

Cooperative Wireless Multicast: Performance Analysis and Power/Location Optimization

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Abstract—The popularity of multimedia multicast/broadcast applications over wireless networks makes it critical to address the error-prone, heterogeneous and dynamically changing nature of wireless channels. A promising solution to combat channel fading is to explore the cooperative diversity in which users may help each other forward packets. This paper investigates cooperative multicast schemes that use a maximal ratio combiner to enhance the received signal-to-noise ratio (SNR), and provides a thorough performance analysis. Two relay selection schemes are considered: the distributed and the genie-aided cooperation schemes. We derive the closed-form formulation and the approximations of their average outage probabilities. We also analyze the optimal power allocation and relay location strategies, and show that allocating half of the total transmission power to the source minimizes the average outage probability. Our analysis and simulation results show that cooperative multicast gives better performance when more relays help forward signals. Cooperative multicast helps achieve diversity order 2, and user cooperation can significantly reduce the outage probability, especially in the high SNR region. Finally, we compare the two cooperation strategies, and show that distributed cooperative multicast is preferred since it achieves a lower outage probability without introducing extra overhead for control messages.

Index Terms—Cooperative wireless multicast, cooperative relaying, optimum power allocation, relay location optimization, outage probability.

I. INTRODUCTION

WITH recent advance in communications, networking and signal processing technologies, we witness the emergence of multimedia broadcast and multicast applications over wireless networks, where multimedia data are delivered to a group of users simultaneously. Examples include the Internet Protocol television (IPTV) over WiMax [1] and multimedia broadcast/multicast service (MBMS) within 3GPP [2]. However, due to the error-prone and dynamically changing characteristics of wireless fading channels, multimedia multicast over wireless medium is very challenging. To further proliferate multimedia applications over wireless networks, it is of crucial importance to combat inherent channel fading, path loss, and shadowing effects in wireless channels in order to provide reliable and satisfactory service.

Various spatial/temporal/frequency diversity techniques provide effective solutions to enhance the reliability of wireless

links, in which a destination receives multiple distorted versions of the original signal and uses these signals collectively to reduce detection error rate and to improve system performance. A well-known approach is to use multiple-input-multiple-output (MIMO) techniques to exploit the spatial diversity [3]. Recently, an emerging concept - *cooperative communication* - provides a new communication view, where users in a wireless network help each other forward packets to improve system performance [4]. Several recent works [5], [6] addressed the information-theoretic aspects of the cooperative communications, while many efforts have also been focused on the design of cooperative communication protocols to combat severe fading in wireless channels. Specifically, various cooperation protocols were proposed for wireless networks [7], [8] in which a user/node may help others to forward information serving as a relay. As users or nodes in a wireless network cooperate with each other by receiving and forwarding information, each user may decode the received information and forward the decoded symbol, which results in a *decode-and-forward* (DF) cooperation protocol. Or it may simply amplify and forward the message, which results in an *amplify-and-forward* (AF) cooperation protocol. The concept of user cooperation was also studied in [9], [10] where a specific two-user cooperation scheme was investigated for CDMA systems and substantial performance gain was demonstrated with comparison to the non-cooperative approach. More recently, the cooperative communication protocols were further analyzed and generalized to multi-node scenario (see [11]–[13] and the references therein).

The above prior works focused on optimization of cooperation schemes for point-to-point communications where the primary consideration is for one intended receiver/user. There have been works on cooperative multicast/broadcast over wireless networks, where a group of intended users receive the same data/video service from the source [14], [15]. To achieve user cooperation in multicast/broadcast applications, users who correctly decode the message sent by the source serve as relays and forward the message to others [16]. Same as the point-to-point cooperative communications, focusing on the 2-hop transmission of data, there are two stages (phases) in cooperative multicast/broadcast: the source (e.g., the base station) transmits the message in the first stage, while the selected relays forward the message in stage 2. The work in [15] assumed that the selected relays transmit in orthogonal channels, e.g., TDMA, FDMA or CDMA, and investigated the optimal relay scheduling and power allocation strategies to minimize the total power consumption. The authors in [17] considered cooperative multicast of videos over wireless

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networks. Same as in [15], [16], the relays used TDMA and took turns to forward packets, and layered video coding was used to provide users with different video quality depending on their channel conditions. The work in [17] studied the optimal rate adaptation and relay selection strategies.

Using orthogonal channels for different relays enables users in the multicast/broadcast group to combine the signals received from all nodes and to maximize the received signal-to-noise ratio (SNR), but it reduces the spectral efficiency. To address the tradeoff between received SNR and spectral efficiency and to minimize the control overhead for relay coordination, the work in [18] adopted the distributed cooperation strategies proposed in [19]. In [18], all users who correctly decode the message transmitted by the source serve as relays, and all relays forward packets simultaneously using randomized distributed space-time code (RDSTC). In [17]–[19], if an end user does not decode the message correctly in stage 1, it uses only data received in stage 2 to decode the message.

Note that utilizing all received signals, including the one received in stage 1, can help further increase the signal-to-noise ratio and improve the system performance. In this paper, we incorporate the maximal ratio combining (MRC) technique [20] into cooperative wireless multicast schemes, and combine the signals received in both stages to jointly detect the message at an intended receiver. We provide a thorough performance analysis for such MRC-based cooperative multicast schemes and optimize power allocation between source and dynamic relays. Same as in [18], [19], we assume that all the selected relays transmit simultaneously in stage 2. We consider two different relay selection schemes: the distributed cooperation strategy in [18], [19], and the one used in [17] with a fixed number of relays at predetermined locations. For each scheme, we analyze its average outage probability and derive the closed-form formulation. We also obtain an asymptotically tight approximation for the average outage probability to study the asymptotic behavior of cooperative multicast schemes in the high SNR regime. Based on the tight outage probability approximation, we are able to determine the optimal power allocation and relay position strategies for cooperative multicast to minimize the average outage probability. Finally, we compare the performance of different multicast schemes, including the direct multicast without cooperation, the distributed and the fixed cooperative multicast schemes, examine when users should cooperate with each other, and study their performance tradeoff.

The rest of the paper is organized as follows. Section II introduces the direct multicast scheme as well as the cooperative multicast schemes that we consider in this paper. Section III analyzes the average outage probability of the multicast schemes and derives the closed-form formulation. In Section IV, we investigate the asymptotic behavior of the cooperative multicast schemes in the high SNR region, and analyze the optimal power allocation and relay position strategies. In Section V, we compare the performance of different multicast schemes, examine their tradeoff, and use simulation results to validate our theoretical analysis. Finally, conclusions are drawn in Section VI.

II. SYSTEM MODEL

In this paper, we consider a wireless network with a circular cell of radius R_2 . The base station/access point (BS/AP) is located at the center of the cell and multicasts to M users who are uniformly distributed in the cell. For user i ($1 \leq i \leq M$), the joint probability density function of the user's distance r_i from the BS/AP and the angle θ_i is $f(r_i, \theta_i) = r_i/(\pi R_2^2)$, with $0 \leq r_i \leq R_2$ and $0 \leq \theta_i \leq 2\pi$. The marginal distribution of r_i is $f(r_i) = 2r_i/R_2^2$ with $0 \leq r_i \leq R_2$, θ_i is uniformly distributed in $[0, 2\pi]$, and r_i and θ_i are independent of each other. We assume that $\{(r_i, \theta_i)\}_{i=1}^M$ for different users are independently and identically distributed (i.i.d.). We also assume that the wireless link between any two nodes is subject to independent narrow-band Rayleigh fading, propagation path loss, and additive white Gaussian noise (AWGN). All nodes in the network work in the half-duplex mode, that is, they cannot transmit and receive in the same frequency band at the same time.

A. Direct Multicast

In the direct multicast scheme, the BS/AP broadcasts a signal x with unit power. For user i located at (r_i, θ_i) , its received signal can be written as

$$y_i^{nc} = \sqrt{Pr_i^{-\eta}}h_i x + n_i, \quad (1)$$

where P is the transmission power in the direct multicast mode, h_i is the channel gain between the BS/AP and user i , and η is the path loss parameter. Here, the subscript i is the user index, and the superscript “nc” means no cooperation (direct multicast). h_i is modeled as a zero-mean circularly symmetric complex Gaussian random variable with unit variance, and n_i is additive white Gaussian noise with variance N_0 . Therefore, the received signal-to-noise ratio (SNR) in (1) is $\gamma_i^{nc} = P|h_i|^2 r_i^{-\eta}/N_0$, which is an exponential random variable with parameter $\lambda_i^{nc} = (N_0 r_i^\eta)/P$ for a given r_i .

B. Cooperative Multicast

In 2-hop cooperative multicast, users receive more than one copy of the message and explore cooperative diversity to improve system performance. It consists of two stages: in the first stage, the BS/AP broadcasts the message; and in the second phase, those relays who decode the message correctly in stage 1 forward the message to other users.¹ In this work, we consider a repetition code in stage 2, i.e., all relays forward the same message coming from the BS/AP *simultaneously* in stage 2. Finally, for those users who decode the message erroneously in stage 1, they use MRC to combine the signal from the BS/AP in stage 1 and that from relays in stage 2, and jointly decode the message. We also assume that all users are willing to cooperate, and there is no selfish free riding or malicious behavior.

¹We assume that all relays are synchronized and the delay spread of arriving signals is negligible, which is valid in narrow-band wireless communications.

1) *Distributed Cooperative Multicast Scheme*: In the distributed cooperative multicast scheme, the BS/AP first broadcasts a unit-power signal x . For user i located at (r_i, θ_i) , its received signal is

$$\begin{aligned} y_i^{sd,d} &= \sqrt{P_{s,d} r_i^{-\eta}} h_i x + n_i \\ \text{with SNR } \gamma_i^{sd,d} &= \frac{P_{s,d} |h_i|^2 r_i^{-\eta}}{N_0}, \end{aligned} \quad (2)$$

where $P_{s,d}$ is the transmission power used by the BS/AP in the distributed cooperation scheme, and other parameters are the same as in (1). Here, the subscript i is the user index and the superscript “ sd,d ” means the source-destination channel in the distributed cooperation scheme. Since $h_i \sim \mathcal{CN}(0, 1)$, for any given r_i , $\gamma_i^{sd,d}$ follows an exponential distribution with parameter $\lambda_i^{sd,d} = (N_0 r_i^\eta) / P_{s,d}$.

Assume that N ($N \leq M$) out of M users decode the message correctly in stage 1 and $C_s = \{i_1, \dots, i_N\}$ denotes the set including their indices. $C_f = \{1, 2, \dots, M\} \setminus C_s = \{i_{N+1}, \dots, i_M\}$ contains the indices of those who decode incorrectly in stage 1. In stage 2, all users in C_s serve as relays and broadcast the message x simultaneously, using the same power $P_{r,d}$. For a user $i_j \in C_f$, its received signal $y_{i_j}^{rd,d}$ in stage 2 is the superposition of all the N signals broadcasted by the relays, subject to narrow-band Rayleigh channel fading, propagation path loss and additive white Gaussian noise. So $y_{i_j}^{rd,d}$ can be written as

$$y_{i_j}^{rd,d} = \sum_{i_l \in C_s} \sqrt{P_{r,d} (r_{i_l i_j})^{-\eta}} h_{i_l i_j} x + n_{i_j},$$

where $(r_{i_l i_j})^2 = r_{i_j}^2 + r_{i_l}^2 - 2r_{i_j} r_{i_l} \cos(\theta_{i_j} - \theta_{i_l})$. (3)

In (3), $r_{i_l i_j}$ is the distance between the relay $i_l \in C_s$ and the user $i_j \in C_f$. Here, the superscript “ rd,d ” means the relay-destination channel in the distributed cooperation scheme. We assume that the channel gains $\{h_{i_l i_j}\}_{i_l \in C_s, i_j \in C_f}$ are i.i.d. following circularly symmetric complex Gaussian distribution $\mathcal{CN}(0, 1)$ and n_{i_j} are additive white Gaussian noise with zero mean and variance N_0 . Therefore,

$$y_{i_j}^{rd,d} = \sqrt{P_{r,d}} h'_{i_j} x + n_{i_j}, \quad (4)$$

where $h'_{i_j} \triangleq \sum_{l=1}^N \sqrt{(r_{i_l i_j})^{-\eta}} h_{i_l i_j} \sim \mathcal{CN}\left(0, \sum_{l=1}^N r_{i_l i_j}^{-\eta}\right)$.

Given $\{(r_i, \theta_i)\}$ and $C_s = \{i_1, i_2, \dots, i_N\}$, the SNR of $y_{i_j}^{rd,d}$ is $\gamma_{i_j}^{rd,d} = (P_{r,d} |h'_{i_j}|^2) / N_0$, which is an exponential random variable with parameter $\lambda_{i_j}^{rd,d} = \left[N_0 \left(\sum_{l=1}^N (r_{i_l i_j})^{-\eta} \right)^{-1} \right] / P_{r,d}$.

We assume that the channel gains h_{i_j} and h'_{i_j} are known to user $i_j \in C_f$, and user i_j uses MRC to combine $y_{i_j}^{sd,d}$ in (2) and $y_{i_j}^{rd,d}$ in (3) and jointly decodes the message. Given (r_i, θ_i) and C_s , the SNR of the combined signal is

$$\gamma_{i_j}^d = \gamma_{i_j}^{sd,d} + \gamma_{i_j}^{rd,d} = \frac{P_{s,d} |h_{i_j}|^2 r_i^{-\eta} + P_{r,d} |h'_{i_j}|^2}{N_0}. \quad (5)$$

For fair comparison, we let the average total transmission power used by the BS/AP and by all relays in the cooperative multicast be the same as the total power used in the

direct multicast. That is, we select $P_{s,d}$ and $P_{r,d}$ such that $P_{s,d} + E[N]P_{r,d} = 2P$ [3], in which $E[N]$ is the expected number of users who decode the message correctly in stage 1 and serve as relays.

2) *Genie-Aided Cooperative Multicast Scheme*: The authors in [17] used a fixed number of relays at fixed positions to help forward packets. Following the work in [17], we consider a genie-aided cooperative multicast scheme in this paper, where we can put any number of relays to help forward the message and the relays can be placed at optimal locations to minimize the outage probability.

Assume that there are N' dedicated relays located on a fixed circle of radius R_1 with equal separation angle, and the l th ($1 \leq l \leq N'$) relay's location is $(R_1, (l-1)2\pi/N')$. In stage 1, the BS/AP broadcasts a unit-power signal x , and for user i located at (r_i, θ_i) , its received signal is

$$\begin{aligned} y_i^{sd,g} &= \sqrt{P_{s,g} r_i^{-\eta}} h_i x + n_i \\ \text{with SNR } \gamma_i^{sd,g} &= \frac{P_{s,g} |h_i|^2 r_i^{-\eta}}{N_0}. \end{aligned} \quad (6)$$

In (6), $P_{s,g}$ is the transmission power used by the BS/AP in the genie-aided cooperative multicast scheme, and the rest terms are the same as in (1). Here, the subscript i is the user index, and the superscript “ sd,g ” means the source-destination channel in the genie-aided cooperation scheme. Given r_i , $\gamma_i^{sd,g}$ is an exponential random variable with parameter $\lambda_i^{sd,g} = N_0 r_i^\eta / P_{s,g}$. For the l th relay ($1 \leq l \leq N'$), given R_1 , its received signal is

$$\begin{aligned} y_l^{sr,g} &= \sqrt{P_{s,g} R_1^{-\eta}} h_l x + n_l \\ \text{with SNR } \gamma_l^{sr,g} &= \frac{P_{s,g} |h_l|^2 R_1^{-\eta}}{N_0}. \end{aligned} \quad (7)$$

For each relay, if it correctly decodes the transmitted signal x in (7), then it will forward the decoded message in stage 2. Otherwise, the relay remains idle in stage 2. Assume that N out of N' relays decode the message correctly in stage 1, and $RC_s = \{l_1, l_2, \dots, l_N\}$ is the set including their indices. In stage 2, relays l_1, l_2, \dots, l_N transmit the message x simultaneously using the same power $P_{r,g}$. Similar to the analysis of the distributed cooperative multicast, for user i at (r_i, θ_i) , in stage 2, its received signal $y_i^{rd,g}$ is the superposition of all N relay messages and can be written as

$$y_i^{rd,g} = \sum_{l_j \in RC_s} \sqrt{P_{r,g} r_{il_j}^{-\eta}} h_{il_j} x + n_i, \quad (8)$$

where $r_{il_j}^2 = r_i^2 + R_1^2 - 2r_i R_1 \cos\left(\theta_i - \frac{(l_j - 1)2\pi}{N}\right)$.

In (8), r_{il_j} and h_{il_j} are the distance and the channel gain between user i and the l_j th relay, respectively. Here, the superscript “ rd,g ” means the relay-destination channel in the genie-aided cooperation scheme. Denote $h'_i \triangleq \sum_{l_j \in RC_s} \sqrt{r_{il_j}^{-\eta}} h_{il_j}$. We assume that all channel gains $\{h_{il_j}\}$ are i.i.d. circularly symmetric complex Gaussian $\mathcal{CN}(0, 1)$, and n_i are additive white Complex Gaussian $\mathcal{CN}(0, N_0)$. Therefore, given (r_i, θ_i) and the indices of the relays who decode correctly in stage 1 $\{l_1, l_2, \dots, l_N\}$, h'_i is also a circularly symmetric complex

Gaussian with zero mean and variance $\sum_{l_j \in RC_s} r_{il_j}^{-\eta}$. Thus, we have

$$\text{with SNR } \begin{cases} y_i^{rd,g} &= \sqrt{P_{r,g}} h_i' x + n_i \\ \gamma_i^{rd,g} &= (P_{r,g} |h_i'|^2) / N_0. \end{cases} \quad (9)$$

$\gamma_i^{rd,g}$ in (9) follows the exponential distribution with parameter $\lambda_i^{rd,g} = \left[N_0 \left(\sum_{l_j \in RC_s} r_{il_j}^{-\eta} \right)^{-1} \right] / P_{r,g}$.

Assume that user i has perfect knowledge of the channel gains h_i and h_i' . Given $y_i^{sd,g}$ and $y_i^{rd,g}$ as in (6) and in (8), respectively, user i uses MRC to combine these two signals, and the SNR of the combined signal is

$$\gamma_i^g = \gamma_i^{sd,g} + \gamma_i^{rd,g} = \frac{P_{s,g} |h_i|^2 r_i^{-\eta} + P_{r,g} |h_i'|^2}{N_0}. \quad (10)$$

Similar to the distributed cooperative multicast, given R_1 and N' , we select $P_{s,g}$ and $P_{r,g}$ such that $P_{s,g} + E[N]P_{r,g} = 2P$, where $E[N]$ is the expected number of relays who decode correctly in stage 1.

III. OUTAGE PROBABILITY ANALYSIS

In this section, we analyze average outage probabilities for the two cooperative multicast schemes. The outage probability is defined as the probability that the maximum mutual information between the message transmitted by the BS/AP and the signal received by a user is smaller than a predetermined threshold T_R , or equivalently, the probability that the received SNR is below a threshold γ_0 [3]. If the received SNR is higher than the threshold γ_0 , the user is assumed to be able to decode the received message with a negligible probability of error.

A. Outage Probability Analysis for Direct Multicast

For comparison, we first derive outage probability for the direct multicast scheme. In the direct multicast mode, for user i at location (r_i, θ_i) , the maximum mutual information between the input x and the output y_i^{nc} (with i.i.d. circularly symmetric complex Gaussian inputs) is $I_i^{nc} = \log_2(1 + \gamma_i^{nc})$, where γ_i^{nc} follows the exponential distribution with parameter $\lambda_i^{nc} = (N_0 r_i^\eta) / P$. Thus, given a threshold T_R on the maximum mutual information and user i 's location r_i , user i 's outage probability is

$$\begin{aligned} Pr [I_i^{nc} < T_R | r_i] &= Pr \left[\log_2 \left(1 + \frac{|h_i|^2 r_i^{-\eta} P}{N_0} \right) < T_R | r_i \right] \\ &= 1 - \exp \left\{ - \frac{(2^{T_R} - 1) N_0 r_i^\eta}{P} \right\}. \end{aligned} \quad (11)$$

For a network with M uniformly distributed users, under the assumption that all channel gains $\{h_i\}$ are i.i.d., the average outage probability of direct multicast is

$$\begin{aligned} \mathcal{P}^{nc} &= \int \dots \int \frac{\sum_{i=1}^M Pr [I_i^{nc} < T_R | r_i]}{M} \\ &\quad \times f(r_1) f(r_2) \dots f(r_M) dr_1 \dots dr_M \\ &= \int_0^{R_2} Pr [I_1^{nc} < T_R | r_1] f(r_1) dr_1 \\ &= \int_0^{R_2} \left(1 - \exp \left\{ - \frac{(2^{T_R} - 1) N_0 r_1^\eta}{P} \right\} \right) \frac{2r_1}{R_2^2} dr_1 \end{aligned}$$

$$= 1 - \frac{2}{\eta R_2^2} \left(\frac{P}{N_0 \gamma_0 d} \right)^{2/\eta} \Gamma \left(\frac{2}{\eta}, \frac{R_2^\eta N_0 \gamma_0 d}{P} \right), \quad (12)$$

where $\gamma_0 d \triangleq 2^{T_R} - 1$ and $\Gamma(a, x) = \int_0^x e^{-t} t^{a-1} dt$ is the incomplete Gamma function. The second equality in (12) is due to the i.i.d. assumption of users' locations $\{r_i\}$.

B. Outage Probability Analysis for the Distributed Cooperative Multicast

1) *Decoding Results in Stage 1*: In distributed cooperative multicast, for user i located at (r_i, θ_i) , the SNR of its received signal in stage 1, $\gamma_i^{sd,d}$, is an exponential random variable with parameter $\lambda_i^{sd,d} = N_0 r_i^\eta / P_{s,d}$. With i.i.d. circularly symmetric complex Gaussian input x , the maximum mutual information between the input and the output $y_i^{sd,d}$ in (2) is $I_i^{sd,d} = \frac{1}{2} \log_2(1 + \gamma_i^{sd,d})$, where the normalization factor $1/2$ is due to the fact that the cooperative multicast scheme uses two time slots to transmit one symbol [21]. If $I_i^{sr,d}$ is larger than the threshold T_R , we assume that user i can decode the message with a negligible probability of error in stage 1. Thus, given r_i , the probability that user i decodes incorrectly in stage 1 is

$$\mathcal{P}_i^{1,d} = Pr \left[I_i^{sd,d} < T_R | r_i \right] = 1 - \exp \left\{ - \frac{\gamma_0 m N_0 r_i^\eta}{P_{s,d}} \right\}, \quad (13)$$

where $\gamma_0 m \triangleq 2^{2T_R} - 1$. Here, the subscript i is the user index, and the superscript "1,d" means the stage-1 decoding in the distributed cooperative scheme. We assume that all channel gains $\{h_i\}$ are i.i.d., so given the locations of the M users $\{(r_i, \theta_i)\}_{i=1, \dots, M}$, the probability that users in $C_s = \{i_1, \dots, i_N\}$ decode correctly and users in $C_f = \{1, 2, \dots, M\} \setminus C_s$ decode incorrectly is

$$\begin{aligned} &\mathcal{P}^{1,d}(C_s, C_f | r_1, \dots, r_M) \\ &= \prod_{i_i \in C_s} \left(1 - \mathcal{P}_i^{1,d} \right) \times \prod_{i_j \in C_f} \mathcal{P}_i^{1,d} \\ &= \prod_{i_i \in C_s} \exp \left\{ - \frac{\gamma_0 m N_0 r_{i_i}^\eta}{P_{s,d}} \right\} \\ &\quad \times \prod_{i_j \in C_f} \left[1 - \exp \left\{ - \frac{\gamma_0 m N_0 r_{i_j}^\eta}{P_{s,d}} \right\} \right]. \end{aligned} \quad (14)$$

2) *Conditional Outage Probability*: For user $i_j \in C_f$ who decodes incorrectly in stage 1, given its location $\{(r_i, \theta_i)\}$ and C_s , the SNR of the maximum-ratio combined signal is $\gamma_{i_j}^d = \gamma_{i_j}^{sd,d} + \gamma_{i_j}^{rd,d}$ in (5), where $\gamma_{i_j}^{sd,d}$ and $\gamma_{i_j}^{rd,d}$ are exponential random variables with parameters $\lambda_{i_j}^{sd,d} = N_0 r_{i_j}^\eta / P_{s,d}$ and $\lambda_{i_j}^{rd,d} = N_0 \left(\sum_{i_i \in C_s} (r_{i_j i_i})^{-\eta} \right)^{-1} / P_{r,d}$, respectively. For user i_j , the maximum mutual information with i.i.d. complex Gaussian inputs is [21]

$$I_{i_j}^d = \frac{1}{2} \log_2 \left(1 + \gamma_{i_j}^d \right) = \frac{1}{2} \log_2 \left(1 + \gamma_{i_j}^{sd,d} + \gamma_{i_j}^{rd,d} \right). \quad (15)$$

Thus, given $\{(r_i, \theta_i)\}$ and C_s , for user $i_j \in C_f$, its conditional outage probability is

$$\begin{aligned} \mathcal{P}_{i_j}^d &= Pr \left[I_{i_j}^d < T_R | \{(r_i, \theta_i)\}, C_s \right] \\ &= Pr \left[\gamma_{i_j}^{sd,d} + \gamma_{i_j}^{rd,d} < \gamma_0 m | \{(r_i, \theta_i)\}, C_s \right] \end{aligned}$$

$$\begin{aligned}
&= 1 - \frac{\lambda_{i_j}^{rd,d}}{\lambda_{i_j}^{rd,d} - \lambda_{i_j}^{sd,d}} \exp\left(-\lambda_{i_j}^{sd,d} \cdot \gamma_{0m}\right) \\
&\quad + \frac{\lambda_{i_j}^{sd,d}}{\lambda_{i_j}^{rd,d} - \lambda_{i_j}^{sd,d}} \exp\left(-\lambda_{i_j}^{rd,d} \cdot \gamma_{0m}\right). \quad (16)
\end{aligned}$$

3) *Average Outage Probability*: Given the locations of the M users $\{(r_i, \theta_i)\}$, we first average $\mathcal{P}_{i_j}^d$ in (16) over all users $i_j \in C_f$ and all possible decoding results (C_s) in stage 1, which is shown in (17). Here, the superscript d' means the conditional outage probability of the distributed cooperative scheme. In (17), the fourth equality is due to the nonnegativity of $\gamma_{i_j}^{rd,d}$ and the fact that $\gamma_{i_j}^{sd,d} + \gamma_{i_j}^{rd,d} < \gamma_{0m}$ implies $\gamma_{i_j}^{sd,d} < \gamma_{0m}$. The last equality comes from the i.i.d. assumption for channel gains.

We then integrate $\mathcal{P}^{d'}$ in (17) with respect to user locations $\{(r_i, \theta_i)\}$, and the average outage probability of the distributed cooperative multicast scheme is in (18). To analyze the average outage probability, we calculate the first term $A(P_{s,d})$ in (18) as follows

$$\begin{aligned}
A(P_{s,d}) &= \int_0^{R_2} \int_0^{2\pi} Pr \left[\gamma_{i_k}^{sd,d} < \gamma_{0m} \mid \{(r_i, \theta_i)\} \right] \\
&\quad \times f(r_{i_k}, \theta_{i_k}) d\theta_{i_k} dr_{i_k} \\
&= \int_0^{R_2} \left(1 - \exp \left\{ -\frac{\gamma_{0m} N_0 r_{i_k}^\eta}{P_{s,d}} \right\} \right) \frac{2r_{i_k}}{R_2^2} dr_{i_k} \\
&= 1 - \frac{2}{\eta R_2^2} \left(\frac{P_{s,d}}{N_0 \gamma_{0m}} \right)^{2/\eta} \Gamma \left(\frac{2}{\eta}, \frac{R_2^\eta N_0 \gamma_{0m}}{P_{s,d}} \right), \quad (19)
\end{aligned}$$

where $\Gamma(a, x)$ is the incomplete Gamma function. Apparently, $A(P_{s,d})$ depends only on the base station's transmission power $P_{s,d}$. It is the same for all possible decoding results in stage 1 and for different user index $i_k \in C_f$.

Then, for a given decoding result in stage 1 (C_s) and user $i_j \in C_f$, we calculate the second term in (18), which is shown in (20). From (20), for a given C_s , if we consider two different users i_j and i_k who decode incorrectly in stage 1, we have $B_N(C_s, i_j) = B_N(C_s, i_k)$. In addition, from (20), if we fix the number of users who decode correctly in stage 1 as N , for two different decoding results (C_s^1, C_f^1) and (C_s^2, C_f^2) where $|C_s^1| = |C_s^2| = N$, we can show that $\sum_{i_k \in C_f^1} B_N(C_s^1, i_k) = \sum_{i_j \in C_f^2} B_N(C_s^2, i_j)$. Given the total number of users M , for a fixed number of users who serve as relays N , there are a total of $\binom{M}{N}$ possible decoding results in stage 1 with $|C_s| = N$. Without loss of generality, we use $C'_s(N) = \{1, 2, \dots, N\}$ and $i_j = N+1 \in C'_f(N)$ as an example, and define $B_N \triangleq B_N(C'_s(N) = \{1, 2, \dots, N\}, i_j = N+1)$. Therefore, we have $\sum_{|C_s|=N} \sum_{i_j \in C_f} B_N(C_s, i_j) = \binom{M}{N} (M-N) B_N$.

Based on the above analysis, \mathcal{P}^d in (18) can be written as

$$\begin{aligned}
\mathcal{P}^d &= [A(P_{s,d})]^M \\
&\quad + \sum_{N=1}^{M-1} \frac{M-N}{M} \binom{M}{N} B_N [A(P_{s,d})]^{M-N-1} \\
&= [A(P_{s,d})]^M \\
&\quad + \sum_{N=1}^{M-1} \binom{M-1}{N} B_N [A(P_{s,d})]^{M-N-1}. \quad (21)
\end{aligned}$$

The first term in (21) corresponds to the scenario where all users decode incorrectly in stage 1, and thus the conditional

outage probability $\mathcal{P}_{i_j}^d$ is 1 for all users. The second term in (21) calculates the average outage probability when N users decode correctly in stage 1 for $1 \leq N \leq M-1$. Note that when $N = M$, that is, all users decode correctly in stage 1, the conditional outage probability is 0. Thus, (21) does not include the term corresponding to $N = M$.

4) *Power Constraint*: In distributed cooperative multicast, the average transmission power used by the base station and all relays are $P_{s,d} + E[N]P_{r,d}$. With i.i.d. channel gains, we have

$$\begin{aligned}
E[N] &= \sum_{i=1}^M \int_0^{R_2} \int_0^{2\pi} Pr [I_i^{sd} > R] (r_i, \theta_i) \\
&\quad \times f(r_i, \theta_i) d\theta_i dr_i \\
&= M \int_0^{R_2} \exp \left\{ -\frac{\gamma_{0m} N_0 r_1^\eta}{P_{s,d}} \right\} \frac{2r_1}{R_2^2} dr_1 \\
&= M [1 - A(P_{s,d})]. \quad (22)
\end{aligned}$$

For fair comparison, we select $P_{s,d}$ and $P_{r,d}$ such that $P_{s,d} + E[N]P_{r,d} = 2P$, where P is the transmission power used in direct multicast to transmit one message. Therefore, given the BS/AP's transmission power $P_{s,d}$ and the total number of users M , the transmission power used by each relay is

$$P_{r,d} = \frac{2P - P_{s,d}}{M \cdot [1 - A(P_{s,d})]} \quad (23)$$

Note that $P_{r,d}$ depends on the *average* (not the instantaneous) number of users who receive correctly in stage 1. It can be precalculated and remains the same during the transmissions.

To summarize, for the distributed cooperative multicast, given the total number of users M and the base station's transmission power $P_{s,d}$, the transmission power used by each relay should be determined by (23), and the average outage probability can be calculated using (21).

C. Outage Probability Analysis for the Genie-Aided Cooperative Multicast

The analysis of the outage probability for the genie-aided cooperative multicast is similar to that in the previous section, and we will present the main results by omitting some details.

1) *Decoding Results in Stage 1*: Similar to that in Section III-B1, in stage 1 of the genie-aided cooperative multicast, for user i located at (r_i, θ_i) , the SNR of its received signal is $\gamma_i^{sd,g}$, which follows the exponential distribution with parameter $\lambda_i^{sd,g} = N_0 r_i^\eta / P_{s,g}$.

For the N' dedicated relays, from (7), given R_1 , the SNR of the received signal by the l th relay in stage 1 $\gamma_l^{sr,g}$ is an exponential random variable with parameter $\lambda_l^{sr,g} = N_0 R_1^\eta / P_{s,g}$. Same as distributed cooperative multicast, we assume that relay l can decode the message correctly when the maximum mutual information $I_l^{sr,g} = \frac{1}{2} \log_2 (1 + \gamma_l^{sr,g})$ is larger than the predetermined threshold T_R . Otherwise, the relay is assumed to decode incorrectly in stage 1. Thus, given R_1 , the probability that relay l decodes erroneously in stage 1 is

$$\begin{aligned}
\mathcal{P}_l^{1,g} &= Pr [I_l^{sr,g} < T_R | R_1] \\
&= Pr \left[\frac{1}{2} \log_2 (1 + \gamma_l^{sr,g}) < T_R | R_1 \right]
\end{aligned}$$

$$\begin{aligned}
 \mathcal{P}^{d'} &= \frac{1}{M} \sum_{C_s} \sum_{i_j \in C_f} \mathcal{P}_{i_j}^d \cdot \mathcal{P}^{1,d}(C_s, C_f | \{(r_i, \theta_i)\}) \\
 &= \frac{1}{M} \sum_{C_s} \sum_{i_j \in C_f} Pr \left[\gamma_{i_j}^{sd,d} + \gamma_{i_j}^{rd,d} < \gamma_{0m} \mid C_s, \{(r_i, \theta_i)\} \right] \\
 &\quad \times Pr \left[\gamma_{i_l}^{sd,d} > \gamma_{0m} \forall i_l \in C_s, \gamma_{i_k}^{sd,d} < \gamma_{0m} \forall i_k \in C_f \mid \{(r_i, \theta_i)\} \right] \\
 &= \frac{1}{M} \sum_{C_s} \sum_{i_j \in C_f} Pr \left[\gamma_{i_j}^{sd,d} + \gamma_{i_j}^{rd,d} < \gamma_{0m}, \gamma_{i_j}^{sd} < \gamma_{0m}, \gamma_{i_l}^{sd,d} > \gamma_{0m} \forall i_l \in C_s, \right. \\
 &\quad \left. \gamma_{i_j}^{sd} < \gamma_{0m} \forall i_k \in C_f \text{ and } i_k \neq i_j \mid \{(r_i, \theta_i)\} \right] \\
 &= \frac{1}{M} \sum_{C_s} \sum_{i_j \in C_f} Pr \left[\gamma_{i_j}^{sd,d} + \gamma_{i_j}^{rd,d} < \gamma_{0m}, \gamma_{i_l}^{sd,d} > \gamma_{0m} \forall i_l \in C_s, \right. \\
 &\quad \left. \gamma_{i_j}^{sd,d} < \gamma_{0m} \forall i_k \in C_f \text{ and } i_k \neq i_j \mid \{(r_i, \theta_i)\} \right] \\
 &= \frac{1}{M} \sum_{C_s} \sum_{i_j \in C_f} Pr \left[\gamma_{i_j}^{sd,d} + \gamma_{i_j}^{rd,d} < \gamma_{0m} \mid C_s, \{(r_i, \theta_i)\} \right] \times \prod_{i_l \in C_s} Pr \left[\gamma_{i_l}^{sd,d} > \gamma_{0m} \mid \{(r_i, \theta_i)\} \right] \\
 &\quad \times \prod_{i_k \in C_f, i_k \neq i_j} Pr \left[\gamma_{i_k}^{sd,d} < \gamma_{0m} \mid \{(r_i, \theta_i)\} \right]. \tag{17}
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{P}^d &= \int \dots \int \mathcal{P}^{d'} f(r_1, \theta_1) \dots f(r_M, \theta_M) dr_1 d\theta_1 \dots dr_M d\theta_M \\
 &= \frac{1}{M} \sum_{N=0}^M \sum_{|C_s|=N} \sum_{i_j \in C_f} \prod_{i_k \in C_f, i_k \neq i_j} \underbrace{\int_0^{R_2} \int_0^{2\pi} Pr \left[\gamma_{i_k}^{sd,d} < \gamma_{0m} \mid \{(r_i, \theta_i)\} \right] f(r_{i_k}, \theta_{i_k}) d\theta_{i_k} dr_{i_k}}_{\triangleq A(P_s, d)} \times \\
 &\quad \underbrace{\left[\int \dots \int \mathcal{P}_{i_j}^d f(r_{i_j}, \theta_{i_j}) \prod_{i_l \in C_s} \left(Pr \left[\gamma_{i_l}^{sd,d} > \gamma_{0m} \mid \{(r_i, \theta_i)\} \right] f(r_{i_l}, \theta_{i_l}) \right) dr_{i_l} d\theta_{i_l} \dots dr_{i_N} d\theta_{i_N} dr_{i_j} d\theta_{i_j} \right]}_{\triangleq B_N(C_s, i_j)} \\
 &= \frac{1}{M} \sum_{N=0}^M \sum_{|C_s|=N} \sum_{i_j \in C_f} [A(P_s, d)]^{M-N-1} B_N(C_s, i_j). \tag{18}
 \end{aligned}$$

$$\begin{aligned}
 B_N(C_s, i_j) &= \int \dots \int Pr \left[\gamma_{i_j}^{sd,d} + \gamma_{i_j}^{rd,d} < \gamma_{0m} \mid C_s, \{(r_i, \theta_i)\} \right] f(r_{i_j}, \theta_{i_j}) \times \\
 &\quad \prod_{i_l \in C_s} \left(Pr \left[\gamma_{i_l}^{sd,d} > \gamma_{0m} \mid \{(r_i, \theta_i)\} \right] f(r_{i_l}, \theta_{i_l}) \right) dr_{i_1} d\theta_{i_1} \dots dr_{i_N} d\theta_{i_N} dr_{i_j} d\theta_{i_j}. \tag{20}
 \end{aligned}$$

$$\underbrace{= 1 - \exp \left\{ -\frac{\gamma_{0m} N_0 R_1^\eta}{P_{s,g}} \right\}}_{\triangleq C(R_1)}. \tag{24}$$

Here, the subscript l is the relay index, and the superscript "l,g" means the stage-1 decoding in the genie-aided cooperation scheme. Assume that N out of N' relays decode correctly in stage 1, and $RC_s = \{l_1, \dots, l_N\}$ contains their indices. Thus, the probability that relays in RC_s decode correctly in stage 1 is

$$\begin{aligned}
 \mathcal{P}^{1,g}(RC_s) &= \prod_{l_i \in RC_s} (1 - \mathcal{P}_{l_i}^{1,g}) \prod_{l_j \notin RC_s} \mathcal{P}_{l_j}^{1,g} \\
 &= [1 - C(R_1)]^N C(R_1)^{N'-N}. \tag{25}
 \end{aligned}$$

2) *Outage Probability*: After maximum-ratio combining, for user i at (r_i, θ_i) , the SNR of the combined signal is

$$\begin{aligned}
 \mathcal{P}_i^g &= Pr [I_i^g < T_R | (r_i, \theta_i), RC_s] \\
 &= P \left[\gamma_i^{sd,g} + \gamma_i^{rd,g} < \gamma_{0m} \mid (r_i, \theta_i), RC_s \right] \\
 &= 1 - \frac{\lambda_i^{rd,g}}{\lambda_i^{rd,g} - \lambda_i^{sd,g}} \exp \left(-\lambda_i^{sd,g} \cdot \gamma_{0m} \right) \\
 &\quad + \frac{\lambda_i^{sd,g}}{\lambda_i^{rd,g} - \lambda_i^{sd,g}} \exp \left(-\lambda_i^{rd,g} \cdot \gamma_{0m} \right). \tag{26}
 \end{aligned}$$

$\gamma_i^g = \gamma_i^{sd,g} + \gamma_i^{rd,g}$, where $\gamma_i^{sd,g}$ and $\gamma_i^{rd,g}$ are exponential random variables with parameter $\lambda_i^{sd,g} = N_0 r_i^\eta / P_{s,g}$ and $\lambda_i^{rd,g} = \left[N_0 \left(\sum_{l_j \in RC_s} r_{il_j}^{-\eta} \right)^{-1} \right] / P_{r,g}$, respectively. For user i , the maximum mutual information with i.i.d. complex Gaussian inputs is $I_i^g = \frac{1}{2} \log_2 (1 + \gamma_i^g)$, and the conditional outage probability is

Note that when $RC_s = \emptyset$ and $N = 0$, that is, all N' relays decode incorrectly in stage 1, then $\gamma_i^{r,d,g} = 0$ and user i 's outage probability is

$$\begin{aligned} & \Pr \left[\gamma_i^{sd,g} < \gamma_{0m} \mid (r_i, \theta_i) \right] \\ &= 1 - \exp \left\{ -\frac{\gamma_{0m} N_0 r_i^\eta}{P_{s,g}} \right\} = A(P_{s,g}), \end{aligned} \quad (27)$$

where $A(\cdot)$ is defined in (19).

Averaging \mathcal{P}_i^g over all possible decoding results in stage 1 (RC_s) and user i 's location (r_i, θ_i) , the average outage probability of genie-aided cooperative multicast is in (28). In (28), $C(R_1)$ and \mathcal{P}_i^g are defined in (24) and (26), respectively. The superscript "g" means the genie-aided cooperation scheme. In (28), the first term corresponds to the scenario where all relays decode incorrectly in stage 1, and the second term considers the scenario where there are $1 \leq N \leq N'$ relays that decode correctly in stage 1.

3) *Power Constraint*: Same as in Section III-B4, in the genie-aided cooperative multicast, we adjust $P_{s,g}$ and $P_{r,g}$ such that $P_{s,g} + E[N]P_{r,g} = 2P$, where $E[N]$ is the average number of relays who decode correctly in stage 1. From (24), $E[N] = N'(1 - \mathcal{P}_i^{1,g}) = N'(1 - C(R_1))$. Therefore, given the base station's transmission power $P_{s,g}$ and R_1 , we have

$$P_{r,g} = \frac{2P - P_{s,g}}{N' \cdot \exp(-\gamma_{0m} N_0 R_1^\eta / P_{s,g})}. \quad (29)$$

Same as $P_{r,d}$ in distributed cooperative multicast, $P_{r,g}$ depends on the average number of relays who decode correctly in stage 1, and can be precalculated.

To summarize, for the genie-aided cooperative multicast, given the total number of relays N' and the relay location R_1 , the transmission power used by the base station and that used by each relay should satisfy (29), and the average outage probability is given by (28).

IV. OPTIMUM POWER ALLOCATION AND RELAY LOCATION

This section considers optimal power allocation for cooperative multicast schemes to minimize their outage probability. For genie-aided cooperative multicast, we also investigate the optimal relay location R_1 to maximize the system performance.

A. Distributed Cooperative Multicast

In distributed cooperative multicast, given the power constraint in (23), we look for the optimal $P_{s,d}$ that minimizes the average outage probability in (21). Due to the complexity of (21), it is difficult to find a closed-form solution for the optimal $P_{s,d}$ directly from (21). In the following, we consider a high SNR scenario. We obtain an asymptotically tight approximation of (21) to study its asymptotic behavior in the high SNR region, and derive an asymptotically optimal power allocation scheme.

1) *Approximation of the Average Outage Probability*: We first analyze the term $A(\cdot)$ in (21). From the definition in (19),

$$\begin{aligned} A(P_{s,d}) &= \int_0^{R_2} \left(1 - \exp \left\{ -\frac{\gamma_{0m} N_0 r_{ik}^\eta}{P_{s,d}} \right\} \right) \frac{2r_{ik}}{R_2^2} dr_{ik} \\ &\approx \int_0^{R_2} \frac{N_0 \gamma_{0m} r_{ik}^\eta}{P_{s,d}} \frac{2r_{ik}}{R_2^2} dr_{ik} \\ &= \frac{N_0 \gamma_{0m}}{P_{s,d}} \cdot \frac{2R_2^\eta}{\eta + 2}. \end{aligned} \quad (30)$$

Here, we use the first order Taylor series approximation $\exp(x) \approx 1 + x$ for x close to 0, which is tight at high SNR and when the ratio $P_{s,d}/N_0$ is large. Similarly, to simplify the term B_N in (21), given $C_s = \{1, 2, \dots, N\}$, with high SNR where $P_{s,d}/N_0$ and $P_{r,d}/N_0$ are large, we use the following approximations:

$$\begin{aligned} & \Pr \left[\lambda_{N+1}^{rd,d} + \lambda_{N+1}^{sd,d} \leq \gamma_{0m} \mid \{(r_i, \theta_i)\}, C_s \right] \\ &= 1 - \frac{\lambda_{N+1}^{rd,d}}{\lambda_{N+1}^{rd,d} - \lambda_{N+1}^{sd,d}} \exp \left(-\lambda_{N+1}^{sd,d} \cdot \gamma_{0m} \right) \\ & \quad + \frac{\lambda_{N+1}^{sd,d}}{\lambda_{N+1}^{rd,d} - \lambda_{N+1}^{sd,d}} \exp \left(-\lambda_{N+1}^{rd,d} \cdot \gamma_{0m} \right) \\ &\approx \frac{1}{2} \lambda_{N+1}^{rd,d} \cdot \lambda_{N+1}^{sd,d} \cdot \gamma_{0m}^2, \end{aligned} \quad (31)$$

$$\begin{aligned} \text{and } \Pr \left[\lambda_i^{sd,d} > \gamma_{0m} \mid (r_i, \theta_i) \right] \\ &= \exp \left\{ -\frac{\gamma_{0m} N_0 r_i^\eta}{P_{s,d}} \right\} \approx 1 - \frac{\gamma_{0m} N_0 r_i^\eta}{P_{s,d}} \approx 1 \end{aligned} \quad (32)$$

for $1 \leq i \leq N$. Therefore, we have (33). Note that in (33), D_N depends only on N and R_2 but not other parameters.

Substituting (30) and (33) into (21), we have

$$\begin{aligned} \mathcal{P}^d &\approx \left(\frac{N_0 \gamma_{0m}}{P_{s,d}} \cdot \frac{2R_2^\eta}{\eta + 2} \right)^M \\ & \quad + \sum_{N=1}^{M-1} \binom{M-1}{N} \frac{N_0^2 \gamma_{0m}^2}{2P_{s,d} P_{r,d}} \\ & \quad \times D_N \left(\frac{N_0 \gamma_{0m}}{P_{s,d}} \cdot \frac{2R_2^\eta}{\eta + 2} \right)^{M-N-1}. \end{aligned} \quad (34)$$

Note that in (34), the lowest order of $N_0/P_{s,d}$ and $N_0/P_{r,d}$ is 2 when $N = M - 1$. Therefore, for high SNR with large values of $P_{s,d}/N_0$ and $P_{r,d}/N_0$, we can ignore the third and all other higher order terms and further simplify \mathcal{P}^d as

$$\mathcal{P}^d \approx \tilde{\mathcal{P}}^d = \frac{N_0^2 \gamma_{0m}^2}{2P_{s,d} P_{r,d}} D_{M-1}, \quad (35)$$

where D_{M-1} is a constant and depends only on the total number of user M and the radius R_2 .

2) *Optimal Power Allocation*: Given the above approximation of the average outage probability, we can determine the optimal power allocation between the BS/AP and the relays. From (35), we have $\tilde{\mathcal{P}}^d = N_0^2 \gamma_{0m}^2 D_{M-1} / (2P_{s,d} P_{r,d})$, where D_{M-1} depends only on the total number of users M . Therefore, minimization of $\tilde{\mathcal{P}}^d$ is equivalent to maximization of the product $P_{s,d} P_{r,d}$ under the constraint that $P_{s,d}$ and $P_{r,d}$ satisfy (23).

$$\begin{aligned}
 \mathcal{P}^g &= \int_0^{R_2} \int_0^{2\pi} \sum_{RC_s} \mathcal{P}_i^g \mathcal{P}_{1,g}(RC_s) f(r_i, \theta_i) d\theta_i dr_i \\
 &= \sum_{N=0}^{N'} \sum_{|RC_s|=N} [1 - C(R_1)]^N [C(R_1)]^{N'-N} \int_0^{R_2} \int_0^{2\pi} \mathcal{P}_i^g \frac{r_i}{\pi R_2^2} d\theta_i dr_i \\
 &= C(R_1)^{N'} A(P_{s,g}) + \sum_{N=1}^{N'} [1 - C(R_1)]^N [C(R_1)]^{N'-N} \sum_{|RC_s|=N} \int_0^{R_2} \int_0^{2\pi} \mathcal{P}_i^g \frac{r_i}{\pi R_2^2} d\theta_i dr_i, \quad (28)
 \end{aligned}$$

$$\begin{aligned}
 B_N &\approx \int \cdots \int \frac{1}{2} \lambda_{N+1}^{rd,d} \cdot \lambda_{N+1}^{sd,d} \cdot \gamma_{0m}^2 \cdot \frac{r_1 \cdots r_{N+1}}{(\pi R_2^2)^{N+1}} dr_1 d\theta_1 \cdots dr_{N+1} d\theta_{N+1} \\
 &= \int \cdots \int \frac{1}{2} \frac{N_0 r_{N+1}^\eta}{P_{s,d}} \cdot \frac{N_0 \left(\sum_{i=1}^N (r_{(N+1)i})^{-\eta} \right)^{-1}}{P_{r,d}} \cdot \gamma_{0m}^2 \cdot \frac{r_1 \cdots r_{N+1}}{(\pi R_2^2)^{N+1}} dr_1 d\theta_1 \cdots dr_{N+1} d\theta_{N+1} \\
 &= \frac{N_0^2 \gamma_{0m}^2}{2P_{s,d} P_{r,d}} D_N, \\
 \text{where } D_N &\triangleq \int \cdots \int r_{N+1}^\eta \left[\sum_{i=1}^N (r_i^2 + r_{N+1}^2 - 2r_i r_{N+1} \cos(\theta_i - \theta_{N+1}))^{-\eta/2} \right]^{-1} \\
 &\quad \times \frac{r_1 \cdots r_{N+1}}{(\pi R_2^2)^{N+1}} dr_1 d\theta_1 \cdots dr_{N+1} d\theta_{N+1}. \quad (33)
 \end{aligned}$$

To simplify the power constraint (23), we need to simplify the denominator in (23). From (30), with high SNR and large values of $P_{s,d}/N_0$, we can use the approximation

$$1 - A(P_{s,d}) \approx 1 - \frac{1}{P_{s,d}} \cdot N_0 \gamma_{0m} \cdot \frac{2R_2^\eta}{\eta + 2} = 1 - \frac{b}{P_{s,d}}, \quad (36)$$

where $b \triangleq N_0 \gamma_{0m} 2R_2^\eta / (\eta + 2)$. Therefore, we have

$$\begin{aligned}
 P_{r,d} &= \frac{2P - P_{s,d}}{M \cdot [1 - A(P_{s,d})]} \\
 &\approx \frac{2P - P_{s,d}}{M \cdot (1 - b/P_{s,d})} = \frac{P_{s,d}(2P - P_{s,d})}{M(P_{s,d} - b)}. \quad (37)
 \end{aligned}$$

Consequently, to minimize the outage probability of distributed cooperative multicast, we should select $P_{s,d}$ to maximize $G^d(p) \triangleq p^2(2P - p)/[M(p - b)]$. To find the optimal $P_{s,d}$, we let

$$\begin{aligned}
 0 &= \left. \frac{\partial G^d(p)}{\partial p} \right|_{p=P_{s,d}^*} = p \frac{2p^2 - (2P + 3b)p + 4bP}{M(p - b)^2} \Big|_{p=P_{s,d}^*}, \\
 \Leftrightarrow & 2P_{s,d}^{*2} - (2P + 3b)P_{s,d}^* + 4bP = 0, \\
 \Leftrightarrow & P_{s,d}^* = \frac{2P + 3b \pm \sqrt{(2P + 3b)^2 - 32bP}}{4}. \quad (38)
 \end{aligned}$$

With high SNR and $P \gg b$, the optimal solution in (38) can be approximated as $P_{s,d}^* \approx P$, that is, the BS/AP uses half of the total transmission power. The other half is evenly distributed among relays in the statistical sense, and we use the average number of relays $E[N]$ and (23) to calculate the transmission power used by each relay $P_{r,d}$.

B. Genie-Aided Cooperative Multicast

To analyze the optimal power allocation and relay locations for the genie-aided cooperative multicast, similar to the previous section, we first consider the high SNR scenario with

large transmission power and find an approximation of the average outage probability. Then, we use the approximated outage probability to find the optimal power allocation and relay location strategies.

1) *Approximation of the Average Outage Probability:* For a given R_1 , for high SNR with large $P_{s,g}/N_0$, we start with the first term corresponding to $N = 0$ in (28), and use (30) to approximate it as

$$\begin{aligned}
 &C(R_1)^{N'} A(P_{s,g}) \\
 &= \left[1 - \exp\left(-\frac{\gamma_{0m} N_0 R_1^\eta}{P_{s,g}}\right) \right]^{N'} A(P_{s,g}) \\
 &\approx \left(\frac{\gamma_{0m} N_0 R_1^\eta}{P_{s,g}} \right)^{N'} \frac{N_0 \gamma_{0m}}{P_{s,g}} \cdot \frac{2R_2^\eta}{\eta + 2} \\
 &= 2 \left(\frac{N_0 \gamma_{0m}}{P_{s,g}} \right)^{N'+1} \frac{(R_1^{N'} R_2)^\eta}{\eta + 2}. \quad (39)
 \end{aligned}$$

To simplify the second term in (28) corresponding to the scenario where $1 \leq N \leq N'$, we first need to find approximations for \mathcal{P}_i^g in (26). Similar to the approximation of (31) in the previous Section, for high SNRs, we can approximate \mathcal{P}_i^g as

$$\begin{aligned}
 \mathcal{P}_i^g &= Pr \left[\gamma_i^{sd,g} + \gamma_i^{rd,g} < \gamma_{0m} \mid (r_i, \theta_i), RC_s \right] \quad (40) \\
 &= 1 - \frac{\lambda_i^{rd,g}}{\lambda_i^{rd,g} - \lambda_i^{sd,g}} \exp\left(-\lambda_i^{sd,g} \cdot \gamma_{0m}\right) \\
 &\quad + \frac{\lambda_i^{sd,g}}{\lambda_i^{rd,g} - \lambda_i^{sd,g}} \exp\left(-\lambda_i^{rd,g} \cdot \gamma_{0m}\right) \\
 &\approx \frac{1}{2} \lambda_i^{rd,g} \lambda_i^{sd,g} \gamma_{0m}^2 = \frac{1}{2} \frac{N_0^2 \gamma_{0m}^2 r_i^\eta}{P_{s,g} P_{r,g}} \left(\sum_{l_j \in RC_s} r_{il_j}^{-\eta} \right)^{-1}.
 \end{aligned}$$

In addition, with large $P_{s,g}/N_0$, $C(R_1) = 1 - \exp\left(-\frac{\gamma_{0m} N_0 R_1^\eta}{P_{s,g}}\right) \approx \gamma_{0m} N_0 R_1^\eta / P_{s,g} \ll 1$ and we can have

$$\begin{aligned}
& \sum_{N=1}^{N'} [1 - C(R_1)]^N [C(R_1)]^{N'-N} \sum_{|RC_s|=N} \int_0^{R_2} \int_0^{2\pi} \mathcal{P}_i^g \frac{1}{\pi R_2^2} d\theta_i dr_i \\
& \approx \sum_{N=1}^{N'} \left(\frac{\gamma_{0m} N_0 R_1^\eta}{P_{s,g}} \right)^{N'-N} \sum_{|RC_s|=N} \int_0^{R_2} \int_0^{2\pi} \frac{1}{2} \frac{N_0^2 \gamma_{0m}^2 r_i^\eta}{P_{s,g} P_{r,g}} \left(\sum_{l_j \in RC_s} r_{il_j}^{-\eta} \right)^{-1} \frac{r_i}{\pi R_2^2} d\theta_i dr_i \\
& = \sum_{N=1}^{N'} \left(\frac{\gamma_{0m} N_0 R_1^\eta}{P_{s,g}} \right)^{N'-N} \frac{1}{2} \left(\frac{\gamma_{0m}^2 N_0^2}{P_{s,g} P_{r,g}} \right) Z_N, \\
\text{where } Z_N & \triangleq \sum_{|RC_s|=N} \int_0^{R_2} \int_0^{2\pi} r_i^\eta \left(\sum_{l_j \in RC_s} r_{il_j}^{-\eta} \right)^{-1} \frac{r_i}{\pi R_2^2} d\theta_i dr_i. \tag{41}
\end{aligned}$$

$1 - C(R_1) \approx 1$. Therefore, the second term in \mathcal{P}^g can be approximated as in (41). Note that Z_N depends on the relay location R_1 and the relay's decoding result in stage 1 RC_s , but not the transmission powers.

Combining the results in (39) and (41), the average outage probability of the genie-aided cooperative multicast scheme is

$$\begin{aligned}
\mathcal{P}^g & \approx 2 \left(\frac{N_0 \gamma_{0m}}{P_{s,g}} \right)^{N'+1} \frac{(R_1^{N'} R_2)^\eta}{\eta + 2} \\
& + \sum_{N=1}^{N'} \frac{1}{2} \left(\frac{\gamma_{0m}^2 N_0^2}{P_{s,g} P_{r,g}} \right) \cdot \left(\frac{\gamma_{0m} N_0 R_1^\eta}{P_{s,g}} \right)^{N'-N} Z_N. \tag{42}
\end{aligned}$$

Same as in (34), the lowest order of $N_0/P_{s,g}$ and $N_0/P_{r,g}$ in (42) is 2 when $N = N'$. Thus, for high SNR with large transmission powers, we can further simplify the expression in (42) by omitting the third and higher order terms, and we have

$$\mathcal{P}^g \approx \tilde{\mathcal{P}}^g \triangleq \frac{1}{2} \frac{N_0^2 \gamma_{0m}^2}{P_{s,g} P_{r,g}} Z_{N'}. \tag{43}$$

Here, $Z_{N'}$ depends on the total number of relays N' and their locations R_1 but not the transmission powers.

2) *Optimal Power Allocation*: To find the asymptotic optimal power allocation scheme, for a given relay location R_1 , similar to the analysis in Section IV-A2, we find the optimal $P_{s,g}$ that minimizes the approximated average outage probability $\tilde{\mathcal{P}}^g$ in (43).

Note that in (43), $Z_{N'}$ does not depend on $P_{s,g}$. Thus, to minimize $\tilde{\mathcal{P}}^g = (N_0^2 \gamma_{0m}^2 Z_{N'}) / (2P_{s,g} P_{r,g})$, it is equivalent to maximize the product $P_{s,g} P_{r,g}$ under the constraint that $P_{s,g}$ and $P_{r,g}$ satisfy (29). In the high SNR region with large $P_{s,g}/N_0$, we can have the approximation $\exp(-\gamma_{0m} N_0 R_1^\eta / P_{s,g}) \approx 1 - \gamma_{0m} N_0 R_1^\eta / P_{s,g}$. Therefore, following the same analysis as in Section IV-A2, to minimize the outage probability, we should select $P_{s,g}$ to maximize $G^g(p) \triangleq p^2 (2P - p) / [N'(p - b')]$, where $b' \triangleq \gamma_{0m} N_0 R_1^\eta$. That is, the optimal $P_{s,d}^*$ should satisfy $\left. \frac{\partial G^g(p)}{\partial p} \right|_{p=P_{s,d}^*} = 0$, which

gives $P_{s,g}^* = \frac{2P+3b' \pm \sqrt{(2P+3b')^2 - 32b'P}}{4}$. In the high SNR region where $P \gg b'$, we can further have the approximation that $P_{s,g}^* \approx P$, that is, the BS/AP uses half of the total transmission power. The rest half is evenly distributed among relays in the statistical sense, and (29) is used to calculate the transmission power used by each relay $P_{r,g}$.

TABLE I
ASYMPTOTIC OPTIMAL RELAY LOCATIONS FOR THE GENIE-AIDED COOPERATIVE MULTICAST SCHEME.

Optimal Relay Location	$N' = 2$	$N' = 3$	$N' = 4$	$N' = 5$	$N' = 6$
$u^* = R_1^*/R_2$	0.485	0.665	0.735	0.765	0.785

3) *Optimal Relay Location*: To find the asymptotic optimal relay location R_1 , with high SNR, following the above discussion on the optimal power allocation, we let $P_{s,g} = P$, and

$$P_{r,g} \approx \frac{(2P - P_{s,g})P_{s,g}}{N'(P_{s,g} - b')} = \frac{P^2}{N'(P - b')}, \tag{44}$$

where $b' = \gamma_{0m} N_0 R_1^\eta$. Thus, given the number of relays N' , we select R_1 to minimize

$$\tilde{\mathcal{P}}^g = \frac{1}{2} \frac{\gamma_{0m}^2 N_0^2}{P_{s,g} P_{r,g}} Z_{N'} \approx \frac{1}{2} \frac{\gamma_{0m}^2 N_0^2 N'}{P^3} (P - b') Z_{N'}, \tag{45}$$

or equivalently, to minimize $(P - b') Z_{N'}$. With high SNR and $P \gg \gamma_{0m} N_0 R_2^\eta > b'$, the problem can be simplified to select R_1 to minimize $Z_{N'}$. With $0 \leq r_i \leq R_2$ and $0 \leq R_1 \leq R_2$, let $t = r_i/R_2 \in [0, 1]$ and $u = R_1/R_2 \in [0, 1]$. Thus, $Z_{N'}$ in (41) can be rewritten as in (46), and minimization of $Z_{N'}$ is equivalent to minimization of $\bar{z}_{N'}$.

Note that $\bar{z}_{N'}$ is a function of the number of relays N' and the normalized relay position u , but not other parameters. For each N' , Monte Carlo and numerical methods are used to find the optimal u^* that minimizes $\bar{z}_{N'}$. Table I lists the optimal u^* when N' takes different values from 2 to 6.

V. SIMULATION RESULTS AND PERFORMANCE COMPARISON

A. Distributed Cooperative Multicast

We first use Monte Carlo simulations to calculate the exact and the approximated average outage probabilities of the distributed cooperative multicast, and compare them in Fig. 1. We assume that the radius of the circle is $R_2 = 100$, the path loss parameter is $\eta = 2.6$, and the outage threshold is $T_R = 4$. We consider three cases with $M = 10$, $M = 100$ and $M = 250$ users, respectively, and let P/N_0 vary from 75dB to 95dB. We let the transmission power used by the BS/AP equal to the average transmission power used by all relays. We use 20,000 Monte Carlo simulations to find B_N

$$\begin{aligned}
 Z_{N'} &= \int_0^{R_2} \int_0^{2\pi} r_i^\eta \left[\sum_{l=1}^{N'} \left(r_i^2 + R_1^2 - 2r_i R_1 \cos\left(\theta_i - \frac{(l-1) \cdot 2\pi}{N'}\right) \right)^{-\eta/2} \right]^{-1} \frac{r_i}{\pi R_2^2} d\theta_i dr_i \\
 &= R_2^{2\eta} \int_0^1 \int_0^{2\pi} t^\eta \underbrace{\left[\sum_{l=1}^{N'} \left(t^2 + u^2 - 2tu \cos\left(\theta_i - \frac{(l-1) \cdot 2\pi}{N'}\right) \right)^{-\eta/2} \right]^{-1}}_{\triangleq \bar{z}_{N'}} \frac{t}{\pi} d\theta_i dt. \quad (46)
 \end{aligned}$$

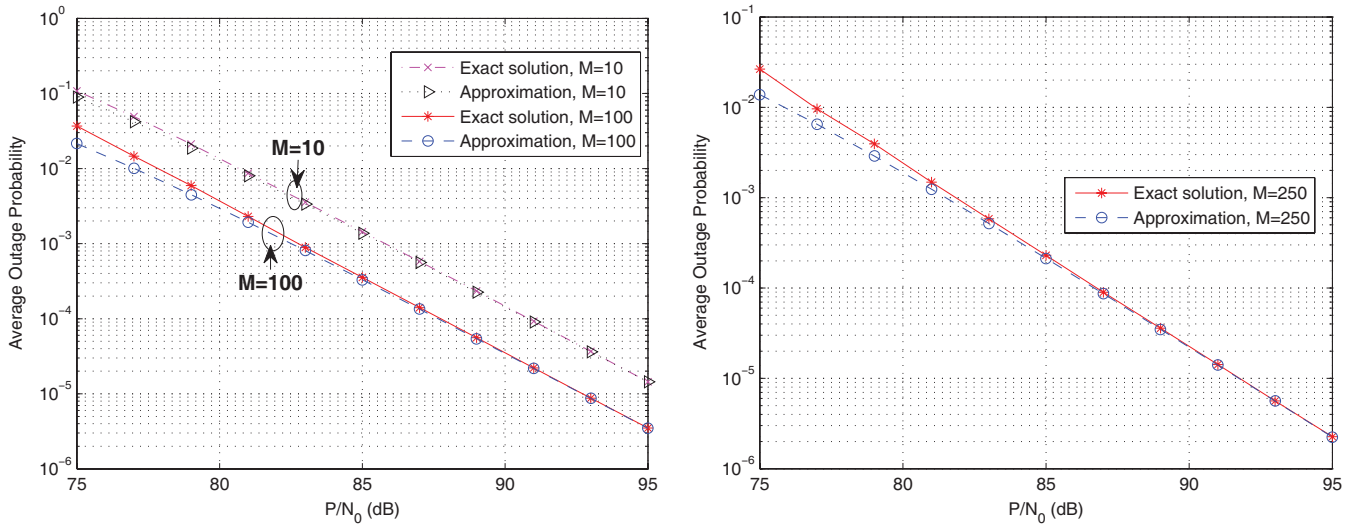


Fig. 1. Comparison of the exact average outage probability \mathcal{P}^d and the approximation $\tilde{\mathcal{P}}^d$ for the distributed cooperative multicast. $R_2 = 100$, $\eta = 2.6$, and $T_R = 4$. $P_{s,d} = P$.

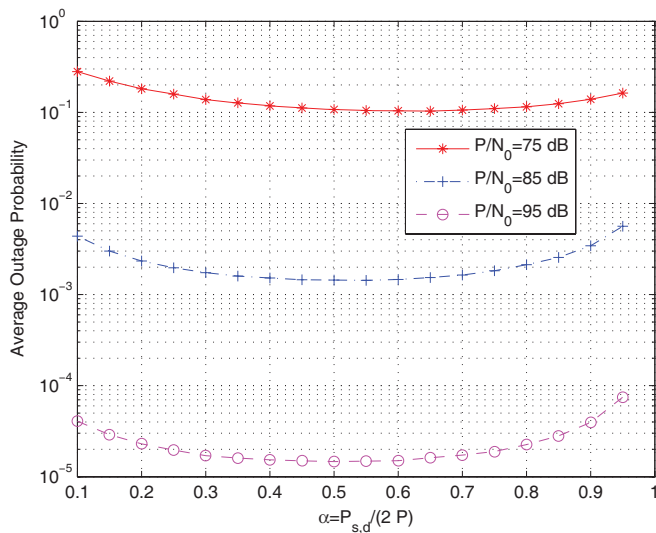


Fig. 2. Optimal power allocation in the distributed cooperative multicast. $R_2 = 100$, $\eta = 2.6$, $T_R = 4$, and $M = 10$. $P/N_0 = 75\text{dB}$, 85dB , 95dB . The x axis is $\alpha \triangleq P_{s,d}/(2P)$.

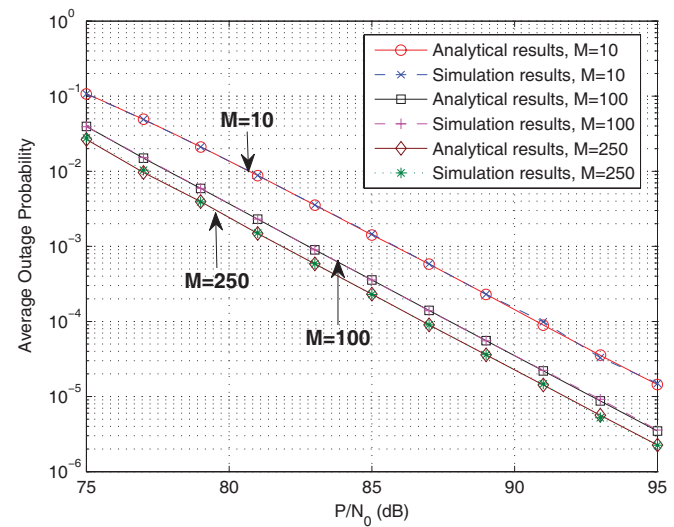


Fig. 3. Simulation results of the distributed cooperative multicast schemes. $R_2 = 100$, $\eta = 2.6$, $T_R = 4$, and $P_{s,d} = P$.

in \mathcal{P}^d for $1 \leq N \leq M-1$ and D_{M-1} in $\tilde{\mathcal{P}}^d$. From Fig. 1, we can see that $\tilde{\mathcal{P}}^d$ matches \mathcal{P}^d very well, especially when P/N_0 is large. We observe similar trend for other values of the parameters. Figure 2 plots the exact average outage probability \mathcal{P}^d versus different power allocation strategies $\alpha \triangleq P_{s,d}/(2P)$

in the distributed cooperative multicast scheme. In Fig. 2, we use $M = 10$ with $P/N_0 = 75\text{dB}$, 85dB , and 95dB as an example, and we observe the same trend for other values of the parameters. From Fig. 2, setting α to be around 0.5 can help minimize the average outage probability, especially with high SNR.

Next, we use simulation results to validate our analysis

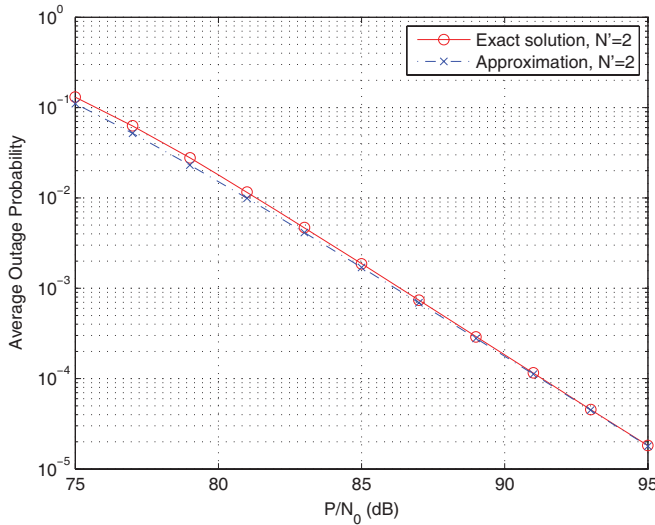
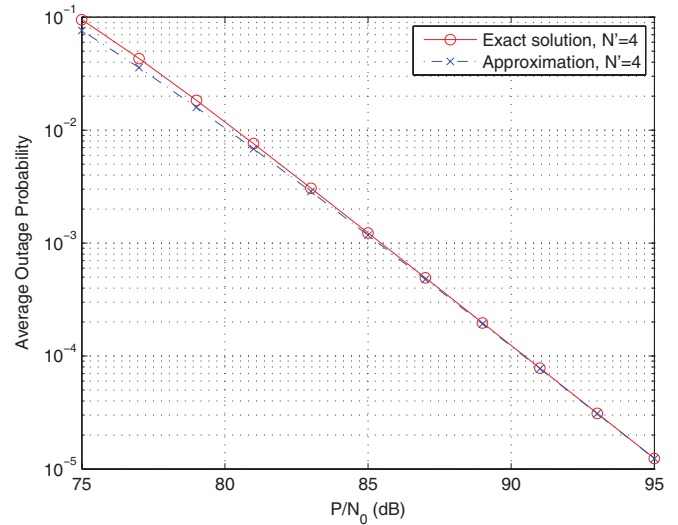
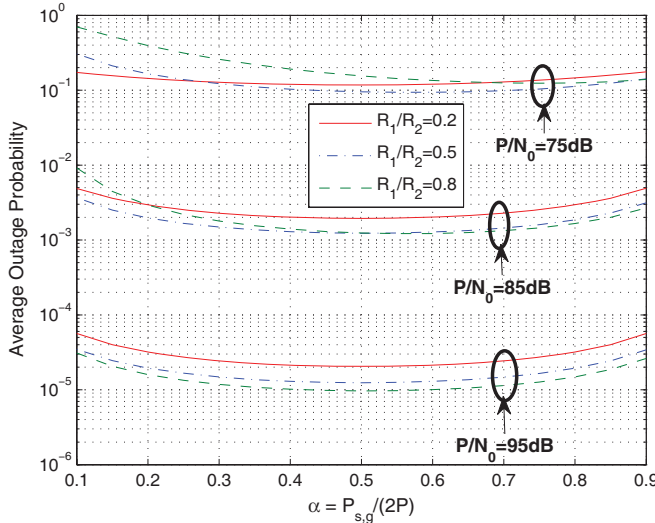
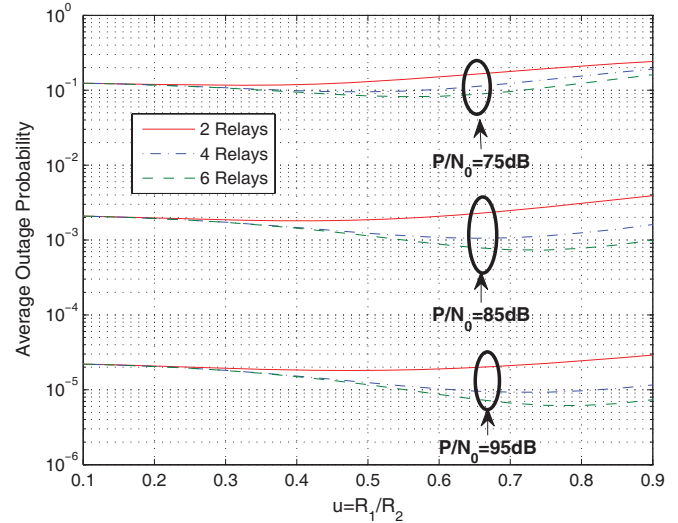
(a): $N' = 2$ relays(b): $N' = 4$ relays

Fig. 4. Comparison of the exact and the approximated average outage probabilities for the genie-aided cooperative multicast scheme. $R_2 = 100$, $\eta = 2.6$, and $T_R = 4$. $P_{s,g} = P$, and $R_1 = R_2/2 = 50$.



(a)



(b)

Fig. 5. (a): Optimal power allocation in genie-aided cooperative multicast with $N' = 4$. The x axis is $\alpha \triangleq P_{s,g}/(2P)$. (b): Asymptotic optimal relay positions for the genie-aided cooperative multicast scheme with $P_{s,g} = P$ and $N' = 2, 4, 6$. In both (a) and (b), $R_2 = 100$, $\eta = 2.6$, $T_R = 4$, and $P/N_0 = 75\text{dB}, 85\text{dB}, 95\text{dB}$.

of the distributed cooperative scheme's outage probability. In our simulations, we randomly generate M user locations uniformly distributed inside a circle with radius $R_2 = 100$. All channel gains are generated independently following the complex Gaussian distribution $\mathcal{CN}(0, 1)$. We let $P_{s,d} = P$ and allocate half of the total transmission power to the BS/AP. Figure 3 compares the simulation results based on 10^7 simulation runs with the analytical results in Section III. It can be seen that the simulation results match our analytical results very well. In addition, from Fig. 3, distributed cooperative multicast achieves a smaller outage probability when the total number of users M is larger and when more users help relay the message in stage 2. For example, there is a 3dB gain if M is increased from 10 to 100, and another 1dB gain if M

is further increased to 250. Therefore, distributed cooperative multicast performs better in denser networks with more users.

B. Genie-aided Cooperative Multicast

For the genie-aided cooperative multicast scheme, Fig. 4 compares the approximated outage probability $\tilde{\mathcal{P}}^g$ with the exact \mathcal{P}^g , both calculated using Monte Carlo simulations. The system setup is similar to that in Fig. 1. We use $P_{s,g} = P$ and $R_1 = R_2/2$ as an example, and we observe the same trend for other power allocation and relay locations. We consider two scenarios with $N' = 2$ and $N' = 4$ dedicated relays, respectively, and other number of N' gives the same trend. It can be seen that the approximation in (43) is very close to the exact outage probability, especially in the high SNR region.

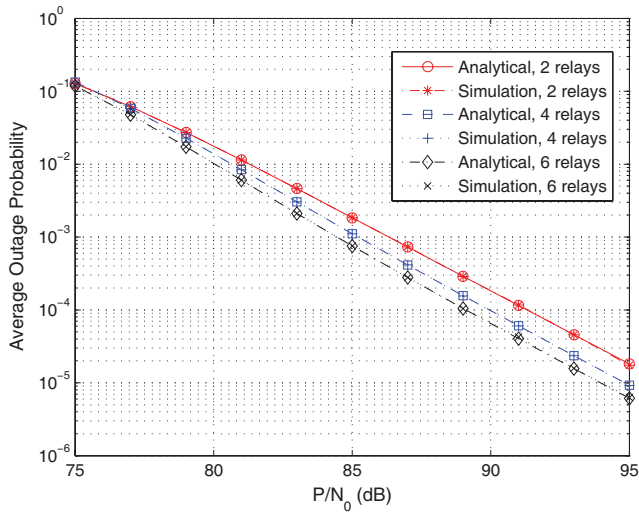


Fig. 6. Simulation results of the genie-aided cooperative multicast schemes. $R_2 = 100$, $\eta = 2.6$, $T_R = 4$, and $P_{s,g} = P$. The radius R_1 is selected according to Table I.

For genie-aided cooperative multicast, Fig. 5(a) plots the exact average outage probability \mathcal{P}^g in (28) for different power allocation strategies $\alpha = P_{s,g}/2P$. The system setup in Fig. 5(a) is similar to that in Fig. 4, and we use $N' = 4$ relays with $R_1 = 0.2R_2$, $R_1 = 0.5R_2$, and $R_1 = 0.8R_2$ as an example. We observe a similar trend for other values of the parameters. From Fig. 5(a), allocating half of the transmission power to the BS/AP helps minimize the average outage probability, especially when P/N_0 is large. Figure 5(b) shows the exact average outage probability \mathcal{P}^g for different relay positions and different SNRs. We can see from Fig. 5(b) that in the high SNR region, the selected u^* in Table I helps minimize the outage probability. Note that we focus on the high SNR region in this work, and our analysis of optimal power allocation and relay position is based on our approximated outage probability $\tilde{\mathcal{P}}^g$, which matches the exact value \mathcal{P}^g very well only in the high SNR region (e.g., when $P/N_0 \geq 85$ dB). Therefore, the optimal power allocation and relay positions in the low SNR region (e.g., when $P/N_0 = 75$ dB) may be different from our analysis in Section IV-B; while our simulation results are consistent with our analysis in the high SNR region (e.g., when $P/N_0 = 85$ dB and $P/N_0 = 95$ dB).

We then use simulation results to validate our analysis on the outage probability of the genie-aided cooperative multicast scheme. Figure 6 compares the simulation results based on 10^7 simulation runs with the analytical results in Section III. The simulation setup is the same as in Fig. 3, and we follow Table I to set the relay location R_1 . The simulation results match our analytical results very well, and the genie-aided cooperative multicast scheme helps reduce the outage probability by a large amount when the number of dedicated relays increases, especially in the high SNR region. In Fig. 6, we observe a maximum of 2 dB gain when N' is increased from 2 to 6.

C. Performance Comparison

We then compare the outage probability of different multicast schemes with different SNRs, as shown in Fig. 7. The system setup in Fig. 7 is the same as that in Fig. 3

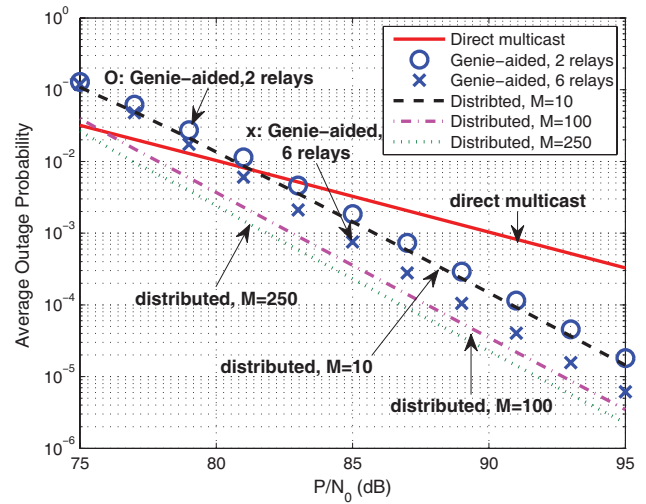


Fig. 7. Performance comparison of different multicast schemes. $R_2 = 100$, $\eta = 2.6$, $T_R = 4$, and $P_{s,d} = P_{s,g} = P$. In the genie-aided cooperative multicast, the relay positions are selected based on the results in Table I.

and 6. Comparing the performance of distributed cooperative multicast with that of direct multicast, we observe that in the high SNR region, user cooperation can significantly help reduce the outage probability. For example, with $P/N_0 = 95$ dB, distributed cooperative multicast can help reduce the average outage probability from $O(10^{-4})$ to $O(10^{-6})$ when there are hundreds of users in the network. We also observe that the simulation curves of different cooperative multicast schemes go in parallel in high-SNR regions and they show a higher diversity order than the direct multicast scheme. This is consistent with the theoretical analysis in (12), (35) and (43), which show that the direct multicast scheme provides only diversity order 1 and the cooperative multicast schemes offer diversity order 2 in the outage probability performance. Furthermore, we observe that cooperation does not always give the best performance. With the system setup as in Fig. 7, for the distributed cooperation strategy, when there are ten users in the network, direct multicast is beneficial when the SNR P/N_0 is below 81 dB; and with $M = 100$ users, user cooperation gives a smaller outage probability when SNR is larger or equal to 76 dB. Similarly, the genie-aided cooperation scheme reduces the outage probability only when P/N_0 is larger than 81 dB.

When comparing the two cooperative multicast schemes, for a sparse network with fewer users, when the number of dedicated relays in genie-aided cooperative multicast N' is small, the two cooperative multicast schemes have similar performance. For denser networks with large number of users, the distributed cooperative multicast scheme achieves a smaller outage probability and has a 1 dB to 4 dB gain compared with genie-aided cooperative multicast. Note that the genie-aided cooperative multicast scheme relies on the existence of dedicated relays that we can put in any locations. Therefore, distributed cooperative multicast is often preferred since it gives a better performance without the help of dedicated relays. Another advantage of the distributed cooperation scheme is that it is easy to implement and does not introduce extra communication overhead for control messages.

VI. CONCLUSIONS

In this paper, we investigated two MRC-based cooperative multicast schemes over wireless networks, where a group of users receive the same data from a BS/AP and they may cooperate with each other to combat channel fading and to achieve more reliable QoS. We analyzed the outage probability performance of the two cooperative multicast schemes and optimized their power allocation.

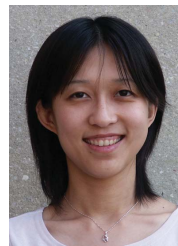
We first analyzed the outage probabilities of the distributed and the genie-aided cooperative multicast schemes. We derived closed-form formulations for the outage probabilities, and provided approximations to show the asymptotic performance of the cooperative multicast schemes. Based on the asymptotically tight outage probability approximations, we obtained the optimal cooperation strategies. It turns out that allocating half of the total transmission power to the BS/AP minimizes the outage probability of cooperative multicast schemes, and the other half is evenly distributed among relays in the statistical sense. We also determined the optimal relay locations for genie-aided cooperative multicast.

We then compared the performance of different multicast schemes. For the distributed cooperative scheme, we observe a smaller outage probability for denser networks with more users and a larger average number of relays. Similarly, the outage probability of the genie-aided cooperation scheme decreases as the number of dedicated relays increases. Compared to the direct multicast scheme, the cooperative multicast schemes achieve diversity order 2, and user cooperation can help significantly improve the performance especially when the signal-to-noise ratio is high. Compared with the genie-aided scheme, the distributed cooperation scheme gives a 1dB to 4dB performance gain and, therefore, it is often preferred to maximize the performance without the help of dedicated relays and without extra overhead for control signals.

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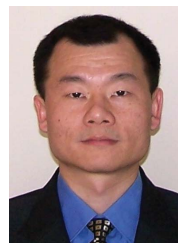
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