On Orthogonal Space-Time Block Codes and Transceiver Signal Linearization

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***Abstract*— We present a necessary and sufficient condition for any orthogonal space-time block code (STBC) to allow transceiver signal linearization (as in the well-known case of the Alamouti scheme). Then, we show that all square orthogonal STBC's that satisfy the condition have rate that goes to zero linearly with the number of transmit antennas. Hence, multiple-antenna systems with orthogonal STBC's and satisfactory rate are possible only if we abandon the linearization property or utilize non-square codes (except for the 2×2 Alamouti code).**

***Index Terms*— Diversity, MIMO systems, multiple antenna, orthogonal designs, space-time block codes (STBC).**

I. Introduction

ORTHOGONAL space-time block codes (STBC's) have received considerable attention in recent open-loop multiple-input-multiple-output (MIMO) wireless communication literature (for example [1]-[11] and references therein) because they allow low complexity maximum-likelihood decoding and guarantee full diversity. An orthogonal STBC is characterized by a code matrix $\mathbf{G}_{p\times n}$ where p denotes time delay or block length and n represents the number of transmit antennas. The entries of $\mathbf{G}$ are linear combinations of k data symbols or their conjugate, $s_1, s_2, \cdots, s_k, s_1^*, s_2^*, \cdots, s_k^*$, that belong to an arbitrary signal constellation. The columns of $\mathbf{G}$ are orthogonal to each other and

$$\mathbf{G}^{\mathcal{H}}\mathbf{G} = (|s_1|^2 + |s_2|^2 + \cdots + |s_k|^2)\mathbf{I}_n \qquad (1)$$

where $\mathbf{A}^{\mathcal{H}}$ denotes the complex conjugate transpose of matrix $\mathbf{A}$, and $\mathbf{I}_n$ is the size-n identity matrix. The code rate of $\mathbf{G}$ is defined as $R = k/p$ (i.e., each codeword with block length p carries k information symbols).

To motivate the developments in this manuscript, consider as an example a communication system with two transmit and one receive antennas that utilizes the Alamouti orthogonal STBC [1]. If y_1 and y_2 denote the received signals at time slot 1 and time slot 2, respectively, then the received signal vector $[y_1 \ \ y_2]^{\mathcal{T}}$ can be expressed as follows:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \mathbf{G}_2(s_1, s_2) \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \qquad (2)$$

where $\mathbf{G}_2(s_1, s_2) \triangleq \begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix}$ is the Alamouti orthogonal STBC, h_1 and h_2 denote the channel coefficients from the two transmit antennas to the receive antenna, and n_1, n_2 represent additive complex Gaussian noise pertinent to time slot 1 and 2, respectively. Due to the special structure of $\mathbf{G}_2(s_1, s_2)$, the received signal in (2) can be rewritten as [7], [9], [10]

$$\begin{bmatrix} y_1 \\ y_2^* \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2^* \end{bmatrix}. \qquad (3)$$

It is interesting to note that in (3) the Alamouti STBC structure is embedded in the channel matrix $\begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}$, while the two data symbols appear as the elements of a 2×1 data input vector and the received signal at time slot 2, y_2, appears conjugated. The *linearized* received signal expression in (3) is appealing as it is backward compatible with existing signal processing techniques and standards [8] and allows, for example, the design of low complexity interference suppressing filters [9], [10] and channel equalizers [11].

A question, then, that arises naturally is whether a similar linearized transceiver signal model (as the one in (3) for the Alamouti scheme) exists for other or all orthogonal STBC's. In this letter, we address exactly this question. In particular, we state and prove a necessary and sufficient condition for an orthogonal STBC to admit a linearized transceiver signal model of the type shown in (3). We then prove that the rate of *square* orthogonal STBC's $\mathbf{G}_n$ that satisfy the above condition goes to zero linearly in the size parameter n. As a result, to obtain high-rate orthogonal STBC's that exhibit the desired linearized signal property, we have to consider non-square orthogonal STBC's, for example the ones proposed in [2], [6].

II. Orthogonal STBC's with Linearized Signal Description

In this section, we first show a necessary and sufficient condition for orthogonal STBC's to have a linearized transceiver signal model. Then, we examine the maximum possible rate of square orthogonal STBC's under which they can have this desired property.

A. Necessary and Sufficient Condition

Without loss of generality, we consider a communication system with n transmit and *one* receive antennas coded with an orthogonal STBC $\mathbf{G}_{p\times n}$ of rate k/n. Then, the received signal vector is given as follows:

$$\begin{bmatrix} y_1 \\ \vdots \\ y_p \end{bmatrix} = \mathbf{G}_{p\times n}(s_1, \cdots, s_k) \begin{bmatrix} h_1 \\ \vdots \\ h_n \end{bmatrix} + \begin{bmatrix} n_1 \\ \vdots \\ n_p \end{bmatrix} \qquad (4)$$

where y_i is the received signal at time slot i, $i = 1, 2, \cdots, p$, h_j is the channel coefficient from the jth, $j = 1, 2, \cdots, n$,

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transmit antenna to the receive antenna, and n_i represents additive Gaussian noise pertinent to time slot i, $i = 1, 2, \cdots, p$. A linearized description of the transceiver signal in (4), if it exists, will have the form

$$\begin{bmatrix} \tilde{y}_1 \\ \vdots \\ \tilde{y}_p \end{bmatrix} = \mathbf{H} \begin{bmatrix} s_1 \\ \vdots \\ s_k \end{bmatrix} + \begin{bmatrix} \tilde{n}_1 \\ \vdots \\ \tilde{n}_p \end{bmatrix} \quad (5)$$

where $\tilde{y}_i$ is either y_i or $y_i^*, i = 1, 2, \cdots, p$, $\mathbf{H}$ is a $p \times k$ matrix whose entries are complex linear combinations of h_j and $h_j^*, j = 1, 2, \cdots, n$, and $\tilde{n}_i$ represents the equivalent noise effect pertinent to time slot i, $i = 1, 2, \cdots, p$.

For a given set of data symbols $s_1, s_2, \cdots, s_k$, we observe that if every row of the STBC code matrix $\mathbf{G}_{p\times n}$ contains data symbols that are either all conjugated or all non-conjugated, then the transceiver signal model in (4) has an equivalent linearized form as in (5). In fact, if the elements of the ith, $i = 1, 2, \cdots, p$, row of $\mathbf{G}_{p\times n}$ are all non-conjugated, then the received signal y_i can be written as $y_i = \sum_{\nu=1}^{k}\sum_{j=1}^{n} \alpha_{\nu,j}(i) h_j s_\nu + n_i$ where $\alpha_{\nu,j}(i)$ are complex coefficients. Then, the ith row of $\mathbf{H}$ can be expressed as $[\sum_{j=1}^{n} \alpha_{1,j}(i)h_j, \cdots, \sum_{j=1}^{n} \alpha_{k,j}(i)h_j]$ and the corresponding elements of the equivalent received signal and noise are $\tilde{y}_i = y_i$ and $\tilde{n}_i = n_i$. If the elements of the ith row of $\mathbf{G}_{p\times n}$ are all conjugated, then the corresponding received signal y_i can be written as $y_i = \sum_{\nu=1}^{k}\sum_{j=1}^{n} \alpha_{\nu,j}(i) h_j s_\nu^* + n_i$. The ith row of $\mathbf{H}$ can now be expressed as $[\sum_{j=1}^{n} \alpha_{1,j}^*(i)h_j^*, \cdots, \sum_{j=1}^{n} \alpha_{k,j}^*(i)h_j^*]$, while the elements of the equivalent received signal and noise are $\tilde{y}_i = y_i^*$ and $\tilde{n}_i = n_i^*$. On the other hand, if one row, say the ith row, of a STBC contains at least one conjugated data symbol and at least one data symbol that is non-conjugated, then the corresponding received signal y_i has the form

$$y_i = \sum_{\nu=1}^{k}\sum_{j=1}^{n} \alpha_{\nu,j}(i) h_j s_\nu + \sum_{\nu=1}^{k}\sum_{j=1}^{n} \beta_{\nu,j}(i) h_j s_\nu^* + n_i \quad (6)$$

where $\alpha_{\nu,j}(i)$ and $\beta_{\nu,j}(i)$ are complex coefficients and at least one of $\alpha_{\nu,j}(i)$ and one of $\beta_{\nu,j}(i)$ are nonzero. By (6), $\tilde{y}_i$ involves both s_ν and s_ν^* regardless of whether $\tilde{y}_i = y_i$ or $\tilde{y}_i = y_i^*$ which makes the model in (5) invalid. Therefore, reformulation of (4) to (5) is possible *if and only if* every row of the STBC has all its elements either conjugated or non-conjugated. An example of an orthogonal STBC that, unfortunately, does not allow a linearized transceiver signal model is the well-known 4×4 design [3]–[5], [7]

$$\mathbf{G}_4(s_1, s_2, s_3) = \begin{bmatrix} s_1 & s_2 & s_3 & 0 \\ -s_2^* & s_1^* & 0 & s_3 \\ -s_3^* & 0 & s_1^* & -s_2 \\ 0 & -s_3^* & s_2^* & s_1 \end{bmatrix}. \quad (7)$$

B. Square Orthogonal STBC's

For convenience, we will use in our presentation the following terminology. For a given set of data symbols $s_1, s_2, \cdots, s_k$, we will call *conjugate* a *row* of an orthogonal STBC whose data symbols are all conjugated. Similarly, the elements of a *non-conjugate row* are all non-conjugated. In the following, we consider the class of all square orthogonal STBC's $\mathbf{G}_n$ of size $n \times n\,(p = n)$ that satisfy the necessary and sufficient condition for the linearized transceiver signal model in (5). We will show that the maximum possible rate for such codes goes to zero linearly with the size parameter n. Specifically, we will prove that the maximum rate of $\mathbf{G}_n$ is $2/n$ if n is even and $1/n$ if n is odd.

For any square orthogonal STBC $\mathbf{G}_n$, (1) implies that $\mathbf{G}_n$ is invertible if s_ν, $\nu = 1, 2, \cdots, k$, are all non-zero, and moreover $\mathbf{G}_n^{\mathcal{H}} = \sum_{\nu=1}^{k} |s_\nu|^2 \mathbf{G}_n^{-1}$. Thus, $\mathbf{G}_n\mathbf{G}_n^{\mathcal{H}} = \sum_{\nu=1}^{k} |s_\nu|^2 \mathbf{G}_n\mathbf{G}_n^{-1} = \sum_{\nu=1}^{k} |s_\nu|^2 \mathbf{I}_n$, i.e. the rows of $\mathbf{G}_n$ are also orthogonal to each other. Let's rewrite $\mathbf{G}_n$ as

$$\mathbf{G}_n = \begin{bmatrix} \mathbf{sE}_1 + \mathbf{s}^*\mathbf{F}_1 \\ \mathbf{sE}_2 + \mathbf{s}^*\mathbf{F}_2 \\ \cdots \\ \mathbf{sE}_n + \mathbf{s}^*\mathbf{F}_n \end{bmatrix}_{n\times n} \quad (8)$$

where $\mathbf{E}_i$ and $\mathbf{F}_i$ are $k \times n$ complex matrices, $\mathbf{s} \triangleq [s_1\, s_2\, \cdots\, s_k]$, and $\mathbf{s}^* \triangleq [s_1^*\, s_2^*\, \cdots\, s_k^*]$. Since the rows of $\mathbf{G}_n$ are orthogonal to each other, we have

$$(\mathbf{sE}_i + \mathbf{s}^*\mathbf{F}_i)(\mathbf{sE}_j + \mathbf{s}^*\mathbf{F}_j)^{\mathcal{H}} = 0, \quad 1 \le i \ne j \le n, \quad (9)$$

$$(\mathbf{sE}_i + \mathbf{s}^*\mathbf{F}_i)(\mathbf{sE}_i + \mathbf{s}^*\mathbf{F}_i)^{\mathcal{H}} = \sum_{\nu=1}^{k} |s_\nu|^2, \quad 1 \le i \le n. \quad (10)$$

Case 1 : $\mathbf{n = 2n_0,\ n_0 > 1}$

Since each row of $\mathbf{G}_n$ is either conjugate or non-conjugate, there are at least n_0 conjugate rows or at least n_0 non-conjugate rows.

(i) Assume first that there are at least n_0 rows of $\mathbf{G}_n$ that are non-conjugate. Since row permutation does not change the orthogonality of $\mathbf{G}_n$, without loss of generality say that the first n_0 rows of $\mathbf{G}_n$ are non-conjugate, i.e. $\mathbf{F}_i = \mathbf{0}_{k\times n}$, $1 \le i \le n_0$. Then, (9) and (10) imply that $\mathbf{sE}_i\mathbf{E}_j^{\mathcal{H}}\mathbf{s}^{\mathcal{H}} = 0$ for $1 \le i \ne j \le n_0$ and $\mathbf{sE}_i\mathbf{E}_i^{\mathcal{H}}\mathbf{s}^{\mathcal{H}} = \sum_{\nu=1}^{k} |s_\nu|^2$ for $1 \le i \le n_0$. Since $\mathbf{s} = [s_1\, s_2\, \cdots\, s_k]$ is supposed to be an arbitrary vector of k complex symbols, we have

$$\mathbf{E}_i\mathbf{E}_j^{\mathcal{H}} = \mathbf{0}_{k\times k}, \quad 1 \le i \ne j \le n_0, \quad (11)$$

$$\mathbf{E}_i\mathbf{E}_i^{\mathcal{H}} = \mathbf{I}_k, \quad 1 \le i \le n_0. \quad (12)$$

If we define $\mathcal{E}^{\mathcal{H}} \triangleq \left[\mathbf{E}_1^{\mathcal{H}}\, \mathbf{E}_2^{\mathcal{H}} \cdots \mathbf{E}_{n_0-1}^{\mathcal{H}}\right]$, then by (11) and (12) we have

$$\mathcal{E}\mathcal{E}^{\mathcal{H}} = \begin{bmatrix} \mathbf{E}_1\mathbf{E}_1^{\mathcal{H}} & \cdots & \mathbf{E}_1\mathbf{E}_{n_0-1}^{\mathcal{H}} \\ \cdots & \cdots & \cdots \\ \mathbf{E}_{n_0-1}\mathbf{E}_1^{\mathcal{H}} & \cdots & \mathbf{E}_{n_0-1}\mathbf{E}_{n_0-1}^{\mathcal{H}} \end{bmatrix} = \mathbf{I}_{(n_0-1)k}$$

which implies that

$$rank(\mathcal{E}) = (n_0 - 1)k. \quad (13)$$

Moreover, since $\mathcal{E}\mathbf{E}_{n_0}^{\mathcal{H}} = \begin{bmatrix} \mathbf{E}_1\mathbf{E}_{n_0}^{\mathcal{H}} \\ \vdots \\ \mathbf{E}_{n_0-1}\mathbf{E}_{n_0}^{\mathcal{H}} \end{bmatrix} = \mathbf{0}_{(n_0-1)k\times k}$, we have

$$rank(\mathcal{E}\mathbf{E}_{n_0}^{\mathcal{H}}) = 0. \quad (14)$$

On the other hand, the rank of the product of the two matrices $\mathcal{E}$ and $\mathbf{E}_{n_0}^{\mathcal{H}}$ satisfies [12]:

$$rank(\mathcal{E}) + rank(\mathbf{E}_{n_0}^{\mathcal{H}}) - n \le rank(\mathcal{E}\mathbf{E}_{n_0}^{\mathcal{H}}). \quad (15)$$

Substituting the exact rank values of (13) and (14) into (15), we are able to obtain the relationship $(n_0-1)k+k-n \le 0$ or, equivalently, $n \ge n_0 k$. Therefore, in this case, the rate of $\mathbf{G}_n$ is upper bounded by

$$R = \frac{k}{n} \le \frac{k}{n_0 k} = \frac{2}{n}. \quad (16)$$

(ii) Assume next that $\mathbf{G}_n$ has at least n_0 conjugate rows. Without loss of generality, say that the first n_0 rows of $\mathbf{G}$ are conjugate, i.e. $\mathbf{E}_i = \mathbf{0}_{k\times n}$ for $1 \le i \le n_0$. Arguing as in (i) above with $\mathbf{F}_i$ in place of $\mathbf{E}_i$, we can reach the same conclusion as in (16).

Case 2 : $\mathbf{n = 2n_0 + 1,\ n_0 \ge 1}$

If n is odd, then at least n_0+1 rows of $\mathbf{G}_n$ are non-conjugate or at least n_0+1 rows are conjugate.

(i) Without loss of generality, assume that the first n_0+1 rows of $\mathbf{G}_n$ are non-conjugate, i.e. $\mathbf{F}_i = \mathbf{0}_{k\times n}$, $1 \le i \le n_0+1$. Then, we have

$$\mathbf{E}_i\mathbf{E}_j^{\mathcal{H}} = \mathbf{0}_{k\times k}, \quad 1 \le i \ne j \le n_0+1, \quad (17)$$

$$\mathbf{E}_i\mathbf{E}_i^{\mathcal{H}} = I_k, \quad 1 \le i \le n_0+1. \quad (18)$$

If we define $\mathcal{E}^{\mathcal{H}} \triangleq \left[\mathbf{E}_1^{\mathcal{H}}\ \mathbf{E}_2^{\mathcal{H}} \cdots \mathbf{E}_{n_0}^{\mathcal{H}}\right]$, we can show that $rank(\mathcal{E}) = n_0 k$ and $rank(\mathcal{E}\mathbf{E}_{n_0+1}^{\mathcal{H}}) = 0$. Since $rank(\mathcal{E}) + rank(\mathbf{E}_{n_0+1}^{\mathcal{H}}) - n \le rank(\mathcal{E}\mathbf{E}_{n_0+1}^{\mathcal{H}})$, we have $n_0k+k-n \le 0$, i.e. $n \ge (n_0+1)k$. But $n = 2n_0+1$ which implies $k < 2$. Therefore, the rate of $\mathbf{G}_n$ is

$$R = \frac{k}{n} \le \frac{1}{n}. \quad (19)$$

(ii) Consider now the case where at least n_0+1 rows of $\mathbf{G}_n$ are conjugate and, without loss of generality, assume that the first n_0+1 rows of $\mathbf{G}_n$ are conjugate, i.e. $\mathbf{E}_i = \mathbf{0}_{k\times n}$, $1 \le i \le n_0+1$. If we replace $\mathbf{E}_i$ by $\mathbf{F}_i$ in (i) above, we reach (19) again.

We have just proved the following theorem.

Theorem 1: *For any square orthogonal STBC $\mathbf{G}_n$, if each row of $\mathbf{G}_n$ is either conjugate or non-conjugate, then the maximum possible rate of $\mathbf{G}_n$ is $2/n$ if n is even and $1/n$ if n is odd.* □

We note that, if $n = 2n_0$ (even), we can reach the rate upper bound $2/n$ in a straightforward manner. We can create a block diagonal version of the code $\mathbf{G}_2(s_1, s_2)$ by $\mathbf{G}_n = I_{n_0} \otimes \mathbf{G}_2(s_1, s_2)$ where $\otimes$ denotes tensor product. On the other hand, if $n = 2n_0+1$ (odd), we may consider $\mathbf{G}_n = s_1\mathbf{I}_n$ as an example a code with rate $1/n$.

As a concluding remark, we note that Theorem 1 implies that to obtain high-rate orthogonal STBC's for more than two transmit antennas that admit the desired linearized signal model as in (5), one has to resort to non-square codes. The non-square orthogonal STBC's proposed in [2] do satisfy the necessary and sufficient condition for linearized signal discription and maintain code rate $1/2$ for any number of transmit antennas. Recently, in [6], a systematic design method for non-square orthogonal STBC's was presented with rate $(n_0+1)/(2n_0)$ if the number of transmit antennas is $n = 2n_0$ or $n = 2n_0 - 1$, which has been conjectured to be optimal. For example, for $n = 4$ transmit antennas, $\mathbf{G}_{8\times 4}$ below [6]

$$\mathbf{G}_{8\times 4} = \begin{bmatrix} s_1 & s_2 & s_3 & 0 \\ -s_2^* & s_1^* & 0 & s_4^* \\ -s_3^* & 0 & s_1^* & s_5^* \\ 0 & -s_3^* & s_2^* & s_6^* \\ 0 & -s_4 & -s_5 & s_1 \\ s_4 & 0 & -s_6 & s_2 \\ s_5 & s_6 & 0 & s_3 \\ -s_6^* & s_5^* & -s_4^* & 0 \end{bmatrix} \quad (20)$$

is a non-square orthogonal STBC of rate $3/4$ where each row of the code is either conjugate or non-conjugate. In contrast to the well-known code $\mathbf{G}_4$ in (7) of the same rate, $\mathbf{G}_{8\times 4}$ in (20) allows linearized signal description.

III. Conclusion and Discussion

We considered the class of orthogonal STBC's (that offer full diversity with low-complexity maximum-likelihood decoding) and presented a necessary and sufficient condition under which an orthogonal STBC enables linearized transceiver signal description. We then proved that square, in particular, orthogonal STBC's for n transmit antennas that satisfy the linearization condition have rate upper-bounded by $2/n$ and $1/n$ for n even and odd, respectively. Hence, regretfully, large $n\ (n = 3, 4, \cdots)$ high-rate orthogonal STBC's that admit transceiver signal linerization are possible only within the class of non-square orthogonal designs. Fortunately, such non-square orthogonal designs already exist in the literature: the fixed $1/2$ rate codes in [2] and the higher rate (conjectured optimal) codes in [6].

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