Single-Block Differential Transmit Scheme for Broadband Wireless MIMO-OFDM Systems

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***Abstract*—In frequency-selective multiple-input multiple-output (MIMO) channel, differential space-time-frequency (DSTF) modulations are known as practical alternatives that are capable of exploiting the available spatial and frequency diversities without the requirement of multichannel estimation at the receiver. However, the encoding nature of the DSTF schemes that expand several OFDM symbol periods makes the DSTF schemes susceptible to fast-changing channel conditions. In this paper, we propose a differential scheme for MIMO-OFDM systems that is able to differentially encode signal within two OFDM symbol periods, and the proposed scheme transmits the differentially encoded signal within one OFDM block. The scheme not only reduces encoding and decoding delay but also relaxes the restriction on channel assumption. The successful differential decoding of the proposed scheme depends on the assumption that the fading channels keep constant over two OFDM symbol periods rather than multiple of them as required in the existing DSTF schemes. We also provide pairwise error probability analysis and quantify the performance criteria in terms of diversity and coding advantages. The design criteria reveal that the existing diagonal cyclic codes can be applied to achieve full diversity. Performance simulations under various channel conditions show that our proposed scheme yields superior performance to previously proposed differential schemes.**

***Index Terms*—Differential modulation, differential space-time-frequency coding, frequency-selective fading channels, MIMO-OFDM systems, multiple antennas.**

I. Introduction

In wireless communication systems, deploying the multiple-input multiple-output (MIMO) concept together with an efficient coding and modulation scheme has been shown to be an effective way to increase link capacity without sacrificing bandwidth [1]. In the case of frequency-nonselective fading channels, space-time (ST) codes [2]–[4] have been proposed to explore spatial and temporal diversities that are available in MIMO links. In the case of frequency-selective channels, the MIMO concept has been deployed with orthogonal frequency-division multiplexing (OFDM) modulation, called MIMO-OFDM, to obtain available diversities and combat frequency selectivity of the channels. Various space-frequency (SF) codes [5]–[7] (and references therein) and space-time-frequency (STF) codes [8]–[10] were proposed for MIMO-OFDM systems. SF coding aims to exploit both spatial and frequency diversities, whereas the additional temporal diversity can be obtained when STF coding is employed under time-varying channels. However, most of the above coding techniques require reliable multichannel estimation, which inevitably increases the cost of frequent retraining and number of estimated parameters to the receiver. Although the channel estimates may be available when the channel changes slowly compared with the symbol rate, it may not be possible to acquire them in fast fading environment. Differential space-time (DST) modulation [11]–[14] has been widely known as one of many practical alternatives that bypasses multichannel estimation in frequency-nonselective MIMO systems. The differential scheme in [11] and [12] utilizes unitary group constellation which can be applied for arbitrary number of transmit antennas. The differential space-time block codes (DSTBC) in [13] and [14] differentially encode the existing space-time block codes in [3] and [4] and allow possible multilevel amplitude modulation to improve the MIMO link performance.

Recently, a technique of incorporating the DST modulation with OFDM transmission, the so-called differential space-time-frequency (DSTF) scheme, was introduced in [16]–[19] for frequency-selective channels. The DSTF modulation simultaneously encodes across spatial, temporal, and frequency domains such that both spatial and frequency diversity can be explored. The DSTF scheme in [16] successfully performs differential transmission by employing the DSTBC in [13] and [14]. However, the scheme does not obtain full frequency diversity since encoding across OFDM tones had not been fully investigated. Later, it was shown in [17] that the available frequency diversity can be obtained by spectral encoding of the information symbols before the DSTBC encoder. The spectral encoder utilizes a linear constellation decimation coding (LCD codes) which encodes across a minimal set of subcarriers such that decoding complexity can be reduced while full frequency diversity is preserved. The DSTF scheme [18], [19], on the other hand, jointly encodes across spatial, temporal, and frequency using unitary signal matrices. The design criteria in [18] and [19] suggest the code structure to have diagonal or block diagonal form in order to achieve maximum diversity order. Therefore, the diagonal cyclic codes [12] were utilized together with subcarrier grouping strategy such that maximum diversity is achieved with high coding gain. Also in [19], block diagonal signal structure has been designed to improve coding advantage by repeating the existing fixed-point-free groups [20], or using reducible constellation constructed from permutation of Lie groups [21].

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Due to the encoding nature of the DSTF scheme, a complete transmission of a DSTF codeword expands several OFDM symbol periods which are proportional to the number of transmit antennas. In order to perform successful differential decoding, previous work in [16]–[19] assumed that fading channels keep constant within several OFDM blocks and slowly change from a duration of several OFDM blocks to another. Nevertheless, such channel condition is not valid in most practical situations since the channel coefficients would change before two entire DSTF codeword matrices are completely transmitted. We, therefore, consider the case that the channels remain constant during one OFDM block period and slowly change from one OFDM block to the next. Note that in [22], a class of noncoherent codes was designed for MIMO-OFDM systems, which, however, does not take advantage of the slow fading channels.

In this paper, we propose a differential encoding and decoding scheme for MIMO-OFDM systems which is able to transmit the differentially encoded signal matrix within one OFDM block, regardless of the number of transmit antennas. The scheme allows us to relax the channel fading assumption to vary from a duration of one OFDM block to the next but remain approximately constant over only two OFDM blocks. The pairwise error probability (PEP) analysis in case of frequency-selective fading channels with arbitrary power delay profiles is also given. We address design criteria of the proposed scheme, and it reveals that the diagonal cyclic codes [12] can be used to achieve the maximum diversity order with high coding gain. The merit of our proposed scheme is demonstrated through computer simulations.

The rest of this paper is organized as follows. Section II outlines the system description. In Section III, we derive the differential encoding and decoding scheme for MIMO-OFDM systems. The pairwise error probability is analyzed, and the design criteria of the proposed scheme are given in Section IV. We show some simulation results and discussions in Section V. Finally, Section VI concludes this paper.

The following notations are adopted throughout this paper: vectors (matrices) are denoted by boldface lower (upper) case letters; all vectors are column vectors; superscript $(\cdot)^T$ and $(\cdot)^H$ denote the transpose and conjugate transpose, respectively; $\mathcal{C}^{A\times B}$ represents the complex field of dimension $A \times B$; $\lfloor\cdot\rfloor$ represents the floor function; $\lceil\cdot\rceil$ denotes the ceiling function; $E[\cdot]$ takes the statistical expectation; $\mathrm{diag}_v(\mathbf{a})$ denotes a diagonal matrix whose all elements of vector $\mathbf{a}$ are main diagonal elements; $\mathrm{diag}_m(\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_N)$ denotes a diagonal matrix whose $\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_N$ are vectors or matrices in the main diagonal; $\mathbf{I}_M$ ($\mathbf{0}_M$) denotes $M \times M$ identity matrix (matrix of all zeroes); $\mathbf{1}_K$ represents the $K\times 1$ vector of all ones; for any $N \times M$ matrix $\mathbf{A}$ we represent $det(\mathbf{A})$ as its matrix determinant and $tr(\mathbf{A})$ as its trace operator; $\mathcal{D}(\mathbf{A})$ converts each column of $\mathbf{A}$, $\mathbf{A} = [\mathbf{a}_1\, \mathbf{a}_2\, \cdots\, \mathbf{a}_M]$ to a diagonal matrix and results in an $N \times NM$ matrix of a form

$$\mathcal{D}(\mathbf{A}) = [\mathrm{diag}_v(\mathbf{a}_1)\ \mathrm{diag}_v(\mathbf{a}_2)\ \cdots\ \mathrm{diag}_v(\mathbf{a}_M)]. \quad (1)$$

$\otimes$ denotes the tensor product [28]; $\circ$ denotes the Hadamard product [28] such that if $\mathbf{A} = (a_{ij})$ and $\mathbf{B} = (b_{ij})$ are two $N \times M$ matrices, then $\mathbf{A}\circ\mathbf{B}$ is the $N\times M$ matrix whose (ij)th entry is $a_{ij}b_{ij}$. Finally, $\|\mathbf{A}\|_F^2$ represents the Frobenius norm [28] of matrix $\mathbf{A}$, which is defined as $\|\mathbf{A}\|_F^2 = \sum_{i=1}^{N}\sum_{j=1}^{M}|a_{ij}|^2$.

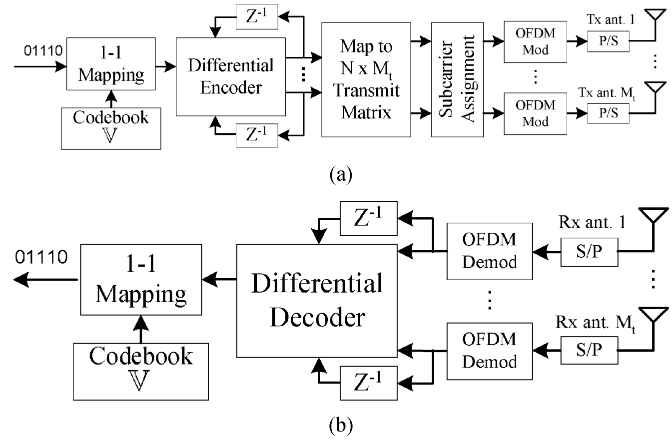

Fig. 1. Description of the differential MIMO-OFDM system. (a) Differential transmitter and (b) differential receiver.

II. System Description

We consider an MIMO wireless communication system equipped with M_t transmit and M_r receive antennas. Each antenna employs an OFDM modulator with N subcarriers, as shown in Fig. 1. The frequency-selective fading channel between transmit antenna i and receive antenna j is assumed to have L independent delay paths with arbitrary power delay profiles. The baseband equivalent channel between the ith transmit antenna and the jth receive antenna is modelled by a finite impulse response (FIR) filter as

$$h_{ij}^k(t) = \sum_{l=0}^{L-1} \alpha_{ij}^k(l)\delta(t-\tau_l) \quad (2)$$

where $\alpha_{ij}^k(l)$ is the multipath channel coefficient from the ith transmit antenna to the jth receive antenna at the kth OFDM block and τ_l represents the lth path delay. The $\alpha_{ij}^k(l)$ is modelled as zero-mean complex Gaussian random variable with variance $E\left|\alpha_{ij}^k(l)\right|^2 = \delta_l^2$. The channel coefficients are assumed to be spatially uncorrelated for different transmit–receive link. In each transmit–receive link, the power of the L independent delay paths is normalized such that $\sum_{l=0}^{L-1}\delta_l^2 = 1$. The frequency response of the channel in (2) is given by

$$H_{ij}^k(f) = \sum_{l=0}^{L-1} \alpha_{ij}^k(l)e^{-j2\pi f\tau_l}. \quad (3)$$

At the transmitter, a bit sequence is differentially encoded and mapped onto an $NM_t \times M_t$ transmit signal matrix

$$\mathbf{X}^k = \begin{pmatrix} x_1^k(0) & x_2^k(0) & \cdots & x_{M_t}^k(0) \\ x_1^k(1) & x_2^k(1) & \cdots & x_{M_t}^k(1) \\ \vdots & \vdots & \ddots & \vdots \\ x_1^k(N-1) & x_2^k(N-1) & \cdots & x_{M_t}^k(N-1) \end{pmatrix} \quad (4)$$

where $x_i^k(n)$ is a differentially encoded data symbol to be transmitted on the nth subcarrier from transmit antenna i during the kth OFDM symbol period. We assume that $\mathbf{X}^k$ is normalized to satisfy the energy constraint $E\|\mathbf{X}^k\|_F^2 = N$. We will explain details of the proposed differential encoding and decoding scheme in Section III. In order to transmit $\mathbf{X}^k$, each of the ith column of matrix $\mathbf{X}^k$ is OFDM modulated using N-point inverse fast Fourier transform (FFT) and augmented by cyclic prefix. The resulting OFDM symbol is transmitted from the ith transmit antenna. Note that all of the M_t OFDM symbols are transmitted simultaneously from different transmit antennas within one OFDM symbol period.

At each receive antenna, the receiver performs match filtering, cyclic prefix removing, and OFDM demodulating by N-point FFT. The received signal is a noisy superposition of transmitted symbols from multiple transmit antennas. We model the received signal of the nth subcarrier at the jth receive antenna during the kth OFDM symbol period as

$$y_j^k(n) = \sqrt{\rho}\sum_{i=1}^{M_t} x_i^k(n)H_{ij}^k(n) + w_j^k(n) \qquad (5)$$

where ρ is the average signal to noise ratio per receiver and

$$H_{ij}^k(n) = \sum_{l=0}^{L-1} \alpha_{ij}^k(l)e^{-j2\pi n\Delta f\tau_l} \qquad (6)$$

is the subchannel gain. Here, $\Delta f = 1/T_s$ is the intersubcarrier spacing and T_s is the OFDM symbol period. The additive complex Gaussian noise $w_j^k(n)$ has zero mean and unit variance $\mathcal{CN}(0,1)$. The additive noise is assumed to be statistically independent for different receive antennas j, subcarriers n, and OFDM symbol periods k. We observe from (5) that OFDM modem converts a frequency-selective fading channel into a set of parallel frequency-flat fading channels. The differential modulation scheme does not require the knowledge of channel state information at either the transmitter or the receiver. However, the subchannel gains are assumed constant over two OFDM symbol periods, i.e., $H_{ij}^k(n) \approx H_{ij}^{k-1}(n)$.

Let $\mathbf{y}_j^k = \left[y_j^k(0), y_j^k(1), \ldots, y_j^k(N-1)\right]^T$ be an $N\times 1$ vector comprising the receive signal at the jth received antenna during the kth OFDM symbol period. We can describe $\mathbf{y}_j^k$ as

$$\mathbf{y}_j^k = \sqrt{\rho}\,\mathcal{D}(\mathbf{X}^k)\mathbf{h}_j^k + \mathbf{w}_j^k, \quad j = 1,2,\ldots,M_r \qquad (7)$$

where $\mathcal{D}(\cdot)$ is defined in (1) and $\mathcal{D}(\mathbf{X}^k)$ represents an $N\times NM_t$ transmit signal matrix. The $NM_t\times 1$ channel gain vector $\mathbf{h}_j^k$ is represented by

$$\mathbf{h}_j^k = \left[(\mathbf{h}_{1j}^k)^T \;\cdots\; (\mathbf{h}_{M_tj}^k)^T\right]^T \qquad (8)$$

in which

$$\mathbf{h}_{ij}^k = \left[H_{ij}^k(0)\cdots H_{ij}^k(N-1)\right]^T \qquad (9)$$

and the noise vector has the form

$$\mathbf{w}_j^k = \left[w_j^k(0)\; w_j^k(1)\;\cdots\; w_j^k(N-1)\right]^T. \qquad (10)$$

By stacking all M_r receive signal vectors together, we obtain the $NM_r\times 1$ receive signal vector

$$\mathbf{y}^k = \sqrt{\rho}\left(\mathbf{I}_{M_r}\otimes\mathcal{D}(\mathbf{X}^k)\right)\mathbf{h}^k + \mathbf{w}^k \qquad (11)$$

where $\mathbf{y}^k = [(\mathbf{y}_1^k)^T\; (\mathbf{y}_2^k)^T\cdots(\mathbf{y}_{M_r}^k)^T]^T$, $\mathbf{h}^k = [(\mathbf{h}_1^k)^T\; (\mathbf{h}_2^k)^T\cdots(\mathbf{h}_{M_r}^k)^T]^T$, and $\mathbf{w}^k = [(\mathbf{w}_1^k)^T\; (\mathbf{w}_2^k)^T\cdots(\mathbf{w}_{M_r}^k)^T]^T$, in which $\mathbf{y}_j^k$, $\mathbf{h}_j^k$, and $\mathbf{w}_j^k$ are specified in (7), (8), and (10), respectively.

III. Single-Block Differential Transmit Scheme

In what follows, we propose a differential scheme for MIMO-OFDM systems under frequency-selective fading channels. By taking advantage of the coding strategy in [24], the proposed scheme is able to completely transmit the differentially encoded signal matrix within one OFDM symbol period. This allows us to relax the channel assumption for efficient differential decoding. Specifically, our scheme requires that the fading channels keep constant within only one OFDM block, and slowly change from one OFDM block to the next.

A. Transmit Signal Structure

We introduce a differential encoding/decoding scheme based on a transmit scheme proposed in [24]. Specifically, for an integer Γ, $1\le\Gamma\le L$, we partition the signal matrix $\mathbf{X}^k$ in (4) into $P = \lfloor N/(\Gamma M_t)\rfloor$ submatrices as follows [24]:

$$\mathbf{X}^k = \left[(\mathbf{X}_1^k)^T\; (\mathbf{X}_2^k)^T\;\cdots\;(\mathbf{X}_P^k)^T\;\; (\mathbf{0}_{N-P\Gamma M_t})^T\right]^T \qquad (12)$$

where $\mathbf{0}_{N-P\Gamma M_t}$ denotes an $(N-P\Gamma M_t)\times M_t$ zero padding matrix to be inserted if N cannot be divided by ΓM_t. The $\Gamma M_t\times M_t$ matrix $\mathbf{X}_p^k$, for $p = 1,2,\ldots,P$, is modelled as

$$\mathbf{X}_p^k = \mathrm{diag}_m(\mathbf{x}_{p,1}^k\;\mathbf{x}_{p,2}^k\;\cdots\;\mathbf{x}_{p,M_t}^k) \qquad (13)$$

where $\mathbf{x}_{p,i}^k$, for $i = 1,2,\ldots,M_t$, is a $\Gamma\times 1$ vector

$$\mathbf{x}_{p,i}^k = \left[s_{p,(i-1)\Gamma+1}^k\; s_{p,(i-1)\Gamma+2}^k\;\cdots\; s_{p,i\Gamma}^k\right]^T \qquad (14)$$

and all $s_{p,m}^k$, $m = 1,2,\ldots,\Gamma M_t$, are differentially encoded symbols that will be specified later.

We now specify information matrices as follows. For each p, $p = 1,2,\ldots,P$, let $\mathbf{V}_p^k$ denote a $\Gamma M_t\times\Gamma M_t$ unitary information matrix having diagonal form as

$$\mathbf{V}_p^k = \mathrm{diag}_v([v_{p,1}^k\; v_{p,2}^k\cdots\; v_{p,\Gamma M_t}^k]^T) \qquad (15)$$

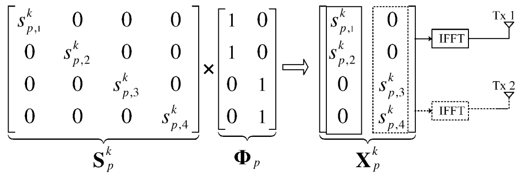

Fig. 2. An example of a code structure for $\Gamma = 2$ and $M_t = 2$ at the pth submatrix.

in which $v_{p,m}^k$, $m = 1, 2, \ldots, \Gamma M_t$, is an information symbol to be transmitted over subcarrier $(p-1)\Gamma M_t + m$ during the kth OFDM symbol period. We will independently design the matrix $\mathbf{V}_p^k$ for different p. The set of all possible information matrices constitutes a constellation $\mathbb{V}_p$. In order to support a data rate of R b/s/Hz, $\mathbb{V}_p$ is designed to have constellation size $\mathcal{L} = |\mathbb{V}_p| = 2^{R\Gamma M_t}$.

B. Differential Encoder and Transmission Matrix

We first encode a sequence of information bits into an information matrix as specified in (15). The information matrix is differentially encoded, and then transformed into a SF code structure in (13). All of the obtained P submatrices are concatenated to construct the code structure in (12) before being transmitted over the M_t transmit antennas. Our proposed differential encoding scheme is composed of a concatenation of two functional blocks, namely, a differential encoder and a multiplicative mapping matrix. An example of a code structure is shown in Fig. 2 for a case of $\Gamma = 2$ and $M_t = 2$ at the pth submatrix.

1) Differential Encoder: Let $\mathbf{S}_p^k$ be a $\Gamma M_t \times \Gamma M_t$ differential encoded signal matrix to be transmitted during the kth OFDM symbol period. We recursively construct $\mathbf{S}_p^k$ from the fundamental differential equation [11], [12]

$$\mathbf{S}_p^k = \begin{cases} \mathbf{V}_p^k\, \mathbf{S}_p^{k-1}, & k \geq 1 \\ \mathbf{I}_{\Gamma M_t}, & k = 0 \end{cases} \tag{16}$$

where the differential transmission initially sends $\mathbf{S}_p^0 = \mathbf{I}_{\Gamma M_t}$. The matrix $\mathbf{S}_p^k$ is also unitary since it results from recursive multiplication of unitary information matrices. Due to the diagonal structure of $\mathbf{V}_p^k$, $\mathbf{S}_p^k$ can be expressed as

$$\mathbf{S}_p^k = \text{diag}_v([s_{p,1}^k, s_{p,2}^k, \ldots, s_{p,\Gamma M_t}^k]^T) \tag{17}$$

where $s_{p,m}^k$, $m = 1, 2, \ldots, \Gamma M_t$, is the differentially encoded complex symbol to be transmitted at subcarrier $(p-1)\Gamma M_t + m$ during the kth OFDM block.

Note that, depending on how the elements of $\mathbf{S}_p^k$ are transmitted over M_t transmit antennas, the differential schemes can be different. The DSTF schemes in [16]–[19] transmit the $\mathbf{S}_p^k$ matrix through M_t OFDM modulators over multiple OFDM blocks. This leads to performance degradation when the fading channels do not stay constant over several OFDM blocks. In what follows, we introduce a multiplicative mapping matrix that allows us to transform $\mathbf{S}_p^k$ into the code structure in (13) and completely transmit $\mathbf{S}_p^k$ within one OFDM block. This not only improves system performance under rapid fading environment but also reduces encoding and decoding delay.

2) Multiplicative Mapping Matrix: We define the $\Gamma M_t \times M_t$ multiplicative mapping matrix as

$$\mathbf{\Phi}_p = [\boldsymbol{\phi}_1\ \boldsymbol{\phi}_2 \cdots \boldsymbol{\phi}_{M_t}] \tag{18}$$

in which $\boldsymbol{\phi}_i$ is a $\Gamma M_t \times 1$ vector

$$\boldsymbol{\phi}_i = \mathbf{e}_i \otimes \mathbf{1}_\Gamma, \qquad i = 1, \ldots, M_t \tag{19}$$

where $\mathbf{e}_i$ is an $M_t \times 1$ unit vector whose ith component is one and all others are zeroes. We postmultiply $\mathbf{S}_p^k$ by $\mathbf{\Phi}_p$, resulting in the $\Gamma M_t \times M_t$ transmission matrix

$$\mathbf{X}_p^k = \mathbf{S}_p^k \mathbf{\Phi}_p. \tag{20}$$

Consequently, the differentially encoded symbol $s_{p,m}^k$, as specified in (17), is transmitted at the $\lceil m/\Gamma \rceil$ transmit antenna.

C. Differential Decoder

From (11) and (12), the receive signal vector corresponding to the transmit signal matrix $\mathbf{X}_p^k$ in (13) is given by

$$\mathbf{y}_p^k = \sqrt{\rho}\left(\mathbf{I}_{M_r} \otimes \mathcal{D}(\mathbf{X}_p^k)\right)\mathbf{h}_p^k + \mathbf{w}_p^k \tag{21}$$

where $\mathcal{D}(\mathbf{X}_p^k)$ is a $\Gamma M_t \times \Gamma M_t M_t$ transmit matrix. The $\Gamma M_t M_t M_r \times 1$ channel vector $\mathbf{h}_p^k = \left[(\mathbf{h}_{p,1}^k)^T\ (\mathbf{h}_{p,2}^k)^T\ \cdots\ (\mathbf{h}_{p,M_r}^k)^T\right]^T$ comprises

$$\mathbf{h}_{p,j}^k = \left[(\mathbf{h}_{p,1j}^k)^T\ (\mathbf{h}_{p,2j}^k)^T\ \cdots\ (\mathbf{h}_{p,M_tj}^k)^T\right]^T \tag{22}$$

where

$$\mathbf{h}_{p,ij}^k = \left[H_{ij}^k((p-1)\Gamma M_t)\ \cdots\ H_{ij}^k(p\Gamma M_t - 1)\right]^T. \tag{23}$$

Similarly, the receive signal vector is $\mathbf{y}_p^k = \left[(\mathbf{y}_{p,1}^k)^T\ (\mathbf{y}_{p,2}^k)^T\ \cdots\ (\mathbf{y}_{p,M_r}^k)^T\right]^T$, where $\mathbf{y}_{p,j}^k = \left[y_j^k((p-1)\Gamma M_t)\ \cdots\ y_j^k(p\Gamma M_t - 1)\right]^T$. The noise vector $\mathbf{w}_p^k$ has the same form as $\mathbf{y}_p^k$ with $y_j^k(n)$ replaced by $w_j^k(n)$.

To perform differential decoding, two consecutive receive signal vectors in (21), i.e., $\mathbf{y}_p^k$ and $\mathbf{y}_p^{k-1}$, are required to recover the information matrix at each OFDM symbol period. Since the two consecutive receive signal vectors are related through the differentially encoded signal matrix $\mathbf{S}_p^k$ [see (16)], we will introduce the equivalent expression of $\left(\mathbf{I}_{M_r} \otimes \mathcal{D}(\mathbf{X}_p^k)\right)\mathbf{h}_p^k$ in terms of $\mathbf{S}_p^k$ for subsequent differential decoding.

From (18) and (20), we can express $\mathcal{D}(\mathbf{X}_p^k)$ as

$$\mathcal{D}(\mathbf{X}_p^k) = \left[\text{diag}_v(\mathbf{S}_p^k \boldsymbol{\phi}_1)\ \cdots\ \text{diag}_v(\mathbf{S}_p^k \boldsymbol{\phi}_{M_t})\right]. \tag{24}$$

According to (22) and (24), we have

$$\mathcal{D}(\mathbf{X}_p^k)\mathbf{h}_{p,j}^k = \sum_{i=1}^{M_t} \mathrm{diag}_v(\mathbf{S}_p^k \boldsymbol{\phi}_i)\mathbf{h}_{p,ij}^k \tag{25}$$

which can be rewritten as

$$\begin{aligned}\mathcal{D}(\mathbf{X}_p^k)\mathbf{h}_{p,j}^k &= \sum_{i=1}^{M_t}(\mathbf{S}_p^k \boldsymbol{\phi}_i) \circ \mathbf{h}_{p,ij}^k \\ &= \mathbf{S}_p^k \sum_{i=1}^{M_t} \boldsymbol{\phi}_i \circ \mathbf{h}_{p,ij}^k \triangleq \mathbf{S}_p^k\, \tilde{\mathbf{h}}_{p,j}^k. \end{aligned}\tag{26}$$

By substituting (23) into (26), we can express $\tilde{\mathbf{h}}_{p,j}^k$ as

$$\tilde{\mathbf{h}}_{p,j}^k = \left[(\tilde{\mathbf{h}}_{p,1j}^k)^T\ (\tilde{\mathbf{h}}_{p,2j}^k)^T \cdots (\tilde{\mathbf{h}}_{p,M_t j}^k)^T\right]^T \tag{27}$$

in which

$$\tilde{\mathbf{h}}_{p,ij}^k = [H_{ij}(n_{p,i}^0)\ H_{ij}(n_{p,i}^1)\ \cdots\ H_{ij}(n_{p,i}^{\Gamma-1})]^T \tag{28}$$

where $n_{p,i}^{\gamma} = (i-1)\Gamma + (p-1)\Gamma M_t + \gamma$ for $\gamma = 0, 1, \ldots, \Gamma-1$. Denoting $\tilde{\mathbf{h}}_p^k = \left[(\tilde{\mathbf{h}}_{p,1}^k)^T\ (\tilde{\mathbf{h}}_{p,2}^k)^T \cdots\ (\tilde{\mathbf{h}}_{p,M_r}^k)^T\right]^T$ as a $\Gamma M_t M_r \times 1$ channel gain vector and using (26), we obtain an equivalent expression

$$\left(\mathbf{I}_{M_r} \otimes \mathcal{D}(\mathbf{X}_p^k)\right)\mathbf{h}_p^k = \left(\mathbf{I}_{M_r} \otimes \mathbf{S}_p^k\right)\tilde{\mathbf{h}}_p^k. \tag{29}$$

For notation convenience, let us define $\boldsymbol{\mathcal{S}}_p^k \triangleq \left(\mathbf{I}_{M_r} \otimes \mathbf{S}_p^k\right)$ and $\boldsymbol{\mathcal{V}}_p^k \triangleq \left(\mathbf{I}_{M_r} \otimes \mathbf{V}_p^k\right)$ such that

$$\boldsymbol{\mathcal{S}}_p^k = \left(\mathbf{I}_{M_r} \otimes \mathbf{V}_p^k\right)\boldsymbol{\mathcal{S}}_p^{k-1} = \boldsymbol{\mathcal{V}}_p^k \boldsymbol{\mathcal{S}}_p^{k-1}. \tag{30}$$

Hence, using (29) and (30), we can rewrite the two consecutive receive signal vectors in (21) as

$$\mathbf{y}_p^{k-1} = \sqrt{\rho}\boldsymbol{\mathcal{S}}_p^{k-1}\tilde{\mathbf{h}}_p^{k-1} + \mathbf{w}_p^{k-1} \tag{31}$$

$$\mathbf{y}_p^k = \sqrt{\rho}\boldsymbol{\mathcal{S}}_p^k\tilde{\mathbf{h}}_p^k + \mathbf{w}_p^k. \tag{32}$$

We relate the equivalent terms of (31) and (32) through (30) and assume that the channel coefficients are almost constant over two consecutive OFDM blocks, i.e., $\tilde{\mathbf{h}}_p^k \approx \tilde{\mathbf{h}}_p^{k-1} \approx \tilde{\mathbf{h}}_p$; then we obtain

$$\mathbf{y}_p^k = \boldsymbol{\mathcal{V}}_p^k \mathbf{y}_p^{k-1} + \sqrt{2}\,\tilde{\mathbf{w}}_p^k \tag{33}$$

where $\tilde{\mathbf{w}}_p^k = (1/\sqrt{2})\left(\mathbf{w}_p^k - \boldsymbol{\mathcal{V}}_p^k\, \mathbf{w}_p^{k-1}\right)$ is a noise vector whose element is $\mathcal{CN}(0,1)$ distributed. Without acquiring channel state information, the differential decoder performs maximum likelihood decoding, and the decision rule can be stated as

$$\hat{\boldsymbol{\mathcal{V}}}_p^k = \arg\min_{\boldsymbol{\mathcal{V}}_p^k \in \mathbb{V}_p} \|\mathbf{y}_p^k - \boldsymbol{\mathcal{V}}_p^k\,\mathbf{y}_p^{k-1}\|_F^2. \tag{34}$$

It is worth mentioning that the detector is able to differentially decode within two OFDM symbol periods regardless of the number of transmit antennas. Therefore, our proposed scheme significantly reduces the decoding delay compared to the DSTF schemes. Note also that the proposed scheme includes the differential scheme in [15] for a single-antenna OFDM system as a special case. In terms of decoding complexity, the proposed scheme yields the same decoding complexity as the previously proposed schemes in [18] and [19]. Specifically, the decoding complexity of the proposed scheme depends exponentially on $R\Gamma M_t$. However, the decoding complexity can be reduced by using the fast decoding technique in [25]. Based on the lattice reduction algorithm, it was shown in [25] that the decoding complexity is polynomial in ΓM_t.

IV. Pairwise Error Probability and Design Criteria

The previous section described the proposed differential encoding and decoding scheme. In this section, we show its average pairwise error probability (PEP) and design criteria under the assumption of frequency-selective channel model in Section II. We provide a PEP formulation based on the results in [26], which showed the asymptotic PEP for differential detection under correlated Rayleigh fading channels.

Suppose that $\boldsymbol{\mathcal{V}}_p^k$ and $\hat{\boldsymbol{\mathcal{V}}}_p^k$ are two different information matrices. With the assumption of slow fading channels, the average PEP is upper bounded by [26, Prop. 7]

$$P\left(\boldsymbol{\mathcal{V}}_p^k \to \hat{\boldsymbol{\mathcal{V}}}_p^k\right) \le \binom{2\nu-1}{\nu}\left(\prod_{m=1}^{\nu}\beta_{p,m}\right)^{-1}\left(\frac{\rho}{2}\right)^{-\nu} \tag{35}$$

where ρ represents the signal-to-noise ratio per symbol, ν is the rank, and $\beta_{p,m}$s are the nonzero eigenvalues of the matrix

$$\boldsymbol{\Psi}_p \triangleq \boldsymbol{\mathcal{S}}_p^{k-1}\boldsymbol{\Sigma}_{\tilde{\mathbf{h}}_p}(\boldsymbol{\mathcal{S}}_p^{k-1})^H\left(\boldsymbol{\mathcal{V}}_p^k - \hat{\boldsymbol{\mathcal{V}}}_p^k\right)^H\left(\boldsymbol{\mathcal{V}}_p^k - \hat{\boldsymbol{\mathcal{V}}}_p^k\right) \tag{36}$$

in which $\boldsymbol{\Sigma}_{\tilde{\mathbf{h}}_p} = E[\tilde{\mathbf{h}}_p\tilde{\mathbf{h}}_p^H]$ denotes the correlation matrix of channel vector $\tilde{\mathbf{h}}_p$. Note that the PEP upper bound in (35) is a function of $\rho/2$, which corresponds to the 3-dB performance loss when compared to its coherent counterpart.

To simplify the expression for matrix $\boldsymbol{\Psi}_p$ in (36), we evaluate the channel correlation matrix $\boldsymbol{\Sigma}_{\tilde{\mathbf{h}}_p}$ as follows. First, we rewrite the channel frequency response in (6) as

$$H_{ij}^k(n) = \boldsymbol{\omega}^T(n)\,\mathbf{a}_{ij}^k \tag{37}$$

where $\mathbf{a}_{ij}^k = [\alpha_{ij}^k(0)\, \alpha_{ij}^k(1) \cdots \alpha_{ij}^k(L-1)]^T$ is an $L\times 1$ matrix of path gain coefficients, $\boldsymbol{\omega}(n) = [\omega^{n\tau_0}\, \omega^{n\tau_1} \cdots \omega^{n\tau_{L-1}}]^T$, and $\omega \triangleq e^{-j2\pi\Delta f}$. Thus, we can represent $\tilde{\mathbf{h}}_{p,ij}^k$ in (28) as

$$\tilde{\mathbf{h}}_{p,ij}^k = \boldsymbol{\Omega}_{p,i}\, \mathbf{a}_{ij}^k \tag{38}$$

where $\boldsymbol{\Omega}_{p,i} = \left[\boldsymbol{\omega}(n_{p,i}^0)\, \boldsymbol{\omega}(n_{p,i}^1) \cdots \boldsymbol{\omega}(n_{p,i}^{\Gamma-1})\right]^T \in \mathcal{C}^{\Gamma\times L}$. Substituting (38) into (27), we have

$$\tilde{\mathbf{h}}_{p,j}^k = \boldsymbol{\Omega}_p\, \mathbf{a}_j^k \tag{39}$$

where $\boldsymbol{\Omega}_p = \mathrm{diag}_m\left(\boldsymbol{\Omega}_{p,1}, \boldsymbol{\Omega}_{p,2}, \ldots, \boldsymbol{\Omega}_{p,M_t}\right) \in \mathcal{C}^{\Gamma M_t \times L M_t}$, and $\mathbf{a}_j^k = \left[(\mathbf{a}_{1j}^k)^T (\mathbf{a}_{2j}^k)^T \cdots (\mathbf{a}_{M_t j}^k)^T\right]^T \in \mathcal{C}^{LM_t\times 1}$. Based on (39) and the assumption that each transmit–receive link has the same power delay profile, we can calculate the correlation matrix of channel vector $\tilde{\mathbf{h}}_{p,j}^k$ as

$$\boldsymbol{\Sigma}_{\tilde{\mathbf{h}}_{p,j}} = E[\tilde{\mathbf{h}}_{p,j}^k (\tilde{\mathbf{h}}^k)_{p,j}^H] = \boldsymbol{\Omega}_p\, \left(\mathbf{I}_{M_t} \otimes \boldsymbol{\Lambda}_{\delta^2}\right) \boldsymbol{\Omega}_p^H \tag{40}$$

where $\boldsymbol{\Lambda}_{\delta^2} = \mathrm{diag}(\delta_0^2, \ldots, \delta_{L-1}^2)$ represents an $L \times L$ diagonal matrix of power delay profile. Observe from (40) that $\boldsymbol{\Sigma}_{\tilde{\mathbf{h}}_{p,j}}$ is the same for all js. Consequently, denoting $\boldsymbol{\Sigma} \triangleq \boldsymbol{\Sigma}_{\tilde{\mathbf{h}}_{p,j}}$, we can express the correlation matrix $\boldsymbol{\Sigma}_{\tilde{\mathbf{h}}_p}$ as

$$\boldsymbol{\Sigma}_{\tilde{\mathbf{h}}_p} = \mathbf{I}_{M_r} \otimes \boldsymbol{\Sigma}. \tag{41}$$

Applying the property of tensor product $(\mathbf{A}_1 \otimes \mathbf{B}_1)(\mathbf{A}_2 \otimes \mathbf{B}_2)(\mathbf{A}_3 \otimes \mathbf{B}_3) = (\mathbf{A}_1\mathbf{A}_2\mathbf{A}_3 \otimes \mathbf{B}_1\mathbf{B}_2\mathbf{B}_3)$ [28, p. 251] to (36), we obtain

$$\boldsymbol{\Psi}_p = \mathbf{I}_{M_r} \otimes \boldsymbol{\Theta}_p \tag{42}$$

in which

$$\boldsymbol{\Theta}_p = \mathbf{S}_p^{k-1} \boldsymbol{\Sigma} (\mathbf{S}_p^{k-1})^H \boldsymbol{\Delta} \tag{43}$$

and $\boldsymbol{\Delta} = \left(\mathbf{V}_p^k - \hat{\mathbf{V}}_p^k\right)^H \left(\mathbf{V}_p^k - \hat{\mathbf{V}}_p^k\right)$. Hence, by (42), the PEP in (35) can be expressed as

$$P\left(\mathbf{V}_p \to \hat{\mathbf{V}}_p\right) \le \binom{2rM_r - 1}{rM_r} \left(\prod_{m=1}^{r} \lambda_{p,m}\right)^{-M_r} \left(\frac{\rho}{2}\right)^{-rM_r} \tag{44}$$

where r is the rank and $\lambda_{p,m}$s are the nonzero eigenvalues of the matrix $\boldsymbol{\Theta}_p$.

The PEP upper bound in (44) suggests two design criteria.

1) *Rank criterion:* For any $\mathbf{V}_p^k \neq \hat{\mathbf{V}}_p^k$, design a constellation set of unitary matrices $\mathbb{V}_p$ such that the minimum rank of $\boldsymbol{\Theta}_p$ is maximized.

2) *Product criterion:* For any $\mathbf{V}_p^k \neq \hat{\mathbf{V}}_p^k$, design a constellation set of unitary matrices $\mathbb{V}_p$ such that the minimum value of the product $\prod_{m=1}^{r} \lambda_{p,m}$ is maximized.

To quantify the maximum achievable diversity order, we substitute (40) into (43) and reexpress $\boldsymbol{\Theta}_p$ as

$$\boldsymbol{\Theta}_p = \mathbf{S}_p^{k-1} \boldsymbol{\Omega}_p \left(\mathbf{I}_{M_t} \otimes \boldsymbol{\Lambda}_{\delta^2}\right) \boldsymbol{\Omega}_p^H \left(\mathbf{S}_p^{k-1}\right)^H \boldsymbol{\Delta}. \tag{45}$$

Observe from (45) that $\mathbf{S}_p^{k-1}$ and $\mathbf{V}_p^k$ are of size $\Gamma M_t \times \Gamma M_t$, the correlation matrix $\boldsymbol{\Omega}_p$ is of size $\Gamma M_t \times LM_t$, and $\mathbf{I}_{M_t} \otimes \boldsymbol{\Lambda}_{\delta^2}$ is an $LM_t \times LM_t$ diagonal matrix. Since $\Gamma \le L$, the rank of $\boldsymbol{\Theta}_p$ is at most ΓM_t. Therefore, the maximum achievable diversity gain is

$$G_d^{\max} = M_r \max\left(\min_{\forall\, \mathbf{V}_p^k \neq \hat{\mathbf{V}}_p^k} rank\left(\boldsymbol{\Theta}_p\right)\right) = \Gamma M_t M_r. \tag{46}$$

When the maximum diversity order is achieved, the maximum product criterion is determined by the normalized coding advantage or the so-called diversity product [12], [24]

$$\zeta = \frac{1}{2} \min_{\mathbf{V}_p^k \neq \hat{\mathbf{V}}_p^k} \left| \prod_{m=1}^{\Gamma M_t} \lambda_{p,m} \right|^{1/2\Gamma M_t}. \tag{47}$$

We can calculate the product of the nonzero eigenvalues of the matrix $\boldsymbol{\Theta}_p$ as

$$\begin{aligned}\prod_{m=1}^{\Gamma M_t} \lambda_{p,m} &= det\left(\mathbf{S}_p^{k-1} \boldsymbol{\Omega}_p \left(\mathbf{I}_{M_t} \otimes \boldsymbol{\Lambda}_{\delta^2}\right) \boldsymbol{\Omega}_p^H \left(\mathbf{S}_p^{k-1}\right)^H\right) \\ &\quad \times det\left(\left(\mathbf{V}_p^k - \hat{\mathbf{V}}_p^k\right)^H \left(\mathbf{V}_p^k - \hat{\mathbf{V}}_p^k\right)\right) \\ &= \prod_{i=1}^{M_t} det\left(\boldsymbol{\Omega}_{p,i} \boldsymbol{\Lambda}_{\delta^2} \boldsymbol{\Omega}_{p,i}^H\right) \prod_{m=1}^{\Gamma M_t} \left|v_{p,m}^k - \hat{v}_{p,m}^k\right|^2\end{aligned} \tag{48}$$

where $\mathbf{V}_p^k - \hat{\mathbf{V}}_p^k = diag(v_{p,1}^k - \hat{v}_{p,1}^k, \ldots, v_{p,\Gamma M_t}^k - \hat{v}_{p,\Gamma M_t}^k)$. Substitute (48) into (47), resulting in

$$\zeta = \left|\prod_{i=1}^{M_t} det\left(\boldsymbol{\Omega}_{p,i} \boldsymbol{\Lambda}_{\delta^2} \boldsymbol{\Omega}_{p,i}^H\right)\right|^{1/2\mathcal{M}} \frac{1}{2} \min_{\mathbf{V}_p^k \neq \hat{\mathbf{V}}_p^k} \prod_{m=1}^{\mathcal{M}} \left|v_{p,m}^k - \hat{v}_{p,m}^k\right|^{1/\mathcal{M}} \tag{49}$$

in which $\mathcal{M} = \Gamma M_t$.

Observe from (49) that ζ can be maximized by designing the two terms on the right-hand side separately. The first term depends only on the power delay profile, and it can be maximized by the use of proper subcarrier selection method, e.g., the subcarrier grouping method [15] or, more generally, the optimum permutation strategy proposed in [24]. In this paper, however, we resort to random permutation strategy to enable fair performance comparison between the proposed scheme and the previously proposed scheme in [19]. The second term in (49) also relies on the code structure. It is a challenging task to find such a good diagonal signal constellation. One method is to adopt the diagonal cyclic group code design in [12], which is systematically designed well and applicable for MIMO systems with any number of transmit antennas and any transmission rates. In particular, for a specific integer $\mathcal{M}$ and transmission rate R such that the constellation size $\mathcal{L} = 2^{R\mathcal{M}}$, a set of parameters used to fully specify the signal constellation $\mathbb{V}_p$ is denoted as $G_{\mathcal{M},\mathcal{L}} = (\mathcal{M}, \mathcal{L}, [u_1, u_2, \ldots, u_{\mathcal{M}}])$ [12].

V. Simulation Results

We provide in this section computer simulation results to illustrate performances of our proposed differential scheme in comparison with the previously existing schemes. In the following simulations, each OFDM modulator utilized $N = 128$ subcarriers with the total bandwidth of 1 MHz. The corresponding OFDM symbol period was $T_s = 1/\Delta f = 128$ μs. We added a guard interval of 20 μs against intersymbol interference due to channel multipath delay spread. We used a simple two-ray and a more realistic typical urban (TU) six-ray power delay profile. Each delay path of the two-ray profile had equal power with delay $\tau = 20$ μs. The fading channels are assumed constant within each OFDM block and slow varying from one OFDM block to another according to the Jakes fading model [29] with f_D representing the maximum Doppler frequency in hertz. The thermal noise was complex Gaussian random variable with zero mean and variance $N_o = 1$.

We simulated the system under different mobile environment by varying the normalized Doppler frequencies, namely, $f_D T_s = 0.0025, 0.005, 0.01,$ and 0.025, which correspond to mobile speeds of 6, 13, 26, and 65 m/s, respectively. The performance curves are demonstrated in terms of average bit error rate (BER) versus average signal energy per bit (E_b/N_o) in decibels. We compare the performance of our proposed differential scheme (showed in solid lines) to that of an existing DSTF scheme in [19] (showed in dashed lines) with the same rate R. The random permutation strategy, in which the nth subcarrier is moved to the $\tilde{n}$th subcarrier, follows the Takeshita–Costello method as [27] $\tilde{n} = mod(n(n+1)/2, N) + 1$, $n = 1, 2, \ldots, N$.

In Fig. 3, we first investigate the effect of varying Γ to the diversity order by simulating the proposed scheme employing $M_t = 2$ and $M_r = 1$ under the TU six-ray power delay profile. For $R = 1$ b/s/Hz (omitting cyclic-prefix and guard interval), we chose $\Gamma = 1, 2,$ and 3. Their corresponding signal constellations are $G_{2,4} = (2, 4, [1, 1])$, $G_{4,16} = (4, 16, [1, 3, 5, 7])$, and $G_{6,64} = (6, 64, [1, 9, 15, 17, 23, 25])$, respectively. In this case, the transmit matrix (20) is given by

$$\mathbf{X}_p^k = \begin{bmatrix} s_{p,1}^k & \ldots & s_{p,\Gamma}^k & 0 & \ldots & 0 \\ 0 & \ldots & 0 & s_{p,\Gamma+1}^k & \ldots & s_{p,\Gamma M_t}^k \end{bmatrix}^T. \quad (50)$$

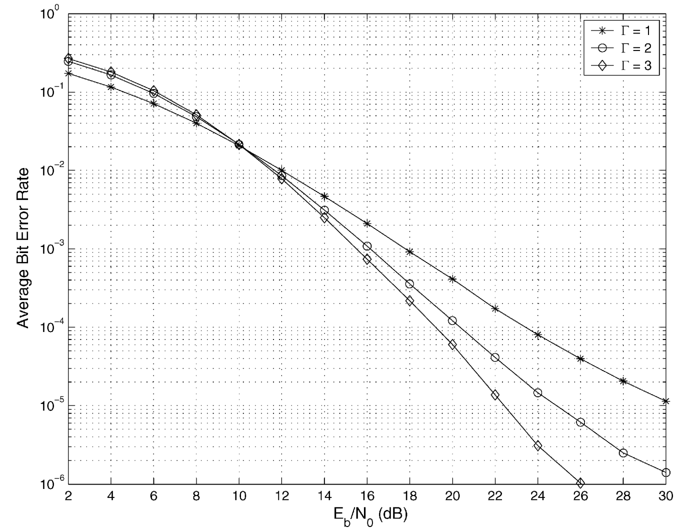

Fig. 3. Performance for $M_t = 2, M_r = 1, R = 1$ b/s/Hz and $\Gamma = 1, 2,$ and 3 under the TU power delay profile.

As clearly see from the figure, the diversity order of the proposed scheme increases with the number of jointly encoded subcarriers Γ. This observation supports our analytical analysis in (46) that the diversity order is proportional to the value of Γ. Hence, increasing Γ results in significant performance improvement especially in the high SNR regime.

Fig. 4 depicts the simulation results for a system with $M_t = 2, M_r = 1$, and $\Gamma = 2$ in two-ray power delay profile. We chose $R = 1.5$ b/s/Hz and generated signal constellation by $G_{4,64} = (4, 64, [1, 17, 45, 53])$. It is apparent that the performances of our proposed scheme are superior to that of the previously proposed scheme [19] in every normalized Doppler frequency. For instance, in the case of fading channels with $f_D T_s = 0.0025$ and 0.005, our proposed scheme yields almost the same performance of BER$\approx 5 \times 10^{-5}$ at E_b/N_0 of 24 dB, which outperforms that of previous schemes that achieved BER$= 1.5 \times 10^{-4}$. When fading rate increases from 0.005 to 0.01, the performances of our proposed scheme and the previous scheme degrade to BER$= 1.22 \times 10^{-4}$ and 4.5×10^{-4}, respectively, at $E_b/N_0 = 24$ dB. Observe that the performance of the previous scheme degrades faster than that of our proposed scheme. In other words, in the case of $f_D T_s = 0.0025$ and 0.005, the proposed scheme outperforms the previous scheme about 2 dB at a BER of 10^{-4}. When the fading rate is 0.01, our proposed scheme achieves more than 6 dB performance improvement at a BER of 10^{-4} compared to that of previous scheme. For a more rapid fading at $f_D T_s = 0.025$, the previous scheme degrades even faster from BER$= 1.5 \times 10^{-4}$ to 6.81×10^{-3} and nearly reaches the error floor, while the performance of our proposed scheme degrades from BER$\approx 5 \times 10^{-5}$ to 5.2×10^{-4}. This confirms our expectation that by coding within only one OFDM block, our proposed scheme is robust to the effect of rapid channel variation. In contrast, the DSTF scheme relies on constant channel over several OFDM blocks, thereby more susceptible to rapid fading condition. Note that in all figures, we provide simulation results for coherent detections of our scheme for $f_D T_s = 0.0025$. The 3 dB performance loss due to differential detection can be observed.

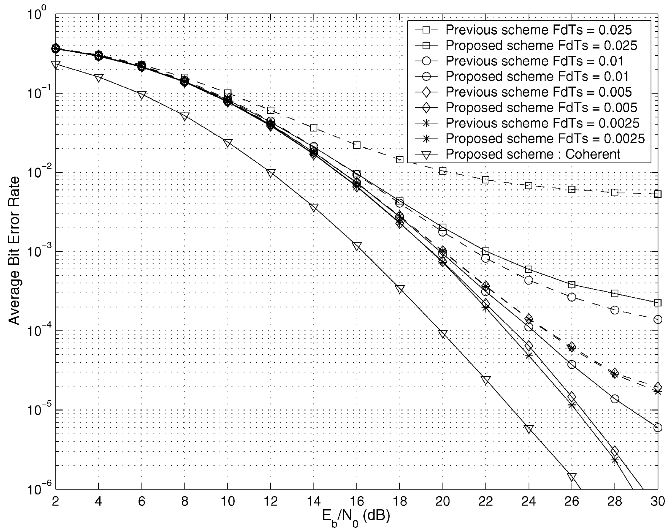

Fig. 4. Performance for $M_t = 2, M_r = 1, \Gamma = 2$, and $R = 1.5$ b/s/Hz under the two-ray power delay profile.

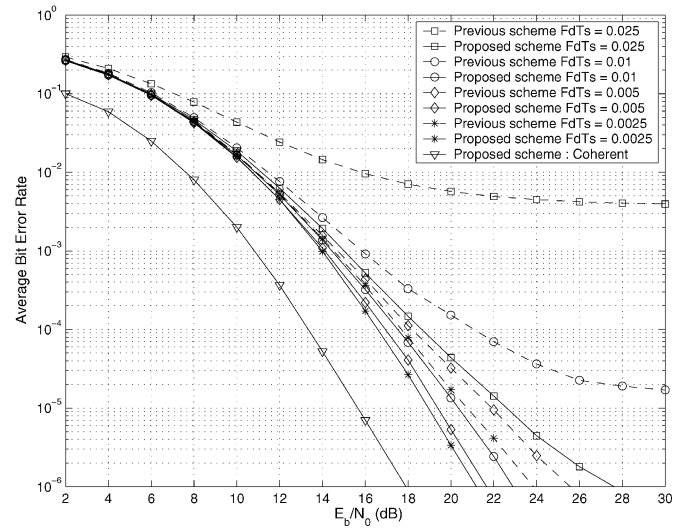

Fig. 6. Performance for $M_t = 3, M_r = 1, \Gamma = 2$, and $R \approx 1$ b/s/Hz under the two-ray power delay profile.

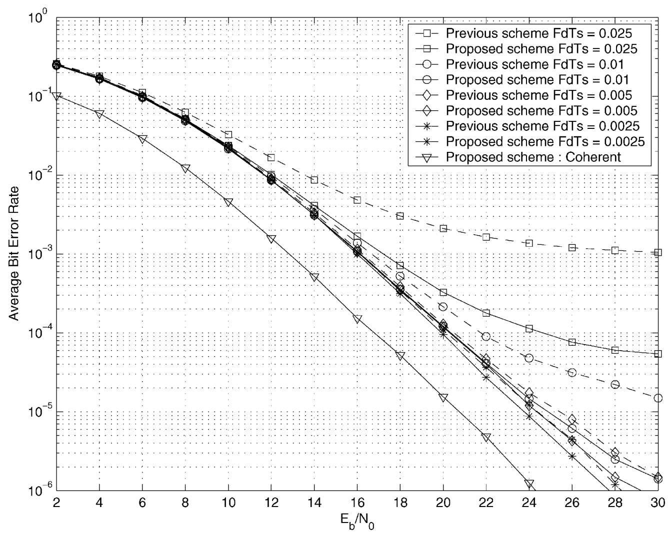

Fig. 5. Performance for $M_t = 2, M_r = 1, \Gamma = 2$, and $R = 1$ b/s/Hz under the TU power delay profile.

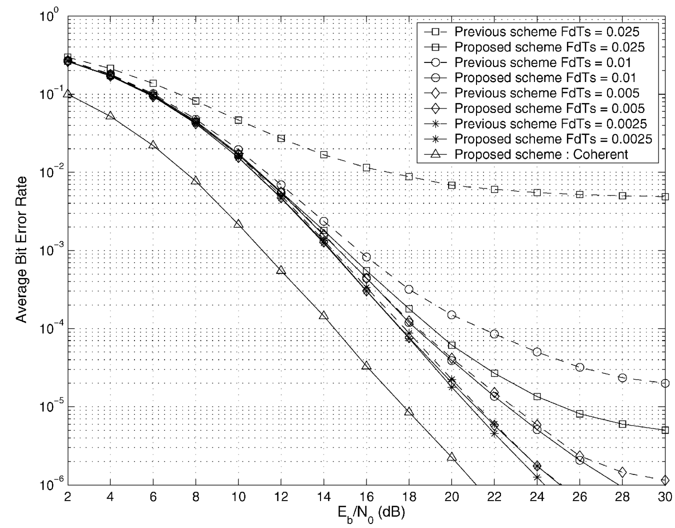

Fig. 7. Performance for $M_t = 3, M_r = 1, \Gamma = 2$, and $R \approx 1$ b/s/Hz under the TU power delay profile.

The performance under the TU power delay profile is shown in Fig. 5 for $M_t = 2$, $M_r = 1$, $\Gamma = 2$, and $R = 1$ b/s/Hz in which $G_{4,16}$ is used. Observe that under slow fade rates, i.e., $f_D T_s = 0.0025$ and 0.005, our scheme yields slightly better performances than those in previous scheme at E_b/N_0 of 22 dB. Significant performance difference can be observed when $f_D T_s = 0.01$. In this case, our proposed scheme achieves an average BER of 4.13×10^{-5} at $E_b/N_0 = 22$ dB, whereas the previous scheme has a BER of 9.0×10^{-5}. The performance of the proposed scheme is better than that of previous scheme about 2 dB at a BER of 10^{-4}. When $f_D T_s$ increases from 0.01 to 0.025, the BER of the previous scheme severely degrades to 1.75×10^{-3} at E_b/N_0 of 22 dB, while the BER of our proposed scheme slightly degrades to 1.92×10^{-4}.

The superior performance of our proposed scheme over the previous scheme can be obviously seen in case of $M_t = 3$ and $M_r = 1$ in Figs. 6 and 7 for the two-ray and the TU power delay profiles, respectively. For $\Gamma = 2$ and $R \approx 1$ b/s/Hz (due to zero padding insertion), we generated the signal constellation by $G_{6,64}$ in which the differential encoded signal matrix has the following structure:

$$\mathbf{X}_p^k = \begin{pmatrix} s_{p,1}^k & s_{p,2}^k & 0 & 0 & 0 & 0 \\ 0 & 0 & s_{p,3}^k & s_{p,4}^k & 0 & 0 \\ 0 & 0 & 0 & 0 & s_{p,5}^k & s_{p,6}^k \end{pmatrix}^T.$$

Similar to the case of two transmit antennas, our scheme yields better performance and is more robust to channel fading conditions than the previous scheme. In the case of fast fading, e.g., $f_D T_s = 0.025$, the performance degradation is significant and high error floor can be observed in the previous scheme. In contrast, the performance of our proposed scheme slightly degrades with an acceptable error floor.

VI. Conclusion

We proposed in this paper a differential scheme for MIMO-OFDM systems that can differentially encode a signal within two OFDM blocks, and where the transmission of differentially encoded signal is completed in one OFDM block. The scheme allows us to relax the channel assumption to keep constant during each OFDM block and slowly change from a duration of one OFDM block to another, rather than multiple OFDM blocks, as assumed in the previously existing works. We formulated the pairwise error probability and design criteria and showed that our scheme achieves maximum diversity order by utilizing an existing diagonal cyclic codes. Compared to the previous scheme, the proposed scheme not only is robust to the effect of rapid channel variation but also reduces encoding and decoding delay. Simulation results showed that our proposed scheme yields better performance than those previously proposed in all of the fading conditions and different power delay profiles. In particular, for an MIMO-OFDM system with two transmit and one receive antennas under the two-ray power delay profile, the proposed scheme outperforms the previous scheme about 2 dB in case of $f_D T_s = 0.0025$ and 0.005 at a BER of 10^{-4}. The performance improvement of more than 6 dB is observed when the fading rate is 0.01. Moreover, in the case of TU power delay profile with $f_D T_s = 0.01$, our proposed scheme achieves 2 dB performance improvement at a BER of 10^{-4} compared to the previous scheme.

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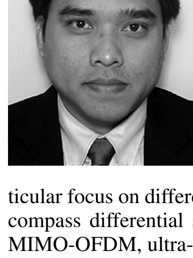

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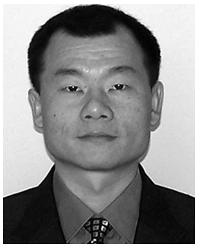
His current research interests include signal processing, wireless communications, cooperative communications, and wireless sensor networks, with particular focus on differential modulation systems. His research contributions encompass differential space-time coding and modulation for MIMO systems, MIMO-OFDM, ultra-wide-band systems, and cooperative communications.



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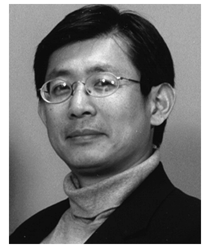
His research interests span a broad range of areas from signal processing to wireless communications and networking, including space-time coding and modulation for MIMO wireless communications, MIMO-OFDM systems, cooperative communications for wireless networks, and ultra-wide-band communications. He has published more than 20 journal papers and 30 conference papers in the related areas. He joined the Department of Electrical Engineering at the State University of New York (SUNY) at Buffalo as an Assistant Professor in March 2005. From June 2002 to March 2005, he was a Postdoctoral Research Associate with the Department of Electrical and Computer Engineering and the Institute for Systems Research (ISR), University of Maryland, College Park.

Dr. Su received the Signal Processing and Communications Faculty Award from the University of Delaware in 2002 as an outstanding graduate student in the field of signal processing and communications. In 2005, he received the Outstanding Invention Award from the Office of Technology Commercialization.



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He is Professor and Director of Communications and Signal Processing Laboratories with the Electrical and Computer Engineering Department and Institute for Systems Research, University of Maryland, College Park. His research contributions encompass broad aspects of wireless communications and networking, information forensics and security, multimedia communications and signal processing, bioinformatics and biomedical imaging, and signal-processing algorithms and architectures. He was founding Editor-in-Chief of the *EURASIP Journal on Applied Signal Processing*.

Dr. Liu has received numerous honors and awards including best paper awards from the IEEE Signal Processing Society, IEEE Vehicular Technology Society, and EURASIP. He has been an IEEE Signal Processing Society Distinguished Lecturer. He received the EURASIP Meritorious Service Award and the National Science Foundation Young Investigator Award. He also received the Poole and Kent Company Senior Faculty Teaching Award from the A. James Clark School of Engineering and the Invention of the Year Award, both from the University of Maryland. Dr. Liu is Vice President–Publication and a member of the Board of Governors of the IEEE Signal Processing Society. He was Editor-in-Chief of IEEE SIGNAL PROCESSING MAGAZINE and the prime proposer and architect of the new IEEE TRANSACTIONS ON INFORMATION FORENSICS AND SECURITY.