial r} = \sum_{l=1}^{L} P_l^* - \sum_{l=2}^{L} P_l^* \left[1 + \frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 \sum_{i=1}^{l-1} r P_i^*}\right] e^{-\frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 \sum_{i=1}^{l-1} r P_i^*}}. \quad (22)$$

Since $e^{-x}(1+x) < 1$ for any positive x, so in (22),

$$\left[1 + \frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 \sum_{i=1}^{l-1} r P_i^*}\right] e^{-\frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 \sum_{i=1}^{l-1} r P_i^*}} < 1. \quad (23)$$

Thus, we have

$$\frac{\partial f(r)}{\partial r} > \sum_{l=1}^{L} P_l^* - \sum_{l=2}^{L} P_l^* = P_1^* > 0, \quad (24)$$

which means that $f(r)$ is an increasing function for any $r \geq \frac{P_0}{P_1^* + P_2^* + \cdots + P_L^*}$, and the minimum of $f(r)$ is achieved when $r = \frac{P_0}{P_1^* + P_2^* + \cdots + P_L^*}$. It implies that the average total transmission power resulting from the new power sequence with $r = \frac{P_0}{P_1^* + P_2^* + \cdots + P_L^*} (< 1)$ is less than that based on the power sequence $P_1^*, P_2^*, \cdots, P_L^*$. This is contradictory to the assumption that the power sequence $P_1^*, P_2^*, \cdots, P_L^*$ minimizes the average total transmission power $\bar{P}$. Therefore, the minimum average total transmission power $\bar{P}$ can be achieved at the boundary of the constraint (with equality) in (10). We note that with $r = \frac{P_0}{P_1^* + P_2^* + \cdots + P_L^*}$, the new power sequence satisfies

$$\tilde{P}_1 + \tilde{P}_2 + \cdots + \tilde{P}_L = P_0, \quad (25)$$

which is the boundary of the constraint in (10).

We note that in general a Lagrangian solution may not guarantee global optimality, i.e. it may lead to a local minima or maxima. Fortunately, the Lagrangian solution in (16) and (18) leads to a global minima as explained as follows. From the Lagrangian solution in (16) and (18) and the total power constraint $P_1 + P_2 + \cdots + P_L = P_0$, we can see that there is only one unique power sequence $P_1, P_2, \cdots, P_L$ that results from the Lagrangian solution. So, the unique power sequence guarantees the global optimality which, however, can be either global minima or maxima. We further examine that with a trivial power assignment $P_1 = P_0, P_2 = P_3 = \cdots = P_L = 0$, the resulting average total transmission power is $\bar{P} = P_0$, which is larger than the average total transmission power resulting from the power sequence associated with the Lagrangian solution. Therefore, the unique power sequence from the Lagrangian solution guarantees the global minima. We summarize the above discussion in the following theorem.

Theorem 1: In the H-ARQ transmission protocol, to minimize the average total transmission power, the optimal transmission power P_k at the kth, $1 \leq k \leq L$, transmission round must satisfy the following

$$P_2 = \frac{\sigma_{sd}^2 P_1^2}{\gamma_0 \mathcal{N}_0}, \quad (26)$$

$$P_k = \frac{\sigma_{sd}^2 (\sum_{i=1}^{k-1} P_i)^2}{\gamma_0 \mathcal{N}_0} \left[1 - e^{-\frac{P_{k-1} \gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 \left(\sum_{i=1}^{k-1} P_i\right)\left(\sum_{i=1}^{k-2} P_i\right)}}\right], \quad (27)$$

for $k = 3, 4, \cdots, L$, and

$$P_1 + P_2 + \cdots + P_L = P_0, \quad (28)$$

where $P_0 \triangleq \frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 \ln \frac{1}{1-p_0}}$, γ_0 is the required SNR for correct decoding, $\mathcal{N}_0$ is the additive white noise variance, σ_{sd}^2 is the Rayleigh fading variance, and p_0 is the targeted H-ARQ outage probability. □

From Theorem 1, we can see that the optimal transmission power sequence is uniquely determined by the set of equations (26)–(28). The optimal transmission power level for each (re)transmission round can be calculated recursively. According to (26) and (27), for any $k = 2, 3, \cdots, L$, the optimal transmission power value P_k can be calculated based on $P_1, P_2, \cdots, P_{k-1}$. So for any given power P_1, all other transmission power $P_k, k = 2, 3, \cdots, L$, can be subsequently determined. The optimal initial power P_1 can be numerically

TABLE I: Algorithm to determine the optimal power assignment sequence $P_k,\ k = 1, 2, \cdots, L$

$$
\begin{aligned}
&Step\ 1: \ Input\ \gamma_0, p_0, \sigma_{sd}^2, \mathcal{N}_0.\ Calculate\ P_0 = \frac{\gamma_0\mathcal{N}_0}{\sigma_{sd}^2 \ln\frac{1}{1-p_0}}.\\
&Step\ 2: \ Set\ lower = 0\ and\ upper = P_0.\\
&Step\ 3: \ Let\ P_1 = (lower + upper)/2,\ and\ calculate\\
&\qquad P_2 = \frac{\sigma_{sd}^2 P_1^2}{\gamma_0\mathcal{N}_0},\\
&\qquad P_k = \frac{\sigma_{sd}^2(\sum_{i=1}^{k-1}P_i)^2}{\gamma_0\mathcal{N}_0}\left[1 - e^{-\frac{P_{k-1}\gamma_0\mathcal{N}_0}{\sigma_{sd}^2\left(\sum_{i=1}^{k-1}P_i\right)\left(\sum_{i=1}^{k-2}P_i\right)}}\right],\\
&\qquad for\ any\ k = 3, 4, \cdots, L.\\
&Step\ 4: \ Check\ if\ abs(P_1 + P_2 + \cdots + P_L - P_0) < \epsilon (= 0.001),\ then\ stop\\
&\qquad and\ output\ power\ sequence\ P_1, P_2, \cdots, P_L; otherwise,\\
&\qquad\quad if\ P_1 + P_2 + \cdots + P_L - P_0 < 0,\ set\ lower = P_1;\\
&\qquad\quad if\ P_1 + P_2 + \cdots + P_L - P_0 > 0,\ set\ upper = P_1;\\
&\qquad and\ go\ to\ Step\ 3.
\end{aligned}
$$

found based on (28) by the Newton method. A complete algorithm to recursively determine the optimal power assignment sequence $P_k,\ k = 1, 2, \cdots, L$, is detailed in Table I.

When $L = 2$, we have a closed-form solution for the optimal power sequence. In this case, $P_1 + P_2 = P_0$ and $P_2 = \frac{\sigma_{sd}^2 P_1^2}{\gamma_0\mathcal{N}_0}$. By solving the two equations, the optimal transmission power P_1 and P_2 are given by

$$P_1 = \frac{2P_0}{1 + \sqrt{1 + \frac{4\sigma_{sd}^2 P_0}{\gamma_0\mathcal{N}_0}}}, \quad (29)$$

$$P_2 = \frac{\gamma_0\mathcal{N}_0}{4\sigma_{sd}^2}\left(\sqrt{1 + \frac{4\sigma_{sd}^2 P_0}{\gamma_0\mathcal{N}_0}} - 1\right)^2. \quad (30)$$

From (29) and (30), we can see that if $\frac{\gamma_0\mathcal{N}_0}{\sigma_{sd}^2} > \frac{P_0}{2}$, then $P_1 > \frac{P_0}{2}$, which implies that $P_1 > P_2$, i.e. the power assigned in the first transmission round should be larger than that for the second retransmission round. The condition $\frac{\gamma_0\mathcal{N}_0}{\sigma_{sd}^2} > \frac{P_0}{2}$ means

$$\frac{\gamma_0\mathcal{N}_0}{\sigma_{sd}^2} > \frac{1}{2}\frac{\gamma_0\mathcal{N}_0}{\sigma_{sd}^2 \ln\frac{1}{1-p_0}},$$

which is true when $p_0 > 1 - e^{-\frac{1}{2}} \approx 0.3935$. On the other hand, if $\frac{\gamma_0\mathcal{N}_0}{\sigma_{sd}^2} < \frac{P_0}{2}$ (*i.e.* $p_0 < 1 - e^{-\frac{1}{2}}$), then $P_1 < \frac{P_0}{2}$, which means P_1 should be less than P_2 (an opposite power assignment strategy compared to that of $P_1 > P_2$). Especially, when the targeted outage probability is $p_0 = 1 - e^{-\frac{1}{2}}$, the optimal power assignment is $P_1 = P_2$, i.e. an equal power assignment, no matter what are the required SNR γ_0, the noise variance $\mathcal{N}_0$ and the channel variance σ_{sd}^2. From the case of $L = 2$, we can see that the optimal power can be assigned either in an increasing, decreasing or equal way depending on the targeted outage probability performance of the H-ARQ protocol. This is different from the case in [12] where the optimal transmission power must be increasing in every retransmission.

For the general case of $L > 2$, numerical results (shown in Figs. 2-4 in Section IV) reveal that the optimal power assignment sequence can be neither increasing nor decreasing. Actually, from the theorem we can see that when $P_1 < \frac{\gamma_0\mathcal{N}_0}{\sigma_{sd}^2}$, the optimal transmission power P_2 is less than P_1 according to (26). On the other hand, when $P_1 > \frac{\gamma_0\mathcal{N}_0}{\sigma_{sd}^2}$, the optimal transmission power P_2 is larger than P_1. This phenomenon is fundamentally different from the case in [12] where the optimal transmission power sequence is always increasing.

We note that if $P_1 > \frac{\gamma_0\mathcal{N}_0}{\sigma_{sd}^2}$, then the optimal power assignment sequence in Theorem 1 is monotonically increasing, i.e. $P_1 < P_2 < \cdots < P_L$. From (26), it is easy to see that $P_2 > P_1$ in this case. For any $k = 3, 4, \cdots, L$, since $1 - e^{-x} > x - \frac{1}{2}x^2$ for any $x > 0$[1], so from (27) we have

$$
\begin{aligned}
P_k &> \frac{\sigma_{sd}^2(\sum_{i=1}^{k-1}P_i)^2}{\gamma_0\mathcal{N}_0}\left[\frac{P_{k-1}\gamma_0\mathcal{N}_0}{\sigma_{sd}^2\left(\sum_{i=1}^{k-1}P_i\right)\left(\sum_{i=1}^{k-2}P_i\right)} - \frac{P_{k-1}^2(\gamma_0\mathcal{N}_0)^2}{2\sigma_{sd}^4\left(\sum_{i=1}^{k-1}P_i\right)^2\left(\sum_{i=1}^{k-2}P_i\right)^2}\right]\\
&= \left[1 + \frac{P_{k-1}}{\sum_{i=1}^{k-2}P_i} - \frac{P_{k-1}\gamma_0\mathcal{N}_0}{2\sigma_{sd}^2\left(\sum_{i=1}^{k-2}P_i\right)^2}\right]P_{k-1}. \quad (31)
\end{aligned}
$$

Since $\frac{\gamma_0\mathcal{N}_0}{\sigma_{sd}^2(\sum_{i=1}^{k-2}P_i)} < \frac{\gamma_0\mathcal{N}_0}{\sigma_{sd}^2 P_1} < 1$, it is easy to see that

$$
\begin{aligned}
&\frac{P_{k-1}}{\sum_{i=1}^{k-2}P_i} - \frac{P_{k-1}\gamma_0\mathcal{N}_0}{2\sigma_{sd}^2\left(\sum_{i=1}^{k-2}P_i\right)^2} =\\
&\qquad \frac{P_{k-1}}{\sum_{i=1}^{k-2}P_i}\left[1 - \frac{\gamma_0\mathcal{N}_0}{2\sigma_{sd}^2\left(\sum_{i=1}^{k-2}P_i\right)}\right] > 0.
\end{aligned}
\quad (32)
$$

Combining (31) and (32), we conclude that $P_k > P_{k-1}$ for any $k = 3, 4, \cdots, L$. Thus, the optimal power assignment sequence in Theorem 1 is monotonically increasing in this

[1]In this footnote, we would like to prove that $G(x) \stackrel{\triangle}{=} (1 - e^{-x}) - (x - \frac{1}{2}x^2) > 0$ for any $x > 0$. We can see that $G'(x) = e^{-x} + x - 1$ and $G''(x) = -e^{-x} + 1$. Since $G'(0) = 0$ and $G''(x) > 0$ for any $x > 0$, so $G'(x) > 0$ for any $x > 0$, i.e. $G(x)$ is monotonically increasing for $x > 0$. With $G(0) = 0$, we conclude that $G(x) > 0$ for any $x > 0$.

case. Actually, in this case the optimal power assignment sequence increases as a function of P_1 roughly in a polynomial way, which is shown in the next section.

IV. Approximation of the Optimal Power Sequence

To reduce calculation complexity in the optimal power assignment, we present in this section a simple and tight approximation for the optimal transmission power sequence. The tight approximation allows us to get more insight understanding of the optimal power assignment strategy for the H-ARQ protocol.

Since $1-e^{-x} \approx x$ for small x, so for any $k = 3, 4, \cdots, L$, the optimal transmission power P_k in (27) can be approximated as

$$P_k \approx \frac{\sigma_{sd}^2 (\sum_{i=1}^{k-1} P_i)^2}{\gamma_0 \mathcal{N}_0} \times \frac{P_{k-1}\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 (\sum_{i=1}^{k-1} P_i)(\sum_{i=1}^{k-2} P_i)} \quad (33)$$

$$= P_{k-1} + \frac{P_{k-1}^2}{\sum_{i=1}^{k-2} P_i}. \quad (34)$$

The approximation in (33) is tight when $\frac{P_{k-1}\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 (\sum_{i=1}^{k-1} P_i)(\sum_{i=1}^{k-2} P_i)}$ is small and it is true in general. We can verify that when $k = 3$,

$$\frac{P_{k-1}\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 (\sum_{i=1}^{k-1} P_i)(\sum_{i=1}^{k-2} P_i)} = \frac{1}{1 + \frac{\sigma_{sd}^2}{\gamma_0 \mathcal{N}_0} P_1}$$

which is strictly less than 1, and it becomes smaller when the power P_1 is larger. When $k > 3$,

$$\frac{P_{k-1}\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 (\sum_{i=1}^{k-1} P_i)(\sum_{i=1}^{k-2} P_i)} < \frac{\frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2}}{\sum_{i=1}^{k-2} P_i}$$

which is small when any of $P_1, P_2, \cdots, P_{k-2}$ is large.

When $k = 3$, substituting $P_2 = \frac{\sigma_{sd}^2 P_1^2}{\gamma_0 \mathcal{N}_0}$ into (34), we have

$$P_3 \approx P_2 + \frac{P_2^2}{P_1} = \frac{\sigma_{sd}^2 P_1^2}{\gamma_0 \mathcal{N}_0}\left(1 + \frac{\sigma_{sd}^2 P_1}{\gamma_0 \mathcal{N}_0}\right). \quad (35)$$

When $k = 4$, substituting $P_2 = \frac{\sigma_{sd}^2 P_1^2}{\gamma_0 \mathcal{N}_0}$ and the above approximation of P_3 into (34), we can approximate P_4 as

$$P_4 \approx P_3 + \frac{P_3^2}{P_1 + P_2} = \frac{\sigma_{sd}^2 P_1^2}{\gamma_0 \mathcal{N}_0}\left(1 + \frac{\sigma_{sd}^2 P_1}{\gamma_0 \mathcal{N}_0}\right)^2. \quad (36)$$

Assume that for any $k \le k_0 (> 2)$, it is true that

$$P_k \approx \frac{\sigma_{sd}^2 P_1^2}{\gamma_0 \mathcal{N}_0}\left(1 + \frac{\sigma_{sd}^2 P_1}{\gamma_0 \mathcal{N}_0}\right)^{k-2}, \qquad k = 3, 4, \cdots, k_0, \quad (37)$$

then for $k = k_0 + 1$, we have

$$\begin{aligned} P_{k_0+1} &\approx P_{k_0} + \frac{P_{k_0}^2}{\sum_{i=1}^{k_0-1} P_i} \\ &= \frac{\sigma_{sd}^2 P_1^2}{\gamma_0 \mathcal{N}_0}\left(1 + \frac{\sigma_{sd}^2 P_1}{\gamma_0 \mathcal{N}_0}\right)^{k_0-2} \\ &\quad + \frac{\left(\frac{\sigma_{sd}^2 P_1^2}{\gamma_0 \mathcal{N}_0}\right)^2 \left(1 + \frac{\sigma_{sd}^2 P_1}{\gamma_0 \mathcal{N}_0}\right)^{2k_0-4}}{P_1 + \frac{\sigma_{sd}^2 P_1^2}{\gamma_0 \mathcal{N}_0} \sum_{i=2}^{k_0-1}\left(1 + \frac{\sigma_{sd}^2 P_1}{\gamma_0 \mathcal{N}_0}\right)^{i-2}} \\ &= \frac{\sigma_{sd}^2 P_1^2}{\gamma_0 \mathcal{N}_0}\left(1 + \frac{\sigma_{sd}^2 P_1}{\gamma_0 \mathcal{N}_0}\right)^{(k_0+1)-2}, \end{aligned}$$

i.e. the result in (37) is also true for $k = k_0 + 1$. Thus, by induction we can conclude that for any $k = 2, 3, \cdots, L$, we have

$$P_k \approx \frac{\sigma_{sd}^2 P_1^2}{\gamma_0 \mathcal{N}_0}\left(1 + \frac{\sigma_{sd}^2 P_1}{\gamma_0 \mathcal{N}_0}\right)^{k-2}. \quad (38)$$

Based on the approximation and the sum-power constraint in (28), we have a constraint on the optimal power P_1 as follows

$$P_1 + \sum_{k=2}^{L} \frac{\sigma_{sd}^2 P_1^2}{\gamma_0 \mathcal{N}_0}\left(1 + \frac{\sigma_{sd}^2 P_1}{\gamma_0 \mathcal{N}_0}\right)^{k-2} = P_0 \quad (39)$$

or equivalently

$$P_1 \left(1 + \frac{\sigma_{sd}^2 P_1}{\gamma_0 \mathcal{N}_0}\right)^{L} = P_0. \quad (40)$$

The left-hand side of the equation (40) is an increasing function in terms of power P_1, so there is a unique solution for the equation. Thus, the optimal power P_1 can be easily determined based on the equation in (40) by using the Newton method. We summarize the above discussion in the form of the following theorem.

Theorem 2: In the H-ARQ transmission protocol, the optimal transmission power at each round can be approximated as

$$P_k \approx \frac{\sigma_{sd}^2 P_1^2}{\gamma_0 \mathcal{N}_0}\left(1 + \frac{\sigma_{sd}^2 P_1}{\gamma_0 \mathcal{N}_0}\right)^{k-2} \quad (41)$$

for $k = 2, 3, \cdots, L$ where P_1 is determined by the equation

$$P_1 \left(1 + \frac{\sigma_{sd}^2 P_1}{\gamma_0 \mathcal{N}_0}\right)^{L-1} = P_0, \quad (42)$$

where $P_0 = \frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 \ln \frac{1}{1-p_0}}$. □

From Theorem 2, we observe that for any $k = 2, 3, \cdots, L$, the optimal transmission power P_k can be approximated as a function of P_1. The optimal transmission power P_1 can be directly determined by the equation (42), then all other optimal transmission power values $P_k, k = 2, 3, \cdots, L$ can be obtained immediately based on the closed-form expression in (41). The procedure is detailed in the algorithm in Table II. We can see that the calculation complexity of the algorithm in Table II is much less than that of the recursive algorithm in Table I. The approximation in Theorem 2 provides some insight understanding that the optimal transmission power in each retransmission round varies in term of P_1 (the transmission power in the first round) in a polynomial way.

When $L = 2$, the constraint in (42) is reduced to $P_1\left(1 + \frac{\sigma_{sd}^2 P_1}{\gamma_0 \mathcal{N}_0}\right) = P_0$. By solving the equation, we have the optimal power value for the first transmission round as $P_1 = \frac{2P_0}{1+\sqrt{1+\frac{4\sigma_{sd}^2 P_0}{\gamma_0 \mathcal{N}_0}}}$, which matches with the exact power value given in (29). When $L > 2$, based on the constraint in (42), the optimal transmission power P_1 can be bounded as follows. Since the geometric mean is not greater than the

TABLE II: Algorithm to determine the approximation of the optimal power sequence $P_k,\ k = 1, 2, \cdots, L$

$$\begin{array}{ll}
Step\ 1: & Input\ \gamma_0, p_0, \sigma_{sd}^2, \mathcal{N}_0.\ Calculate\ P_0 = \frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 \ln \frac{1}{1-p_0}}. \\
Step\ 2: & Set\ lower = 0\ and\ upper = P_0. \\
Step\ 3: & Let\ P_1 = (lower + upper)/2,\ and\ calculate \\
& \quad temp = P_1 \left(1 + \frac{\sigma_{sd}^2 P_1}{\gamma_0 \mathcal{N}_0}\right)^{L-1} - P_0. \\
& Check\ if\ abs(Temp) < \epsilon (= 0.001),\ then\ output \\
& the\ optimal\ power\ P_1\ and\ go\ to\ Step\ 4;\ otherwise \\
& \quad if\ temp < 0,\ set\ lower = P_1; \\
& \quad if\ temp > 0,\ set\ upper = P_1; \\
& and\ repeat\ Step\ 3. \\
Step\ 4: & Calculate\ the\ optimal\ transmission\ power\ P_k\ as\ follows: \\
& \quad P_k \approx \frac{\sigma_{sd}^2 P_1^2}{\gamma_0 \mathcal{N}_0} \left(1 + \frac{\sigma_{sd}^2 P_1}{\gamma_0 \mathcal{N}_0}\right)^{k-2}, \quad k = 2, 3, \cdots, L.
\end{array}$$

arithmetic mean, we have

$$\begin{aligned}
\frac{\sigma_{sd}^2}{\gamma_0 \mathcal{N}_0} P_0 &= \frac{\sigma_{sd}^2 P_1}{\gamma_0 \mathcal{N}_0} \left(1 + \frac{\sigma_{sd}^2 P_1}{\gamma_0 \mathcal{N}_0}\right)^{L-1} \\
&< \left(\frac{(L-1) + L \frac{\sigma_{sd}^2 P_1}{\gamma_0 \mathcal{N}_0}}{L} \right)^L,
\end{aligned}$$

so,

$$P_1 > \frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2} \left[\left(\frac{\sigma_{sd}^2 P_0}{\gamma_0 \mathcal{N}_0} \right)^{\frac{1}{L}} - \frac{L-1}{L} \right]. \tag{43}$$

On the other hand, since

$$P_0 = P_1 \left(1 + \frac{\sigma_{sd}^2 P_1}{\gamma_0 \mathcal{N}_0}\right)^{L-1} > \left(\frac{\sigma_{sd}^2}{\gamma_0 \mathcal{N}_0} \right)^{L-1} P_1^L, \tag{44}$$

so we have

$$P_1 < \frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2} \left(\frac{\sigma_{sd}^2 P_0}{\gamma_0 \mathcal{N}_0} \right)^{\frac{1}{L}}. \tag{45}$$

Therefore, the optimal transmission power P_1 is bounded as follows

$$\frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2} \left[\left(\frac{\sigma_{sd}^2 P_0}{\gamma_0 \mathcal{N}_0} \right)^{\frac{1}{L}} - \frac{L-1}{L} \right] < P_1 < \frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2} \left(\frac{\sigma_{sd}^2 P_0}{\gamma_0 \mathcal{N}_0} \right)^{\frac{1}{L}}, \tag{46}$$

in which the upper bound is tight when P_0 is large. The difference between the lower bound and the upper bound is less than $\frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2}$.

In Figs. 2 and 3, we show comparisons of the approximation of the optimal transmission power sequence by Theorem 2 with the exact optimized power sequence by Theorem 1. In these two figures, we assumed that the targeted SNR is $\gamma_0 = 10$ dB, the required outage performance is $p_0 = 10^{-3}$, $\sigma_{sd}^2 = 1$ and $\mathcal{N}_0 = 1$. The maximum number of transmission rounds is $L = 3$ in Fig. 2, and $L = 5$ in Fig. 3. We can see that the approximations of the optimal transmission power values (solid line with '$*$') match very well with those based on exactly numerical calculation (solid line with '$\circ$'). For comparison, we also include in the figures the transmission power level of the equal-power assignment strategy. We observe that in the first few (re)transmission rounds, the optimal power assignment strategy assigns significantly less transmission power compared to the equal-power assignment strategy.

The optimum transmission power sequence is increasing in both cases in Figs. 2 and 3. Actually, the optimum transmission power sequence can be neither increasing or deceasing, which is shown in Fig. 4. In this case, the maximum number of retransmission rounds is $L = 10$, the targeted SNR is $\gamma_0 = 10$ dB and the required outage performance is $p_0 = 10^{-1}$. We can see that the optimal power assignment is decreasing in the first two rounds and increasing after that. Moreover, we observe that in this case there is a gap between the approximations of the optimal transmission power sequence and the exactly calculated sequence. Since when $k = 3$, the term $\frac{P_{k-1} \gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 (\sum_{i=1}^{k-1} P_i)(\sum_{i=1}^{k-2} P_i)} = \frac{1}{1 + \frac{\sigma_{sd}^2 P_1}{\gamma_0 \mathcal{N}_0}} \approx 0.63$ which is not small enough in this case, so the approximation of the exponential term in (33) is not tight. The difference between the approximated sequence and the exactly calculated power sequence can be more significant when the the maximum number of retransmission rounds L goes to infinity. Fortunately, this is not the case in practice where a reasonable L is normally less than 10 due to delay consideration.

Finally, based on the approximation of the optimal transmission power sequence, the average total transmission power of the H-ARQ protocol accounting for requested retransmissions can be approximated as follows

$$\begin{aligned}
\bar{P}_{opt} &\approx P_1 + \sum_{l=2}^{L} \frac{\sigma_{sd}^2 P_1^2}{\gamma_0 \mathcal{N}_0} \left(1 + \frac{\sigma_{sd}^2 P_1}{\gamma_0 \mathcal{N}_0}\right)^{l-2} \\
&\quad \times \left[1 - e^{-\frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 \sum_{i=1}^{l-1} \frac{\sigma_{sd}^2 P_1^2}{\gamma_0 \mathcal{N}_0} \left(1 + \frac{\sigma_{sd}^2 P_1}{\gamma_0 \mathcal{N}_0}\right)^{i-2}}} \right] \\
&= P_1 + \sum_{l=2}^{L} \frac{\sigma_{sd}^2 P_1^2}{\gamma_0 \mathcal{N}_0} \left(1 + \frac{\sigma_{sd}^2 P_1}{\gamma_0 \mathcal{N}_0}\right)^{l-2} \\
&\quad \times \left[1 - e^{-\frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 P_1} \left(1 + \frac{\sigma_{sd}^2 P_1}{\gamma_0 \mathcal{N}_0}\right)^{-l+2}} \right].
\end{aligned} \tag{47}$$

The approximation of the average total transmission power is

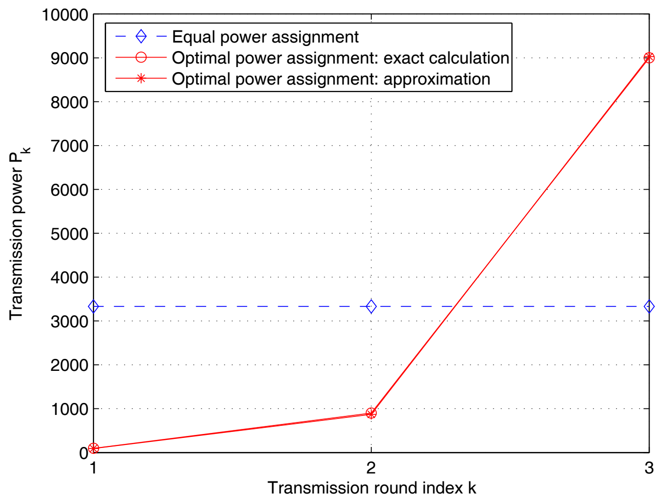

Fig. 2. Transmission power sequence of the optimal power assignment strategy with $L = 3$, $\gamma_0 = 10$ dB, $p_0 = 10^{-3}$.

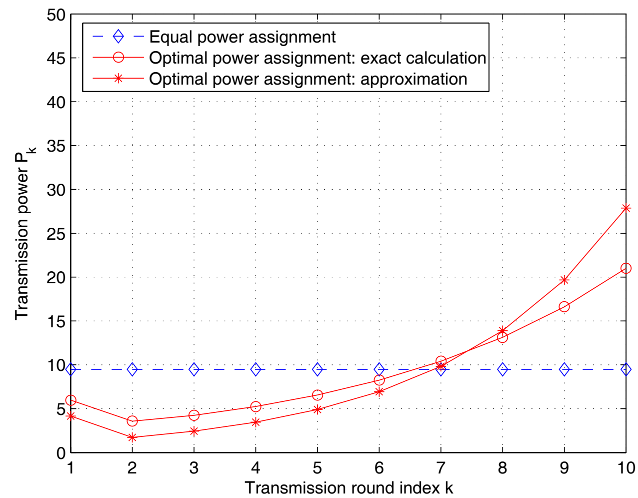

Fig. 4. Transmission power sequence of the optimal assignment strategy with $L = 10$, $\gamma_0 = 10$ dB, $p_0 = 10^{-1}$.

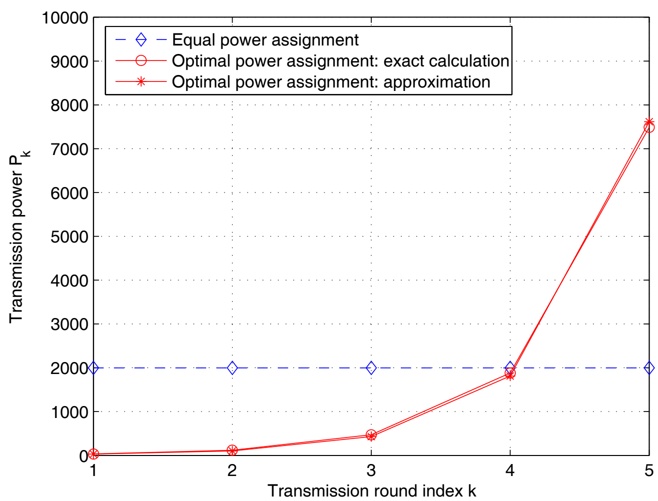

Fig. 3. Transmission power sequence of the optimal power assignment strategy with $L = 5$, $\gamma_0 = 10$ dB, $p_0 = 10^{-3}$.

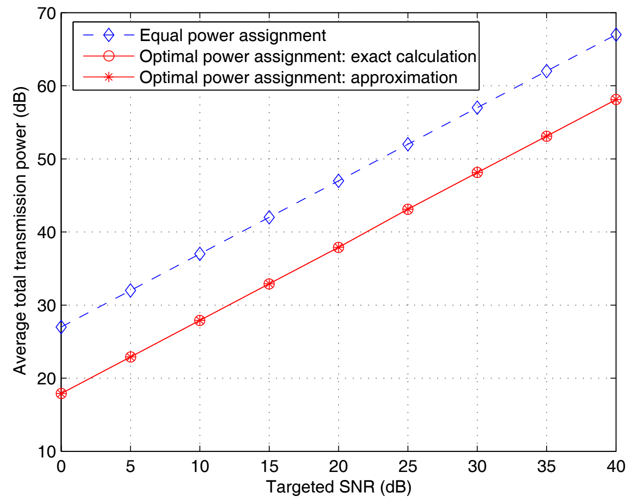

Fig. 5. Comparisons of the average total transmission power of the equal and optimal power assignment strategies with different targeted SNRs. $L = 2$, $p_0 = 10^{-3}$.

a function of P_1. Moreover, when $P_1 > \frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2}$, then using the approximation $1 - e^{-x} \approx x$ for small x, the average total transmission power across requested retransmissions can be further approximated as

$$\bar{P}_{opt} \approx P_1 + \sum_{l=2}^{L} P_1 = LP_1. \tag{48}$$

This approximation is tight when L is small, since P_1 is normally larger than $\frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2}$ in this case. It shows that the average total transmission power is roughly the product of the transmission power in the first round and the number of retransmission rounds allowed in the H-ARQ protocol. If we approximate P_1 as $\frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2} \left(\frac{\sigma_{sd}^2 P_0}{\gamma_0 \mathcal{N}_0} \right)^{\frac{1}{L}}$ based on (45), then the average total transmission power with the optimal power assignment sequence can be approximated as

$$\bar{P}_{opt} \approx L \frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2} \left(\frac{\sigma_{sd}^2 P_0}{\gamma_0 \mathcal{N}_0} \right)^{\frac{1}{L}} = \frac{L}{(\ln \frac{1}{1-p_0})^{\frac{1}{L}}} \frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2}. \tag{49}$$

V. Performance Comparisons between the Equal and Optimal Power Assignments

In this section, we compare the power efficiency of the H-ARQ protocols with the optimal power assignment strategy derived in this work and the conventional equal-power assignment approach. In numerical calculation, we assume that the variance of the channel h_{sd} is $\sigma_{sd}^2 = 1$ and the noise variance is $\mathcal{N}_0 = 1$.

For a targeted outage probability p_0, according to (9), the equal-power assignment approach should also follow the

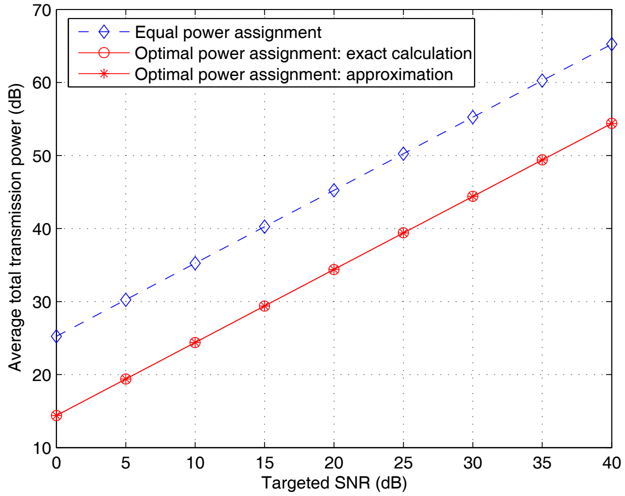

Fig. 6. Comparisons of the average total transmission power of the equal and optimal power assignment strategies with different targeted SNRs. $L = 3$, $p_0 = 10^{-3}$.

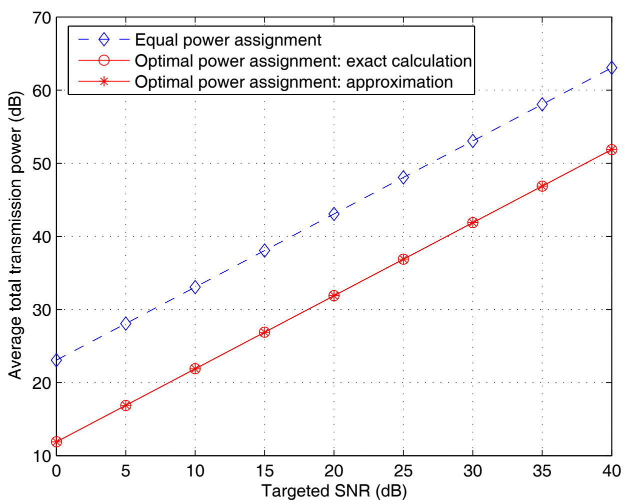

Fig. 7. Comparisons of the average total transmission power of the equal and optimal power assignment strategies with different targeted SNRs. $L = 5$, $p_0 = 10^{-3}$.

power constraint

$$\sum_{l=1}^{L} P_l \geq P_0 = \frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 \ln\frac{1}{1-p_0}}. \tag{50}$$

Thus, the equal-power assignment is $P_l = P_0/L$ for each $l = 1, 2, \cdots, L$. The corresponding average total transmission power is

$$\begin{aligned} \bar{P}_{equ} &= P_1 + \sum_{l=2}^{L} \frac{P_0}{L}\left[1 - e^{-\frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 \sum_{i=1}^{l-1} \frac{P_0}{L}}}\right] \\ &= P_0 - \frac{P_0}{L} \sum_{l=2}^{L} e^{-\frac{L}{l-1}\frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 P_0}}. \end{aligned} \tag{51}$$

When P_0 is large, the average total transmission power of the equal-power assignment strategy can be approximated as

$$\begin{aligned} \bar{P}_{equ} &\approx P_0 - \frac{P_0}{L}\sum_{l=2}^{L}\left(1 - \frac{L}{l-1}\frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2 P_0}\right) \\ &= \frac{P_0}{L} + \frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2}\sum_{l=2}^{L}\frac{1}{l-1}. \end{aligned} \tag{52}$$

Therefore, the power efficiency of the optimal power assignment strategy compared to the equal-power assignment approach can be quantified by the following ratio

$$\begin{aligned} \frac{\bar{P}_{equ}}{\bar{P}_{opt}} &= \frac{\frac{P_0}{L} + \frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2}\sum_{l=2}^{L}\frac{1}{l-1}}{L\frac{\gamma_0 \mathcal{N}_0}{\sigma_{sd}^2}\left(\frac{\sigma_{sd}^2 P_0}{\gamma_0 \mathcal{N}_0}\right)^{\frac{1}{L}}} \\ &= \frac{1}{L^2}\left(\ln\frac{1}{1-p_0}\right)^{\frac{1}{L}-1} \\ &\quad + \frac{1}{L}\left(\ln\frac{1}{1-p_0}\right)^{\frac{1}{L}}\sum_{l=2}^{L}\frac{1}{l-1}. \end{aligned} \tag{53}$$

According to $\ln\frac{1}{1-p_0} \approx p_0$, the ratio can be approximated as

$$\frac{\bar{P}_{equ}}{\bar{P}_{opt}} \approx \frac{p_0^{\frac{1}{L}}}{L}\left(\frac{1}{Lp_0} + \sum_{l=2}^{L}\frac{1}{l-1}\right). \tag{54}$$

For small targeted outage probability p_0, the ratio can be further approximated as $\frac{\bar{P}_{equ}}{\bar{P}_{opt}} \approx \frac{p_0^{\frac{1}{L}-1}}{L^2}$. We can see that the smaller the targeted outage probability p_0, the more power saving the optimal power assignment strategy compared to the equal-power assignment strategy.

For different targeted SNR γ_0 (from 0 dB to 40 dB), we compare the average total transmission power of the optimal power assignment strategy and the equal-power assignment scheme in Figs. 5, 6, and 7 for the cases of $L = 2$, $L = 3$, and $L = 5$, respectively. The required outage performance of the H-ARQ protocol is set at $p_0 = 10^{-3}$. When $L = 2$, from Fig. 5 we observe that the optimal power assignment saves about 9 dB in average total transmission power compared to the equal-power H-ARQ. When $L = 3$, we can see from Fig. 6 that the optimal power assignment shows about 10 dB gain compared to the equal-power assignment scheme. When $L = 5$, Fig. 7 shows that the optimal power assignment strategy significantly outperforms the equal-power assignment scheme with a performance improvement of about 11 dB. Moreover, it is interesting to observe that in each figure, the performance gain of the optimal power assignment strategy is almost constant for different targeted SNR γ_0 (from 0 dB to 40 dB). This is consistent with the theoretical approximation $\frac{\bar{P}_{equ}}{\bar{P}_{opt}}$ in (54) which does not rely on the targeted SNR γ_0. When $L = 2$ and $p_0 = 10^{-3}$, the ratio in (54) is $\frac{\bar{P}_{equ}}{\bar{P}_{opt}} = 8.99$ dB (the observed power saving in Fig. 5 is 9 dB). When $L = 3$ and $p_0 = 10^{-3}$, the ratio in (54) is $\frac{\bar{P}_{equ}}{\bar{P}_{opt}} = 10.48$ dB (the observed power saving in Fig. 6 is 10 dB).

We also compare the average total transmission power required in the two power assignment strategies with different targeted outage probability values. We assume the required SNR is $\gamma_0 = 10$ dB. Figs. 8, 9 and 10 present comparison results for the cases of $L = 2$, $L = 3$, and $L = 5$,

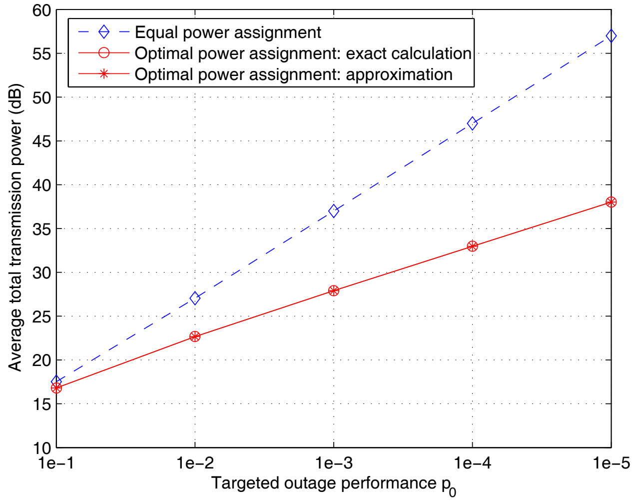

Fig. 8. Comparisons of the average total transmission power of the equal and optimal power assignment strategies with different targeted outage probabilities. $L = 2$, $\gamma_0 = 10$ dB.

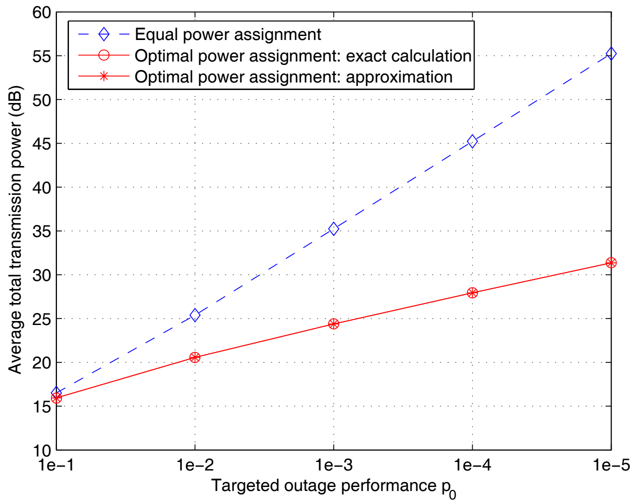

Fig. 9. Comparisons of the average total transmission power of the equal and optimal power assignment strategies with different targeted outage probabilities. $L = 3$, $\gamma_0 = 10$ dB.

respectively. From the three figures, we can see that for an outage performance of $p_0 = 10^{-4}$, the power savings of the optimal power assignment strategy compared to the equal-power assignment scheme are 15 dB when $L = 2$, 17 dB when $L = 3$, and 19 dB when $L = 5$. The lower the required outage probability, the more important optimization of the power sequence becomes. Moreover, we also observe that with the same targeted outage performance, the larger the number of retransmission rounds allowed in the H-ARQ protocol, the larger the performance gain between the optimal power assignment scheme and the equal-power assignment scheme.

We also show the average total transmission power based on the approximated optimal power sequence in the six figures. We can see that the average total transmission power based on the approximated power sequence matches tightly with that from the exact optimal power sequence in each case. Note

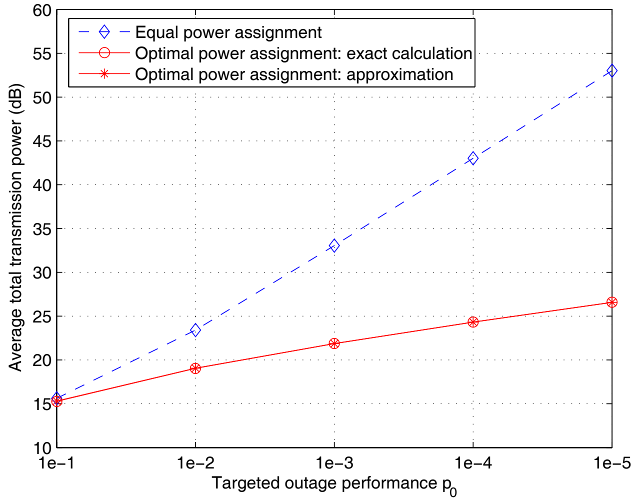

Fig. 10. Comparisons of the average total transmission power of the equal and optimal power assignment strategies with different targeted outage probabilities. $L = 5$, $\gamma_0 = 10$ dB.

that the exact calculation of the optimal transmission power sequence is based on Theorem 1 and the approximated power sequence comes from Theorem 2.

VI. Conclusion

In this paper, we determined the optimal transmission power assignment strategy for the H-ARQ protocol to minimize the average total transmission power in quasi-static Rayleigh fading channels. The optimal transmission power sequence is described by a set of equations which allow an exact recursive calculation of the optimal power sequence. To reduce calculation complexity, we also developed an approximation to the optimal power sequence that is close to the numerically calculated exact result. It is interesting to observe that the optimal transmission power sequence is neither increasing nor decreasing; its form depends on given total power budget and targeted outage performance levels. The optimal power assignment sequence reveals that conventional equal-power assignment is far from optimal. For example, for a targeted outage performance of 10^{-5} and maximum number of transmissions $L = 5$, the average total transmission power by the optimum assignment is about 27 dB less than that of using the equal-power assignment. We also observe that with the same targeted outage performance, the larger the number of retransmission rounds allowed in the H-ARQ protocol, the higher the total power gain of the optimal power assignment scheme compared to equal-power allocation. For the same cap on retransmission rounds, the lower the required outage probability, the higher the total power gain of the optimal power assignment strategy over equal-power assignment.

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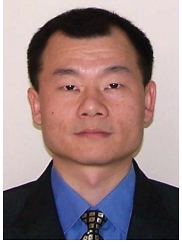

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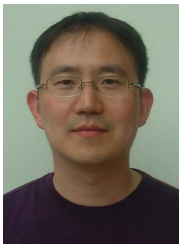

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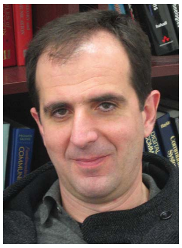

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