EVALUATION OF SYSTEM-RELIABILITY METHODS FOR CABLE-STAYED BRIDGE DESIGN

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ABSTRACT: Probabilistic system-reliability methods have the potential to assist designers in better understanding the ultimate global behavior of bridges, thus leading to more economical, rational, and reliable structures. Nonetheless, these techniques are not currently broadly used in North American bridge engineering practice. Using a sample cable-stayed bridge design, a study is conducted to assess the practicability of system-reliability analytical methods to assist in the design of cable-stayed bridges. Ductile and brittle cables are considered in series and mixed system analyses, respectively. The sample bridge selected is found to be very reliable; the most likely failure mode identified is somewhat counterintuitive. More importantly, considerable insight into global ultimate behavior is provided by these analyses, and the effect of various design assumptions on global structural safety can be assessed. Some obstacles to the transfer of system-reliability procedures to the state of practice are identified.

INTRODUCTION

The theory of structural system reliability is well established; its fundamental analytical methods are comprehensively presented by many authors (Ang and Tang 1984; Thoft-Christensen and Murotsu 1986; Melchers 1987; Harr 1987). These powerful procedures allow the evaluation of the safety of a total structural system as opposed to that of its constituent components, although that global assessment invariably depends on the local ones.

Structural reliability methods, considering applied loads and resistance of individual structural elements as random variables, have been used extensively for the development of limit states design (LSD) bridge codes, including the *Ontario Highway Bridge Design Code* (1983) and the CSA standard S6-M88 ("Design of" 1988). This implicitly reflects that design is normally performed at the component level.

There may be instances, however, where structural system-reliability may assist the designer in better understanding the ultimate global behavior of a structure, which could lead to a more rational and economical final design. Such benefits would be particularly expected for complex and highly redundant structural systems. Nonetheless, these structural system-reliability analytical techniques are apparently not broadly used in North American bridge engineering practice. It is conjectured that the absence of those well-established analytical procedures from the state of practice is attributable to a deficient knowledge of system reliability theory by the designers' community, a poor data base of the needed pertinent information, and the lack of incentives provided by traditional structural engineering approaches and the associated competitiveness pressures.

Methods to rationally quantify structural redundancy in bridges, in a system-reliability perspective, have been presented by some researchers

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(Tharmabala and Nowak 1987; Frangopol and Nakib 1990). Some mathematical models focusing on the study of bridge reliability have also been proposed (Tharmabala and Nowak 1987), and used in combination with results from load tests to improve system-reliability assessments of an existing steel truss bridge (Nowak and Tharmabala 1988). Such studies remain few at this time.

Cable-stayed bridges are perceived to be relatively safe under gravity loads, as commented by Tang (1984) based on deterministic observations. Nonetheless, system-reliability studies are essential to probabilistically quantify this confidence. Such advanced analytical methods, which can lead to more economical, rational, and reliable structures, should be of great interest to designers. Since it is already recognized and anticipated that the engineering efforts required for the design of cable-stayed bridges exceed those required for more standard bridge configurations, other technical barriers are currently preventing the integration of these methodologies into state of practice. These must be identified.

The objective of this study is to assess the practicability of system-reliability analytical methods to assist in the design of cable-stayed bridges. In an attempt to meet this objective, a sample cable-stayed bridge is designed by the writer, and system-reliability methods are applied to the resulting design. The usefulness and practicability of the exercise is assessed. However, the conduct of a complete sensitivity analysis is beyond the scope of the reported work.

It is noteworthy that this research simultaneously provides some rough preliminary estimates of the structural safety index, β, germane to cable-stayed bridge design practice in North America, but the emphasis of this limited investigation does not actually lie in an accurate determination of this index.

Description of Sample Cable-Stayed Bridge Studied

The bridge geometry (Fig. 1) is selected to be representative of a medium-span cable-stayed bridge for which a single tower construction at midspan is assumed possible. The total length of the bridge is 180 m. An increasing number of short- and moderate-span cable-stayed bridges with only a few cables are being constructed in North America.

For simplicity, the deck is selected to be a multispine steel box girder of constant cross section, for which the plastic moment is attainable and large inelastic deformations possible without undesirable local or global instability effects. The central tower is chosen to actually consist of two towers of uniform cross section, joined by a horizontal cross-beam immediately located below the roadway, to form an H-shape in cross-elevation; full plasticity can also be developed for this simple tower configuration.

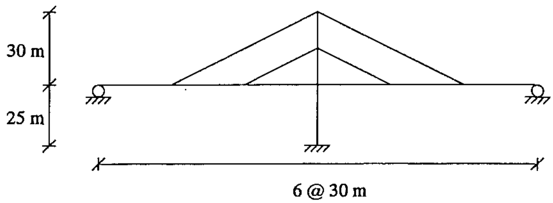

FIG. 1. Geometry of Sample Cable-Stayed Bridge Studied

Other particulars of that deterministic design are as follows.

- The bridge is designed in compliance to the *CAN3-S6-M78* ("Design of" 1978) Canadian standard, a working stress design earlier edition of the current CSA standard.
- The deck width is selected to accommodate two lanes of traffic, both physically and as per the codified design model.
- Symmetry along the centerline of the roadway permits consideration of only half of the total structure. All loads and properties herein are specified accordingly.
- Uniform dead load is 45 kN/m.
- Uniform live load is 26 kN/m. This is representative of the Canadian Standard Association (CSA) specified lane loading for a MS-250 truck considering most unfavorable lateral placement of lane loading, impact considerations, reduction for multiple lane loadings, and conversion, for simplicity, of the point-load that *CSA-S6-M78* specifies must be simultaneously applied with the lane load, into an additional contribution to the uniformly distributed load (UDL).
- The lane load model governs over the truck model for the current design.
- Only the design of the major structural elements is performed in this exercise.
- G40.21-M 350W steel is selected for the deck and tower with a resulting allowable design stress of 210 MPa in both tension and compression. In the final design, the sum of bending and axial stresses divided by this allowable is 0.71 for the towers and 1.0 for the beams. More iterations to further optimize the tower design were not conducted. Reductions in the provided tower strength, and concurrent increases in tower flexibility, would also lead to increased girder strength demand under unbalanced live loads, with little overall benefits.
- High-strength steel cables are selected with 2,250 MPa ultimate stress capacity. The allowable stresses are determined using a factor of safety of 3. Cables are prestressed to eliminate deflections due to dead loads at the cable/deck attachment point. Wind loads, ice accretion, or other loads on the cables are neglected in this design. For analysis purposes, the cables are modeled as tension-only truss elements.
- The deck is connected at the tower such that it is restrained against longitudinal movements, but free to rotate.
- P-Δ and nonlinear geometric effects for this design are very small; thus, they are neglected in both the original design and subsequent reliability analyses.
- Shear lag effects are not significant for the selected multispine double box-girder cross section of the deck, and have been neglected.
- No composite action is assumed to exist between the concrete deck and steel box girders.
- No efforts are made to optimize the design beyond the efficient design of individual structural elements.

A commercial linear-elastic structural analysis program (Wilson and Habibullah 1989) is used to assist in the design of the structure. Some properties of the resulting half-structure are presented in Table 1. A detailed pres-

TABLE 1. Numerical Values of Random Variables

Item (1)	Deterministic value (2)	Distribution type (3)	μ (mean) (4)	σ standard deviation (5)	COV $= \mu/\sigma$ (6)
F_Y tower and deck	350 MPa	Normal	367.5 MPa	36.75 MPa	0.10
Fu cables ($\approx F_{Y\ \mathrm{nominal}}$)	2,250 MPa	Normal	2,363 MPa	236.3 MPa	0.10
Cable area (A)	6,500 mm^2	Normal	6,500 mm^2	10,000 mm^2	0.05
Deck area (A)	200,000 mm^2	Normal	200,000 mm^2	10,000 mm^2	0.05
Tower area (A)	250,000 mm^2	Normal	250,000 mm^2	12,500 mm^2	0.05
Plastic modulus deck (Z)	0.0682 m^3	Normal	0.0682 m^3	0.00341 m^3	0.05
Plastic modulus tower (Z)	0.219 m^3	Normal	0.219 m^3	0.0109 m^3	0.05
Dead load ω_D	45 kN/m	Normal	47.25 kN/m	4.73 kN/m	0.10
Live load ω_L	26 kN/m	Normal	32.7 kN/m	4.27 kN/m	0.13

entation of additional parameters considered or resulting from the static design is beyond the scope of this paper.

Reliability Analysis

Random Variables

Sectional properties (A,Z), material yield stress (F_Y), and applied, uniformly distributed dead and live loads (ω_D and ω_L, respectively) are chosen as the random variables of interest for this study. Plastic capacities are direct functions of those variables.

The probability distributions and statistical moments of common structural engineering random variables presented by Ellingwood (1983) and Ellingwood et al. (1980) are deemed appropriate for this study; nonetheless, bridge live loads are not covered by those documents and require a separate assessment. The results of studies by Agarwal and Wolkowicz (1976), Buckland and Sexsmith (1981), Foster et al. (1981), and Harman and Davenport (1979) are considered to define the live-load random parameters.

In the survey of individual truck weights, conducted by the Ontario Ministry of Transportation and Communication (MTC), and reported by Agarwal and Wolkowicz (1976), a superposition of three normal curves was found to best fit the curve of collected data (Foster et al. 1981). The mean and standard deviation from the normal curve fitting the zone of heaviest trucks are 480 kN and 62.7 kN, respectively.

For the spans under consideration herein, a uniformly distributed live load governs design. The uniform lane load model selected for this reliability analysis is that suggested by Harman and Davenport (1979). It has been derived recognizing that live load, which must simulate accurately the effects of multiple trucks on a bridge, depends mainly on the random weight of each truck, the random sequence of truck along spans, the aspect of influence lines on which it is acting, the random distance between truck queuing, and may be dependant of traffic-jam loading.

According to this model, the corresponding magnitude of the mean maximum 50-year uniform lane live load, q, is obtained by spreading the weight of the mean heaviest truck load over a length of 18.3 m, and multiplying it by a factor, K, which accounts for the effect of rare occurrence of multiple truck presence in a given lane on long spans. The value of K depends on the span; for a length of 90 m, it is 0.6. Thus, for the case at hand

$$q = K \times \left(\frac{480\ \text{kN}}{18.3\ \text{m}}\right) = 0.6 \times (26.2\ \text{kN/m}) = 15.7\ \text{kN/m} \quad \ldots\ldots\ldots (1)$$

over the length and for each lane of the bridge. The reader is referred to the aforementioned papers if additional probabilistic information on this topic is sought.

Finally, q is multiplied by the same factors previously used during the design phase to take into account the most unfavorable lateral placement of lane loading, impact considerations, and reduction for multiple lane loading, to obtain the resulting mean 50-year maximum value of the uniformly distributed live-load value, $\omega_L = 32.7$ kN/m. This load is applied over whatever length necessary to produce the most critical effect. Using the coefficient of variation (COV) measured in MTC's survey, the corresponding standard deviation is 4.27 kN/m.

Table 1 summarizes the resulting values calculated for the selected random variables. Not surprisingly, mean values do not necessarily equal the specified or calculated deterministic values. Also, for simplicity in this study, uncertainties associated with the structural analysis and with the design equations for resistance have been neglected.

It is noteworthy that traffic live-load characteristics and model definitions used in many countries, states, or provinces vary substantially. In a cable-stayed bridge, the bending moments in the deck and towers under permanent loads are held to a minimum by cable adjustments, and consequently, live load has a more considerable effect in this kind of structure. Therefore, special care must be taken before extrapolating the findings of this study in an international perspective.

Ductile Cable Systems—Series System Analysis

When all structural components, including the cables, are ductile, structural system failure will occur by formation of a collapse plastic mechanism. The performance functions can be directly obtained from the virtual work expressions describing the failure plastic mechanisms. The identification of these mechanisms, along with plastic-hinge locations and corresponding plastic capacities, is involved, but once this is accomplished, the determination of the structural system reliability simply consists of identifying, in a probabilistic sense, the weakest mechanism, much like series system analysis.

Effects of Axial Loads

Both deck and tower behave as beam-column components. The significance of axial force on plastic capacities must therefore be assessed. From simple plastic analysis theory and assuming that the neutral axis at ultimate stays within the webs of an hollow rectangular section, the well-known axial bending interaction curve is defined as:

$$\frac{M}{M_P} = 1 - \left(\frac{P}{P_P}\right)^2 \frac{A^2}{4wZ_x} \quad \ldots\ldots\ldots (2)$$

where M = the applied moment; M_P = the plastic moment capacity in the absence of axial loads; P = the applied axial force; P_P = the plastic axial force capacity in the absence of applied moment; A = the cross-sectional area; w = the sum of the web thicknesses; and Z_X = the plastic modulus around the axis of bending.

For the bridge's deck, reductions in plastic moments due to axial force are neglected in the formulation of performance functions since bending moment effects are found to be largely dominant. Deterministic evaluations of the maximum reductions of M_p at ultimate are found to be at worst 5% in this case.

For the tower, except for the formulation of one pure axial force performance function (compression failure) under maximum balanced loads, bending is a dominant factor in spite of the large axial forces, and simpler plastic capacity expressions ignoring the bending/axial interaction curve are again found to be acceptable.

Probabilistic Failure Plastic Analysis

The first-order second-moment (FOSM) method (in which two measures, the mean value and the standard deviation of probability density functions, are considered) was used for this study (*Special Publication* 1981; Ellingwood et al. 1980; Nowak and Lind 1979). This method uses a linear approximation of the performance functions at the most likely failure point. Reliability is measured by a safety index, β, which can be translated into notional probabilities of failure, p_f, according to the relationship:

$$p_f = \Phi(-\beta) \qquad (3)$$

where $\Phi(\cdot)$ = the standard normal cumulative probability, available in most mathematical handbooks. Alternatively, the following closed-form equation (MacGregor 1976) can be used to relate β and P_f for very small probabilities of failure:

$$p_f = 460\, e^{-4.3\beta} \qquad (4)$$

Conceptually, a performance function, Z, is expressed as the difference of the structural resistance, R, and effect of the applied loads, S, random variables. Failure occurs if $Z = R - S < 0$. The safety index is related to the mean and the standard deviation of the performance function, by the following ratio:

$$\beta = \frac{\mu_z}{\sigma_z} \qquad (5)$$

Actual failure functions for structural components or systems must include the effect of various loads and resistances, the latter dependant on different material strengths and geometry, each with its own probability curve. A generalized formulation of the limit state failure equation, or performance function, can be written as:

$$Z(x'_1, x'_2, \ldots, x'_n) = 0 \qquad (6)$$

where x'_i = a set of uncorrelated reduced variates

$$x'_i = \left(\frac{x_i - \mu_{xi}}{\sigma_{xi}}\right) \quad i = 1, 2, \ldots, n \qquad (7)$$

where x'_i = the basic random variables; and μ_{x_i} and σ_{x_i} = their respective means and standard deviations.

The performance function can be visualized as a multidimensional surface in an n-dimension space, where β is the minimum distance from the origin to that failure surface. The solution of the structural reliability problem essentially consists in finding the coordinates $x_i'^*$ of that point on that surface, for which

$$x_i'^* = -\alpha_i^* \beta \quad i = 1, 2, \ldots, n \qquad (8)$$

and where the direction cosines, α_i^*, perpendicular to the failure surface, are:

$$\alpha_i^* = \frac{\left(\dfrac{\partial Z}{\partial x_i'}\right)_*}{\sqrt{\sum_i \left(\dfrac{\partial Z}{\partial x_i'}\right)_*^2}} \qquad (9)$$

As the pertinent point of tangency on the failure surface is initially unknown, a numerical iteration strategy must be implemented to achieve convergence for any nonlinear performance function and probability distribution. An existing computer program, developed to perform these structural-reliability operations, is used in this study (Liu et al. 1989).

Fourteen different plastic collapse mechanisms are considered in the formulation of performance functions. These functions are tabulated in Table 2, and corresponding failure mechanisms are illustrated in Fig. 2. The program considers all these performance functions, as well as the numerical values and type of probability distributions of the related random variables, and calculates their individual first-order probabilities and safety indices.

Four different sets of numerical values for the key random variables are considered. In each case, mean and standard deviations of all variables listed in Table 1 have been used, unless indicated otherwise for some specific

TABLE 2. Performance Functions Considered for Series System Analysis

$Z_i(X) = C_1 M_{P\ \mathrm{Deck}} + C_2 M_{P\ \mathrm{Tower}} + C_3 \omega_L + C_4 \omega_D + C_5 T_{P\ \mathrm{Cable}} + C_6 P_{P\ \mathrm{Tower}}$

i (1)	C_1 (2)	C_2 (3)	C_3 (4)	C_4 (5)	C_5 (6)	C_6 (7)
1	1.0	—	−56.25	−56.25	—	—
2	1.0	—	−75.0	−75.0	—	—
3	3.0	—	−900.0	−900.0	13.42	—
4	4.0	—	−900.0	−900.0	13.42	—
5	3.0	—	−2,025.0	−2,025.0	26.82	—
6	3.0	0.928	−900.0	−900.0	12.44	—
7	4.5	0.928	−900.0	+450.0	—	—
8	2.0	0.464	−1,350.0	−1,350.0	18.66	—
9	3.0	0.464	−1,350.0	—	—	—
10	4.0	2.782	−900.0	−900.0	12.44	—
11	4.5	4.172	−900.0	+450.0	—	—
12	6.0	0.695	−1,350.0	—	34.22	—
13	3.0	0.253	−1,350.0	—	9.05	—
14	—	—	−154.0	−154.0	—	1.0

Note: $M_{P\ \mathrm{Deck}} = Z_{\mathrm{Deck}} F_{Y\ \mathrm{Deck}}$; $M_{P\ \mathrm{Tower}} = Z_{\mathrm{Tower}} F_{Y\ \mathrm{Tower}}$; $T_{P\ \mathrm{Cable}} = A_{\mathrm{Cable}} F_{U\ \mathrm{Cable}}$; $P_{P\ \mathrm{Tower}} = A_{\mathrm{Tower}} F_{Y\ \mathrm{Tower}}$; compatible units are kN and meter.

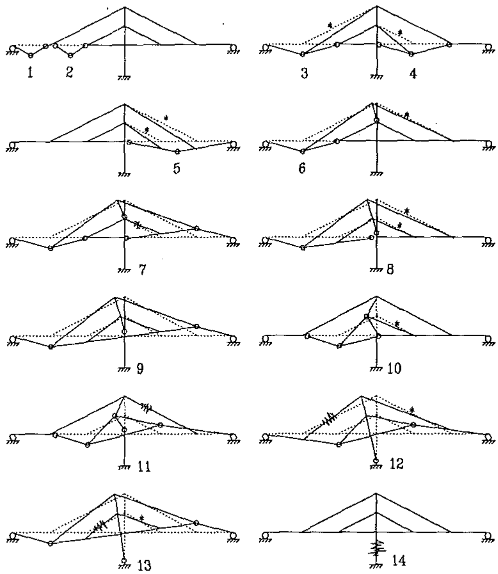

FIG. 2. Plastic Collapse Mechanisms Considered in Formulation of Performance Functions—Live Load Only Applied to Portion of Span Producing Positive Work when Undergoing Plastic Mechanism Action (*Indicates Cable Yielding; Broken Line Symbolizes Plastic Compression Failure of Tower for Case 14, or Cables in Compression, i.e., Not Contributing, for All Other Cases)

TABLE 3. Values of Random Variables Modified in Cases 1–4 of This System-Reliability Study; μ and σ are Mean and Standard Deviations Symbols

Item (1)	Case 1 (2)	Case 2 (3)	Case 3 (actual design) (4)	Case 4 (5)
Dead load μ (kN/m)	45.0	45.0	47.25	47.25
Dead load σ (kN/m)	0.0	0.0	4.73	4.73
Live load μ (kN/m)	26.0	26.0	32.7	32.7
Live load σ (kN/m)	0.0	0.0	4.27	4.27
Cable area (m^2)	0.0065	0.0044	0.0065	0.0044

variables in Table 3. In the first two sets (cases 1 and 2), the mean dead loads and live loads are equal to the specified deterministic design values, and the respective standard deviations null. This is used to provide benchmarks where the variability of the loads has been eliminated, the measure of the safety index being only dependant on the random variables related to material properties and geometry. In the last two sets (cases 3 and 4), the probabilistic system-reliability evaluations proceed using the actual values of the live- and dead-load random variables derived in a previous section.

While cases 1 and 3 provide probabilistic assessments of the original design and the relative effects of loads variability, cases 2 and 4 are included to roughly assess the potential consequences on this bridge's system reliability of a reduction in the cable safety factor, from 3 to 2, as modeled by a 33%

reduction in the area of all cables. It is acknowledged that this reduction of safety factor would be of large consequence on the safety index, β, of the cable element alone, but its influence on system reliability remains to be investigated. Nonetheless, the reduction in area is implemented only parametrically in the reliability analysis; the original structural configuration is unchanged.

Analysis of Results

Results for the 14 previously described performance functions are presented in Table 4. Failure modes are ranked in order of increasing safety index, i.e., from the highest probabilities of failure to the lowest. A large number of observations on those findings is following.

- In all cases, the first failure mode does not include any cable failure; it involves overall rocking of the superstructure by plastic hinging in the deck and in the tower just above the deck, in spite of the presumed conservative tower design. This mode of structural system failure is somewhat counterintuitive, and could have been overlooked if not for this system-reliability analysis. In fact, increases in cable strength would not improve the system reliability of that bridge.
- Variation of the safety factor of cables has a greater impact on the remaining noncritical failure modes and their relative ranking. Failure modes involving cable failure(s) are identified in Table 4.
- Although changes in the ranking of the various plastic collapse mechanisms are modest when comparing the relative effects of loads variability (e.g., cases 1 versus case 3), this could be of profound significance if a capacity design philosophy is sought by strengthening portions of the structure to either delay or prevent some undesirable failure modes. While consequences of strengthening the tower, or other parts of the

TABLE 4. Safety Indices for All Structural System Plastic Collapse Mechanisms

Case 1		Case 2		Case 3		Case 4	
Failure mode (1)	Safety index (2)	Failure mode (3)	Safety index (4)	Failure mode (5)	Safety index (6)	Failure mode (7)	Safety index (8)
9	6.83	9	6.83	9	5.32	9	5.32
2	7.83	5[a]	7.02	5[a]	7.32	5[a]	5.70
5[a]	8.13	2	7.82	2	7.40	8[a]	7.27
1	8.38	1	8.38	1	8.08	2	7.40
14	8.81	8[a]	8.57	8[a]	8.46	1	8.08
8[a]	9.24	14	8.81	14	8.61	3[a]	8.19
7	9.83	3[a]	9.36	3[a]	9.14	14	8.61
3[a]	9.84	7	9.83	7	9.31	4[a]	9.01
11	9.93	11	9.93	11	9.77	7	9.31
4[a]	10.49	4[a]	10.09	4[a]	9.82	13	9.74
6[a]	11.37	6[a]	10.72	13	10.61	11	9.77
13	11.79	10[a]	11.00	6[a]	10.72	6[a]	9.84
10[a]	11.93	13	11.43	10[a]	11.57	10[a]	10.62
12[a]	12.27	12[a]	12.74	12[a]	12.06	12[a]	12.42

[a]Plastic collapse mechanism involving cable failure(s).

bridge, on the global safety can be inferred from the results tabulated in Table 4, a redesign under any changed conditions would be mandatory to validate such deductions. However, the superiority of the probabilistic evaluation, over the deterministic one, lies largely in the capacity to calculate credible safety indices.

- In spite of the limited redundancy of this structure as compared with that of large cable-stayed bridges, safety indices for the structure as a whole are considerably larger than that of individual structural components.
- The critical β is 5.32. A conservative original design may be partly accountable for this high level of safety. Most other plastic-collapse mechanisms, with safety indices above 6, are highly unlikely to occur. It is noteworthy that the interaction of bending and axial loads is partly instrumental to the high structural system reliability calculated. While the performance functions are developed from plastic collapse mechanisms in which the reduction in plastic moments due to the presence of axial force is negligible, the bridge is designed using working stress philosophy in which the axial effect is weighted with the same importance as the bending effect; the difference between the axial-bending elastic and inelastic interaction curves is considerable. In addition, the previously mentioned conservative tower design also contributes to the high overall reliability.
- The substantial insight into the ultimate behavior of the structure provided by these analyses can be used advantageously to prevent undesirable failure modes. A capacity design philosophy could be implemented such that a local noncollapse failure mechanism would develop at a higher probability of failure than the first structural system-failure mode, thus warning of structural distress locally without endangering the structure globally. Stringers or other local deck components could be designed to a lower safety index to serve for that purpose. In other words, by providing an upper bound on the strength of local structural components (relatively to that of other key structural elements), easily repairable local failure modes could be ensured to be more probable than the first global structural failure mode whose occurrence would be of dramatic consequences.
- The determination of the performance functions for this simple structure is a lengthy exercise. The lack of computerized assistance in this task may preclude the applicability of system-reliability to more complex multi-cable/multi-tower bridges.
- Finally, the large safety factors traditionally used in the design of individual cables are also intended to provide protection against a number of factors which are difficult to quantify by system-reliability studies at this time. Corrosion at the cable anchorage is one of these problems, especially for bridges that remain un-maintained for years (the author has inspected structures where severe corrosion affected more than a third of the original cable effective area). Fatigue, loss of prestress due to anchorage slippage, stress relaxation, and dynamic effects are some of the other problems anticipated by designers, leading to generous allocation of safety factors in cable design. It is noteworthy that there is no standard practice in this regard, and that some designers will

instead affix safety factors of different magnitudes to various aspect of behavior to protect against unsatisfactory performance.

Brittle Cables—Mixed System Analysis

Cables used in bridges often cannot be considered ductile, contrary to the assumption adopted in a previous section. In some cases, the nonlinear inelastic behavior, which may occur due to compaction of the strand, will be eliminated by prestretching operations performed by the manufacturer to linearize the behavior up to a very high resistance. In spite of this, while the stress-strain diagram of cable steel does show a genuine plastic plateau, the elongation at rupture is considerably smaller than for mild steel (Gimsing 1983). In fact, the plastic strains of the cable steel may not be sufficient to allow the attainment of the plastic collapse mechanisms previously considered. Thus, the study of the bridge as a mixed system, where cables are brittle but the rest of the structure is ductile, is necessary.

Analysis

For this mixed systems, an event tree approach is adopted. This methodology is well described by Ang and Tang (1984), Thoft-Christensen and Murotsu (1986), and others. Points of possible component failures for the chosen simple bridge structure are identified in Fig. 3. Points A–E identify regions of potential negative moment plastic hinges in the deck, and F–H that of potential plastic hinges in the tower. Points I–L represent failure of the brittle cables, and M–R localize where positive moment plastic hinging is possible in the deck.

Some engineering assumptions are necessary as, even for this simple structure, calculation of probabilities of failure in all possible branches of the event tree represent an unreasonable computational effort. A considerable amount of judgement is used to limit the cases studied to a smaller representative subset; this may be more difficult on a more complex structure, and computer programs providing assistance for this decision making would be most useful. For the case at hand, results from the series analysis provided some guidance to identify dominant failure modes.

For each identified structural element, a component reliability analysis is performed to calculate a probability of failure for this single element. Each cable failure or local plastic hinge modifies the previous load-resistance path, effectively triggering a new load redistribution among the remaining structural elements; in each case, the bridge is reanalyzed using a properly modified model. At each step, the process is repeated for the updated resulting structural behavior, i.e., new performance functions are derived, and new probabilities of failure are calculated for some selected components. For expediency, progression along a given path on the event tree is stopped when probabilities of failure for the subsequent events are expected to be considerably high, even if the complete failure mechanism has not yet been

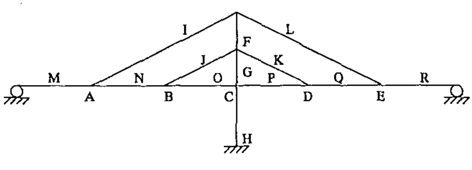

FIG. 3. Points of Possible Component Failures for Mixed System Analysis

attained. Still, the number of manual operations remains large, especially since there is no existing interface currently available to bridge the static analysis and reliability based programs. The prioritary workstation-based structural reliability program PROBAN (1989) has apparently been successfully linked to a structural analysis program to automatically handle these operations in the perspective of a fault-tree analysis for brittle structural systems (Mehta, Private Communication, Veritas Sesam Systems A. S., Det Norske Veritas, Høvik, Norway). However, this feature is not available in the commercial version of PROBAN at the time of this writing, but its inclusion in future releases is apparently being considered.

Analysis of Results

The partial event tree resulting from this analysis is presented in Fig. 4. It is believed that all of the paths significantly contributing to the probability of failure have been identified. The summation of the probability of failures corresponding to each path give the forecasted total system probability of failure. This calculated value is $p_f = 1.53 \times 10^{-6}$. The effect of correlation between modes of failure is not considered. Not surprisingly, due to the brittleness of the cables, the failure probability of the system is raised beyond that obtained with ductile cables, to a resulting safety index of $\beta = 4.54$.

The determination of system reliability for mixed systems is found to require considerably more engineering effort than for comparable series

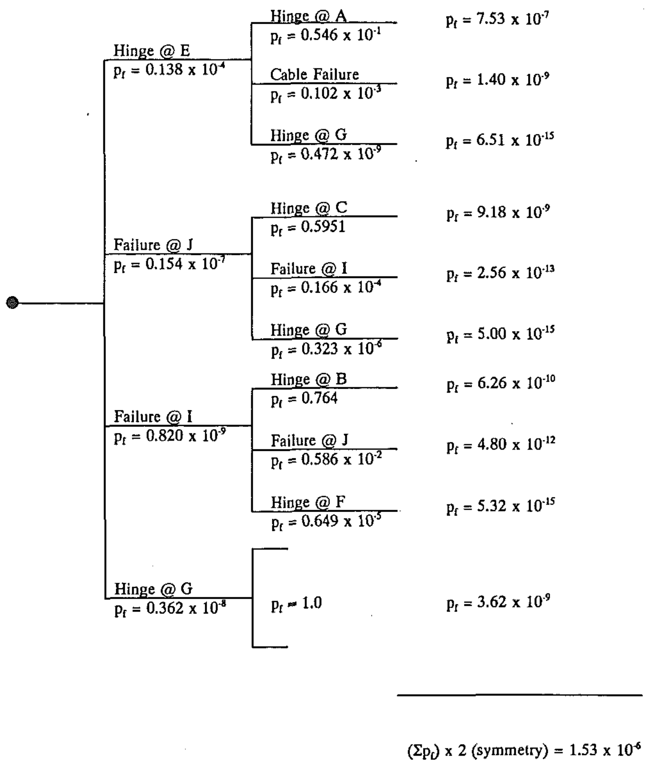

FIG. 4. Partial Event Tree Resulting from Mixed System Analysis of Sample Bridge

systems. As mentioned previously, without a computer-aided environment tailored to address this problem, the incremental complexity inherent to more realistic multicable bridges will hinder the transfer of this technology to the state of practice. An efficient algorithm to address this problem in a systematic manner is not available in the public domain at this time; although the aforementioned strategy could undoubtedly be imitated, it may not be an approach suited to the implementation of a sophisticated integrated production tool. It is speculated that the algorithm used in the aforementioned PROBAN software package will likely not be distributed given its prioritary nature. More research on this topic is required.

Given the laborious effort required for the system-reliability assessment of an already designed structure, it appears unrealistic to expect these analytical methods to routinely assist in the design of new structures at this time. These state-of-the-art analytical methods may nonetheless prove justified for the evaluation of existing structures for which reliability is questioned and which are found to be deficient using traditional deterministic analysis procedures. It could be a rational alternative to the use of reduced load factors proposed elsewhere (Foster et al. 1981).

Conclusions

The application of the system-reliability theory to a simple cable-stayed bridge provides useful insight into the potential failure modes of such structures and reveals how additional strength can be best apportioned to further enhance overall reliability. The sample structure selected has proven to be very reliable.

Nonetheless, the assessment of this system reliability for the simple selected bridge is a rather involved exercise. While an improved understanding of the ultimate global behavior logically leads to more economical, rational, and reliable structures, the absence of an integrated computer program combining both the standard static and reliability analyses significantly deters from conducting such assessment of structural reliability, even more so in a design perspective. Additional research is needed to determine how this integration of complex engineering software is best achieved. The resulting reduction in engineering efforts (and costs) required would hopefully provide the needed incentive to bring these most valuable reliability analysis techniques to the state of practice for the design of such complex structures having numerous potential failure modes.

Further research is also required for integrating the deterioration effect of corrosion, fatigue, and other practical problems, in the formulation of the performance functions, particularly if aiming at the evaluation of existing bridges. Future system-reliability studies incorporating wind and seismic loading, P-Δ effects and geometric nonlinearities of such bridges are also desirable.

Finally, this study confirms that system reliability has the potential to become a useful engineering analytical tool. It not only provides a quantification of global structural safety, but may also be used to avoid needlessly reinforcing structural components who do not contribute to the most likely failure mode, or similarly, to limit the upper-bound strength of local structural components (relatively to that of other key structural elements), to ensure that easily repairable local structural failure modes are more probable than the first more dramatic global failure mode, in a perspective similar to that of capacity design in earthquake engineering.

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